ADVANCED ARITHMETIC

McCLYMONDS AND JONES

CALIFORNIA STATE SERIES

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ADVANCED ARITHMETIC

BY

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PREFACE

THIS text is designed for use in the grammar grades, following the completion of the Elementary Arithmetic of the same series. In the preparation of this text the authors have aimed (*a*) to secure skill in numerical computations and (*b*) to develop the power necessary to the solution of any practical problem that may arise in the common experiences of life.

The following are some of the distinguishing features of this text:

1. The text contains an unusually large number of exercises that are designed to give facility in numerical computations.

2. In the presentation of each topic an effort has been made to stimulate thought and to develop self-reliance on the part of the pupils. Whenever the nature of the work admits, it calls for action on the part of the pupils, as in making measurements, engaging in business relations with others in the class, etc.

3. The scope of the work is restricted to the needs of the majority of persons in the common experiences of life. Traditional materials that make no contribution to the mastery of the essentials of arithmetic have been carefully eliminated. All of the work prescribed in the text proper is easily within the capacity of pupils in the grammar grades. Certain topics that are prescribed in some courses of study but purposely omitted from other courses have

been presented in an Appendix, so that they may be used or omitted, as desired in each case, without destroying the continuity of the other work.

4. The problems of the text have been drawn from the common field of everyday experience. The necessary arithmetical training is had from dealing with practical problems within the experience of the pupils. No unreal problems, or problems dealing with artificial situations, or problems treating of situations remote from the experiences of the average pupil in the grammar grades, are introduced. The text aims to teach arithmetic only.

5. The text contains an unusual amount of oral work, including oral problems under every topic treated. The oral problems are everywhere related to the written work. No additional text in "mental" arithmetic need be used in conjunction with this text.

6. The methods of the text are those commonly employed in business life.

7. The work in fractions and compound numbers is limited to the practical needs of life. Special attention is given in fractions to the use of those fractions which pupils must handle later on as the fractional equivalents of certain per cents. Commission, Taxes, Insurance, etc., are made part of the work in Percentage and are not treated as separate topics. The work in Interest has been considerably reduced, and but one method of finding interest is recommended.

8. A constant review of all previous work is maintained throughout the text.

Finally, the aim of the authors has been to present a course in arithmetic that will secure a thorough knowledge of the essentials of this subject.

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ESSENTIALS OF ARITHMETIC

PART I

REVIEW OF INTEGERS AND DECIMALS

1. The Decimal System.

1. A **unit** is a single thing, or a group of things regarded as a single thing, as a book, an apple, a box of apples, etc. A unit is represented by the least whole number, one (1).

2. Point to several units of the same thing in your schoolroom. Can you think of a way by which you could tell your parents *how many* children there are in your room without using number?

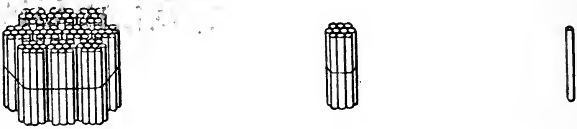
3. Any definite quantity used to measure quantity of the same kind is called a **unit of measure.**

The unit of 6 is 1; of 6 cows is 1 cow; of 9 ft. is 1 ft. The inch, foot, yard, rod, and mile are units used to measure length or distance. Name the units used to measure areas. What is the unit of 10? of $10? In finding the number of hats at $2 each that can be bought for $10, the unit of measure is $2. What is the unit of measure in finding the number of 4-ft. shelves that can be made from a board 12 ft. long?

4. Name the units used to measure liquids; time; weight.

5. In the number 111, the 1 at the right denotes some unit, and the 1 next toward the left denotes a unit ten times as great, and the 1 at the left denotes a unit ten times the second unit, or one hundred times the first unit. This may be shown thus:

one hundreds' unit   one tens' unit   one unit



100   10   1

6. In 236, the 6 represents 6 units; the 3 represents 3 units, each of which is ten times each of the units represented by 6; and the 2 represents 2 units, each of which is ten times each of the units represented by 3, or one hundred times each of the units represented by 6.

7. Tell what each figure represents in 125, 47, 352.

8. In 30, the 0 shows that there are no units of *ones;* and the 3 represents 3 units of *tens.* What does each figure represent in 60, 600, 405, 530, 203, 478, 700, 520?

9. In 324, the units represented by 4 are called units of the *first order*, or of *units' order;* the units represented by 2 are called units of the *second order*, or of *tens' order;* and the units represented by 3 are called units of the *third order*, or of *hundreds' order.*

10. Our number system is a *decimal system. Decimal* means *tens.* A **decimal system** is one in which ten units of one order are equal to one unit of the next higher order.

The decimal system is believed to have had its origin in the practice of using the fingers for counting.

11. Beginning at the left of 111, the 1 in the third order represents some unit; the 1 in the second order represents a unit one tenth as great; and the 1 in the first order represents a unit one tenth as great as that represented by a unit of the second order. A unit one tenth as great as that represented by the 1 in the first order may be represented by 1 written to the right of a **decimal point** (.) placed to the right of units' order, thus: .1 (111.1). A unit one tenth as great as this last unit may be represented by 1 written in the second place to the right of the decimal point, thus: .01 (111.11).

12. .1 is read *one tenth;* .01 is read *one hundredth;* .11 is read *eleven hundredths;* 1.1 is read *one and one tenth;* .4 is read *four tenths.* Read 6.7; 8.05; 56.25.

13. The decimal point is placed after the figure that represents whole units. The figures to the right of the decimal point represent decimal parts of units. The parts thus represented are tenths, hundredths, thousandths, etc.; and are called **decimals.**

14. A whole number is called an **integer.** Write an integer. On which side of the decimal point are integers written?

15. What is the meaning of the word decimal? Why is our number system called a decimal system?

16. What does each 2 in 222.222 represent?

17. Write the following so that units of the same order are below one another: 45.5, 214.25, 347, 4.315, 17.

18. Compare the value of 2 in 24 with the value of 2 in 240; with the value of 2 in .24.

19. Is the system of United States money a decimal system? Explain your answer.

NOTATION AND NUMERATION OF INTEGERS AND DECIMALS

2. **1.** Numbers are commonly expressed by means of figures (or digits) as 5, 10, etc.; by means of words, as *five*, *ten*, etc.; and by means of letters, as V, X, etc. The art of writing numbers by means of symbols is called **notation.**

The word *digit* means finger. Why were the figures called digits?

2. The figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, are called *Arabic numerals*, as they were introduced into Europe by the Arabs, who borrowed most of them from the Hindus. The system of denoting numbers by means of figures is called **Arabic notation.**

3. The figure 0 is called *naught*, *cipher*, or *zero*. It has no value. It is used to fill out places that are not occupied by other figures. Using figures, write six; six tens; six hundreds.

4. The art of reading numbers is called **numeration.**

5. Integers of more than three places are read more easily when the figures are separated by commas into groups of three each, beginning at the right. The groups are called **periods**, and each period is named after the order of the right-hand figure in the group.

6. The names of the first four periods, and the orders in each, are as follows:

ORDERS:	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units
	7	**8**	**9,**	**6**	**0**	**5,**	**8**	**4**	**2,**	**0**	**3**	**1**
PERIODS:		Billions			Millions			Thousands			Units	

3. Reading Integers.

To read an integer of more than three figures, begin at the right of the number and point off periods of three places each. Read the part occupying the left-hand period as though it stood alone, and add the name of the period; then read the part occupying the next period as though it stood alone, and add the name of the period. Continue until units' period is reached; there omit the name of the period.

Read the following:

	a	*b*	*c*	*d*
1.	3,625*	35,205	825,380	7,125,380
2.	8,017	20,007	308,016	6,000,150
3.	9,008	45,500	950,025	8,040,075
4.	6,303	12,012	404,040	5,505,050

5. What is the name of the first period? of the second period? of the third period? of the fourth period?

6. How many periods are there in the numbers in column *a*? *b*? *c*? *d*?

7. How many places do the numbers in column *c* occupy? in column *d*?

8. What is the name of the left-hand period in the numbers in column *a*?

9. Read the left-hand period of the first number in column *d*. Read the middle period of the same number. Read the number.

10. Read the third period of the fourth number in column *d*. Read the second period. Read the number.

11. When a number consists of three periods, how many places must there be in the first period? in the second? How many places may there be in the third period?

* 625 is read *six hundred twenty-five.*

4. Writing Integers.

To write a number in figures, begin with the highest period and write it as though it stood alone, and add a comma; then write the next highest period as though it stood alone, and add a comma; continue until units' period has been written, thus: 5,807,050.

Write in figures:

1. Three thousand, two hundred four.

2. One hundred two thousand, eight hundred ninety.

3. Twelve million, eight hundred seven thousand, eighty-four.

4. Seven hundred two million, sixteen thousand.

5. Write numbers dictated by your teacher.

Read aloud the following statements:

6. The area of Rhode Island is 1,250 sq. mi.; of Massachusetts is 8,315 sq. mi.; of Illinois is 56,650 sq. mi.; of California is 158,360 sq. mi.; of Texas is 265,780 sq. mi.

7. In 1900 the population of Rhode Island was 428,556; of Massachusetts was 2,805,346; of Illinois was 4,821,550; of California was 1,485,053; of Texas was 3,048,710.

8. The total number of votes cast for president in 1900 was 13,964,812. The five states polling the largest number of votes were: New York, 1,548,042; Pennsylvania, 1,173,210; Illinois, 1,131,894; Ohio, 1,040,073, and Missouri, 683,656.

9. The grain production of the United States in 1902 in measured bushels was as follows: Indian corn, 2,523,648,312; wheat, 670,063,008; oats, 987,842,712; barley, 134,954,023; rye, 33,630,592; buckwheat, 14,529,770.

ROMAN NOTATION

5. **1.** The letters used in Roman notation are:

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

2. The above letters are called **Roman numerals.** Other numbers are represented by combinations, thus:

a. Repeating a numeral repeats its value. XXX denotes 30, CCC denotes 300. The numerals V, L, and D are not repeated. Why?

b. If a numeral is followed by another of less value, the sum of their values is denoted. XXVI denotes the sum of 10, 10, 5, and 1.

c. If a numeral is followed by another of greater value, the difference of their values is denoted. XC denotes the difference of 10 and 100, or 90; CD is 500–100, or 400.

d. A bar placed over a numeral increases its value 1000 times. $\overline{\mathrm{V}}$ denotes 5000; $\overline{\mathrm{IX}}$ denotes 9000.

3. Read the following and tell which of the above rules is illustrated in each: LX, XL, CIX, MDCC, $\overline{\mathrm{IV}}$, MDCCCCVI, LXXIV, MMDXL, MDCXX, XLII.

4. Write 1776 in Roman numerals.

Model: 1776 may be divided into the parts 1000 — 700 — 70 — 6. These parts expressed in order, beginning at the left, are M — DCC — LXX — VI. 1776 is written MDCCLXXVI.

5. Write in Roman numerals: 18, 27, 68, 1492, 1907.

6. Write in Arabic figures XCVI, XLVII, XIX, LXXIV, MDCCCXII.

Roman numerals are frequently used to designate chapter numbers in books, the hours on the clock face, dates on monuments and public buildings, etc. The M is used to designate a thousand feet of lumber.

UNITED STATES MONEY

6. **1.** The units of United States money are decimal units. The standard unit of value is the **dollar.** The other units are derived from it. The **dime** is one tenth part of the dollar, and the units that represent dimes are therefore written in the first place to the right of the decimal point. The **cent** is one hundredth part of the dollar, and the units that represent cents are therefore written in the second place to the right of the decimal point.

2. Dimes are written as cents. Two dollars and four dimes is written thus: $2.40. This is read *two dollars and forty cents.*

3. The unit one dollar is written $1. The unit one dime is written $.10. The unit one cent is written $.01. The unit one mill is written $.001.

7. Reading United States Money.

Read the following:

1. $425.15	**5.** $30.755	**9.** $8340.05
2. $301.08	**6.** $ 7.057	**10.** $9015.807
3. $220.20	**7.** $10.105	**11.** $7200.50
4. $100.10	**8.** $ 4.005	**12.** $1306.065

8. Writing United States Money.

Write the following in columns:

1. Six dollars and seventy-five cents.

2. Twenty-five dollars and fifty cents.

3. Eighty-five dollars and six cents.

4. Three hundred forty dollars and eighty cents.

5. One hundred dollars and fifty-two cents.

6. Eight cents. **7.** Thirty-five cents and eight mills.

9. Reading Decimals.

ORDERS:	Hundreds	Tens	Units		Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
	9	1	3	.	4	5	2	8	7	6
	Integers				Decimals					

1. Memorize the number of decimal places required for each of the first six orders.

Tenths (first)	.5
Hundredths (second)	.45
Thousandths (third)	.367
Ten-thousandths (fourth)	.6745
Hundred-thousandths (fifth)	.62789
Millionths (sixth)	.346329

To read a decimal, read the number without reference to the decimal point, and add the name of the order of the right-hand figure.

2. .375 is read *three hundred seventy-five thousandths.* 3.08 is read *three and eight hundredths.* Read: .125, .875, 4.625, 37.075, 670.005, 3.1416, 2150.42, .7854.

10. Writing Decimals.

1. Write sixty-two thousandths. As thousandths is the name of the third order to the right of the decimal point, three figures will be required in writing the number. Two figures are necessary to denote sixty-two; so one cipher must be supplied. To write sixty-two thousandths, first write the decimal point, then write 0, and then write 62 (.062).

2. Write the following: Sixty-nine ten-thousandths; forty-eight hundred-thousandths; thirteen thousandths.

ADDITION OF INTEGERS AND DECIMALS

11. **1.** A number that is not applied to any particular thing, as 6, 43, etc., is called an **abstract number.**

2. A number that is applied to some particular thing, as 6 ft., 43 lb., etc., is called a **concrete number.**

3. Quantities that are expressed in the same unit of measure, as 3 lb. and 6 lb., are called **like quantities.**

4. Quantities that are expressed in different units of measure, as 5 lb. and 4 hr., are called **unlike quantities.**

5. Write two abstract numbers; two concrete numbers; two unlike quantities. Like quantities can be combined and expressed as a single quantity. 3 ft. and 2 ft. may be combined and expressed as 5 ft. Can the unlike quantities 5 lb. and 4 hr. be combined and expressed as a single quantity?

6. Units of the same order may be combined and expressed as single numbers. 3 tens and 2 tens are 5 tens.

7. When two or more numbers are combined and expressed as a single number, this number is called their **sum,** or **amount.**

8. The process of finding the sum of two or more numbers is called **addition.** The numbers that are added are called **addends.**

9. The sign of addition is + and is read *plus.*

10. This sign = is the sign of equality, and when placed between two numbers is read *equals* or *is equal to,* thus: $6 = 4 + 2$ means that 6 is equal to the sum of 4 and 2.

12. Oral Exercises.*

To each number in Exs. 1–4, add in succession 3, 2, 7, 6, 9, 4, 8, 5.

1.	23	35	84	69	26	88	82	57	47	60
2.	39	76	48	87	65	74	33	22	81	30
3.	52	86	49	61	73	95	40	18	67	94
4.	90	66	38	17	41	93	55	74	12	99

13. Add each column as written. Add each column, increasing the number at the bottom of the column by 10, by 20, etc., to 90; thus for column *a*, having increased the number at the bottom of the column by 20: 22, 25, 29, etc.

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*	*n*
9	8	9	5	8	7	9	7	7	7	8	8	9	6
3	7	8	8	7	9	3	3	7	9	9	9	5	8
6	5	9	2	5	7	9	6	8	8	9	5	6	4
4	6	3	7	4	6	5	8	6	6	5	9	4	9
3	8	9	6	8	9	5	2	4	9	1	8	8	8
8	9	7	5	9	6	8	7	8	7	7	7	3	9
7	7	6	4	5	8	5	5	6	8	1	9	4	2
7	7	3	5	7	6	9	9	4	4	8	7	3	7
2	4	5	9	8	8	7	8	5	9	4	6	9	6
3	7	2	3	7	9	6	5	8	8	2	8	5	5
4	3	9	5	6	4	9	7	9	7	6	3	7	7
3	8	9	2	9	6	6	7	8	5	8	2	7	7
2	6	8	4	4	5	7	9	6	2	8	9	7	4

* If the pupils require a more extended drill upon addition than is provided in the above exercises, the method indicated in the elementary text should be followed.

14. Written Exercises.

Numbers to be added or subtracted must be written so that units of the same order are directly below one another, units under units, tens under tens, and tenths under tenths, etc. Why?

When numbers are written so that the decimal points are directly below one another, units of the same order are directly below one another. Explain.

Add:

1.	2.	3.	4.	5.
$ 345.67	$ 58.06	$ 9.045	$405.27	$ 68.
84.075	275.936	590.	73.435	125.87
650.	83.07	5.15	487.50	45.369
70.004	342.457	69.075	50.258	845.075
572.806	34.08	610.75	250.50	8.75
6.605	8.125	57.246	.375	100.
852.451	64.	540.375	62.50	58.268
2581.611	865.728	1881.641	1329.938	1251.332

6. Read aloud each of the above.

7. Write the above from dictation.

8. Add 74.06 mi., 6.8 mi., 320.45 mi., 17.04 mi.

9. Add 64.5 A., 79.14 A., 160.75 A., 321.15 A.

10. Add 60.5 cu. in., 352.24 cu. in., 80.125 cu. in.

11. Add $68.05, $107.98, $730.04, $9.75, $894, $80, $740.40, $375.15, $486.75, $836.95, $.95.

12. Add six and nine hundredths, thirty-seven and six tenths, eighty-five thousandths, seven hundredths.

13. Find the sum of nine hundred eighty and five tenths, seventy and seven hundredths, one hundred and five thousandths, six hundred twenty-five.

14. Write five addition exercises similar to Exs. 1–5 above and add each. Read each answer.

15. Oral Exercises.

Add:

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
1.	40 50	60 20	130 90	120 30	60 140	150 40	90 30	140 60	80 20
2.	70 25	40 54	140 63	50 139	60 42	80 59	70 96	20 192	70 36
3.	29 90	55 80	79 70	56 90	54 80	89 70	46 80	92 90	66 60
4.	23* 89	43 52	64 95	36 94	59 43	54 46	68 43	94 36	39 27

5. Frank weighs 95 lb. and his little brother weighs 34 lb. How much do they both together weigh?

6. A farmer has 46 sheep and his neighbor has 54 sheep. How many have both together?

7. A man paid $94 for a wagon and $36 for a harness. How much did both cost him?

8. Mr. White had 23 head of cattle and bought 39 more. How many had he then?

9. A girl spent 50¢ for cloth and 45¢ for lace. How much did she spend for both?

10. A boy placed 60¢ into his bank one week and 46¢ the next week. How much did he put into the bank in the two weeks?

11. A girl spent 20¢ for stamps, 25¢ for some meat, and 50¢ for sugar. How much did she spend for all?

12. Make and solve ten oral problems in addition.

* **Add: 89, 109, 112.**

SUBTRACTION OF INTEGERS AND DECIMALS

16. 1. Like quantities, such as 5 marbles and 9 marbles, may be compared, and the difference between them found, thus:

9 marbles: • • • • • • • • •

5 marbles: • • • • •

2. If there is added to 5 marbles a quantity that will make it equal to 9 marbles, how much is added? This amount is the difference between the two quantities.

3. If that part of 9 marbles that is equal to 5 marbles is taken from 9 marbles, how many will remain? This remainder is the difference between the two quantities.

4. How does the difference as found in Ex. 3 compare with the difference as found in Ex. 2?

5. The difference between the two quantities may be found by answering either of the following questions:

a. 5 marbles and how many marbles are 9 marbles?

b. 5 marbles from 9 marbles leaves how many marbles?

In either case, the answer is known by recalling that the sum of 5 marbles and 4 marbles is 9 marbles.

6. The **difference** between two numbers is the number which when added to one number makes the other number.

7. The process of finding the difference between two numbers is called **subtraction.**

8. The number to which the difference is added is called the **subtrahend.**

9. The sum of the subtrahend and difference is called the **minuend.**

Or the subtrahend is the number which is subtracted, and the minuend is the number from which the subtrahend is taken.

17. Oral Exercises.

	a	*b*	*c*
1.	6 and — are 11	8 and — are 12	9 and — are 16
2.	9 and — are 14	7 and — are 13	8 and — are 14
3.	8 and — are 11	4 and — are 11	7 and — are 11
4.	7 and — are 12	9 and — are 15	5 and — are 14
5.	5 and — are 13	5 and — are 11	6 and — are 15
6.	4 and — are 12	3 and — are 12	9 and — are 13
7.	8 and — are 15	5 and — are 12	8 and — are 16
8.	7 and — are 16	6 and — are 14	7 and — are 14
9.	9 and — are 17	8 and — are 13	8 and — are 17
10.	6 and — are 12	7 and — are 15	9 and — are 11
11.	9 and — are 12	9 and — are 18	6 and — are 13

12. $\begin{array}{r} 9 \\ -5 \\ \hline \end{array}$ is read 5 *and how many are* 9? Or, 5 *from* 9 *leaves how many?* Use the form with which you are familiar.

13. The sign of subtraction is --, and is called *minus.* It indicates that the number that follows it is to be subtracted from the number that precedes it. $7 - 4$ is read *seven minus four.*

18. Explanation of Subtraction.

1. Find the missing addend.

$$\begin{array}{rl} & \text{(one addend)} \\ 2874 & \text{(one addend)} \\ \hline 5236 & \text{(sum of two addends)} \end{array}$$

The other addend may be found by adding to the given addend the number that will give the sum, thus: 4 and **2** are 6; 7 and **6** are 13; carry 1 to 8, making it 9; 9 and **3** are 12; carry 1 to 2, making it 3; 3 and **2** are 5. Missing addend, 2362.

2. From 5236 subtract 2874.

MODEL *a*:

$$\begin{array}{r} 5236 \\ \underline{2874} \\ 2362 \end{array}$$

Add to the subtrahend the number that will give the minuend, thus: 4 and **2** are 6; 7 and **6** are 13; carry 1 to 8 as in addition, making it 9; 9 and **3** are 12; carry 1 to 2 as in addition, making it 3; 3 and 2 are 5. Write the answer as in the model.

This is known as the **Austrian**, or **additive**, method.

MODEL *b*: Subtract thus: 4 from 6 leaves 2; as 7 tens cannot be taken from 3 tens, 1 hundred is "borrowed" from 2 hundreds and called 10 tens; 10 tens and 3 tens are 13 tens; 7 tens from 13 tens leaves 6 tens; as 1 hundred was borrowed from 2 hundreds, there is left 1 hundred; as 8 hundreds cannot be taken from 1 hundred, 1 thousand is borrowed from 5 thousands and called 10 hundreds; adding 10 hundreds to 1 hundred gives 11 hundreds; 8 hundreds from 11 hundreds leaves 3 hundreds; as 1 thousand was taken from 5 thousands, there are left 4 thousands; 2 thousands from 4 thousands leaves 2 thousands.

MODEL *c*: If the same number is added to both the minuend and the subtrahend, the difference remains unchanged. Subtract thus: 4 from 6 leaves 2; as 7 tens cannot be taken from 3 tens, add 10 tens to 3 tens, making 13 tens; 7 tens from 13 tens leaves 6 tens; as 10 tens were added to the minuend, the same number must be added to the subtrahend, so 1 hundred (10 tens) is added to 8 hundreds, making 9 hundreds; as 9 hundreds cannot be taken from 2 hundreds, 10 hundreds are added to 2 hundreds, making 12 hundreds; 9 hundreds from 12 hundreds leaves 3 hundreds; as 10 hundreds were added to the minuend, the same number must be added to the subtrahend, so 1 thousand (10 hundreds) is added to 2 thousands, making 3 thousands; 3 thousands from 5 thousands leaves 2 thousands.

19. Written Exercises.

Solve:

1. 38,256 – 21,359

2. 40,175 – 19,688

3. 85,430 – 41,856

4. 93,950 – 17,275

5. 97,204 – 57,240

6. 1,106,800 – 289,060

7. 4,083,453 – 613,757

8. 3,256,845 – 465,868

9. 4,741,242 – 572,847

10. 2,814,004 – 935,940

20. Oral Exercises.

Subtract:

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
1.	40	140	150	100	120	150	120	90	110
	20	30	20	40	30	60	90	40	50
2.	95	126	83	142	149	155	153	129	124
	40	90	50	60	80	70	20	40	80
3.	124 *	109	138	96	75	139	136	88	99
	92	44	85	13	24	63	45	16	44
4.	75 †	142	34	57	83	74	42	36	52
	38	96	19	29	68	18	27	19	29

5. Harry bought 120 yd. of string and used 85 yd. for a kite string and gave the rest to George. How many yards did he give to George?

6. A farmer had 52 head of cattle and sold 29. How many had he left?

7. Mary read 87 pages in a book that contained 124 pages. How many more pages must she read to complete the book?

8. There are 38 pupils in Room A and 47 in Room B. How many pupils are there in both rooms? How many more pupils are there in Room B than in Room A?

9. The frontage of a certain city lot is 40 ft. and its depth is 135 ft. Find the difference between the depth and frontage of the lot.

* Suggestion. The difference between 92 and 124 is 30 and 2, or 32.

† Suggestion. The difference between 38 and 75 is 30 (38 to 68) and 7 (68 to 75), or 37; or 40 less 3, or 37.

21. Before solving, represent each by a diagram.

1. Two boys started from the same place. One boy rode east 32 mi. and the other boy rode west 24 mi. How far apart were they then?

From S. to E. is 32 mi. and from S. to W. is 24 mi. From E. to W. is the sum of 32 mi. and 24 mi., or 56 mi.

2. Two boys started from the same place. One rode east 32 mi. and the other rode east 24 mi. How far apart were they then?

3. How far apart are two places, if one is 40 mi. north of the center of a certain city, and the other is 65 mi. south of the center of the same city?

4. Mary lives 8 blocks east of the schoolhouse, and Ethel lives 14 blocks west of the schoolhouse. How far apart do the girls live?

5. Two trains left a certain station at the same time, going in opposite directions. How far apart were they at the end of 2 hours, if one traveled at the average rate of 42 mi. an hour, and the other at the average rate of 36 mi. an hour?

6. How far apart would the trains mentioned in Prob. 5 be at the end of 2 hours, if both traveled in the same direction?

7. In a bicycle race Frank and Henry rode around a park 400 ft. long and 200 ft. wide. When Frank had ridden once around the park, Henry had gained 200 ft. on him. At the same rate of gain, how many times will Frank ride around the park before Henry overtakes him?

22. United States Money.

Write units of the same kind below one another. Do not supply unnecessary 0's.

1. Subtract: *a.* \$12.75 from \$37.25; *b.* \$12 from \$37.25; *c.* \$12.75 from \$37.

Model *a*:	Model *b*:	Model *c*:
\$37.25	\$37.25	\$37.
12.75	12.	12.75
\$24.50	\$25.25	\$24.25

Solve:

2. \$307.57 – \$200.69
3. \$925.07 – \$570.80
4. \$700.40 – \$180.05
5. \$860.455 – \$280
6. 120.375 – 93
7. 690.125 – 209
8. 542 – 45.78
9. 640 – 70.65
10. Read aloud each of the above amounts.
11. Write the above amounts from dictation.

23. Decimals.

Subtract:

1.	2.	3.	4.
320.564	450.125	35.7	600.
206.7	86.75	6.875	57.375

5. A man owned 158.15 acres of land. He sold 79.5 acres. How many acres had he left?

6. If it is 844.7 mi. from San Francisco to Ogden and 1004.7 mi. from Ogden to Omaha, how far is it from San Francisco to Omaha? How much farther is it from Ogden to Omaha than from San Francisco to Ogden?

7. A cubic foot of rain water weighs 62.5 lb. and a cubic foot of petroleum weighs 54.875 lb. How much heavier is a cubic foot of rain water than a cubic foot of petroleum (kerosene)?

24. **1.** Show the effect, if any, upon the difference: (*a*) of adding the same number to both minuend and subtrahend; (*b*) of subtracting the same number from both minuend and subtrahend. Illustrate each with several exercises.

2. Write ten exercises in subtraction of decimals and solve each.

25. Oral Exercises.

1. Name five combinations whose sums are 10.

When these combinations occur in a column, they should be treated as 10. Exercise *a* below may be added: 15, 25, 32, 42, 48, 58, 66, 76. Add the following exercises in a similar manner:

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*
5	7	8	4	1	5	7	4	5	8	7	8	9
5	3	2	6	9	5	2	6	5	7	3	9	8
8	6	9	7	6	4	8	5	6	7	7	1	2
5	3	8	4	1	3	4	7	6	3	8	9	7
5	7	2	6	9	7	6	3	4	8	2	4	6
6	8	7	9	8	8	6	6	7	6	6	6	4
5	7	2	6	9	7	8	2	1	4	5	6	5
5	3	8	4	1	3	2	8	9	8	5	5	9
7	5	5	8	6	9	9	7	7	2	3	2	1
5	3	2	6	9	5	4	1	3	7	9	8	5
5	7	8	4	1	5	6	9	8	9	1	7	9
6	6	7	8	9	9	9	9	9	7	7	8	3
9	8	9	7	8	6	5	2	9	6	8	8	4

2. Write ten columns, in each of which some of the five combinations whose sums are 10 occur several times. Add these columns.

26. Written Exercises.

New England States	Area in Sq. Miles	Great Lakes	Area in Sq. Miles
Maine	33,040	Superior	31,200
New Hampshire . .	9,305	Huron	23,800
Vermont	9,565	Michigan	22,450
Massachusetts . . .	8,315	Erie	9,960
Rhode Island . . .	1,250	Ontario	7,240
Connecticut	4,990		

1. Find the combined area of the New England states; of the Great Lakes.

2. Find the difference between the combined area of the New England states and of the Great Lakes.

3. Compare the area of Vermont with the combined area of Massachusetts and Rhode Island.

4. Find the difference between the area of Lake Ontario and the combined area of Rhode Island and Connecticut.

5. Find the difference between the area of Lake Superior and the combined area of Lakes Huron, Erie, and Ontario.

6. Compare the area of Maine with the area of Lake Superior.

7. Find the difference between the area of Maine and the combined area of the other five New England states.

8. The area of Missouri is 69,415 sq. mi. Compare the area of Missouri with the combined area of the New England states.

27. Written Exercises.

In solving a problem, follow these steps in the order given:

a. Read the problem *carefully*, if convenient, aloud.

b. State what facts are given in the problem and what fact you are asked to find.

c. Determine what relation the given facts have to one another, and state what operation you must use in finding the facts that are asked for, — whether you must add or subtract, etc.

d. Make an estimate of the answer. When you have found the answer, compare it with this estimate.

1. A man bought a house for \$2400 and sold it for \$3000. Find the amount gained.

2. A farmer sold his farm for \$7500, which was \$1800 more than it cost him. How much did he pay for the farm?

3. A farmer bought a farm for \$6250 and sold it at a gain of \$1200. How much did he get for the farm?

4. By selling a farm for \$4500, a farmer received \$900 less than it cost him. How much did it cost him?

5. A room is 24 ft. long and 18 ft. wide. Find how many feet of picture molding it will require for the room.

6. After drawing out \$2300 from a bank, a merchant had \$760 left in the bank. How much had he on deposit in the bank?

7. A merchant had \$1600 on deposit in a bank on Jan. 1, 1907. On Jan. 2 he drew out \$200. On Jan. 5 he deposited \$750. On Jan. 15 he drew out \$2000. How much had he left in the bank?

28. Written Exercises.

1. The total production of corn in the United States in 1899 was 2,666,440,279 bu. In 1889 the total production was 2,122,327,547 bu. How much had the production increased during the decade (10 years)?

2. In 1899 the production of corn in Illinois was 398,149,144 bu. The production in 1889 was 289,697,256 bu. Find the increase in production during the decade.

3. From the amounts given in Probs. 1 and 2, find the total number of bushels produced in all states other than Illinois in 1899.

4. The total production of rice in the United States in 1899 was as follows:

Louisiana	172,732,430 lb.
South Carolina	47,360,128 lb.
Hawaii	33,442,400 lb.
Georgia	11,174,562 lb.
North Carolina	7,892,580 lb.
Texas	7,186,863 lb.
Florida	2,254,492 lb.
Alabama	926,946 lb.
Mississippi	739,222 lb.
Arkansas	8,630 lb.
Virginia	4,374 lb.

a. Read aloud the above quantities.

b. Write the above from dictation.

c. Find the total number of pounds produced.

d. Was the amount produced by Louisiana more or less than that produced by all others combined, and how much?

e. Compare the amount produced in South Carolina with the total amount produced in Hawaii and Georgia.

29. Written Exercises.

1. In 1890 the population of San Francisco was 298,997, and in 1900 it was 342,782. Find the increase in population between 1890 and 1900.

2. In 1898 the population of London was 4,504,766, and in 1900 the population of New York was 3,437,202 and of Chicago was 1,698,575. Find the difference between the population of London and the combined population of New York and Chicago.

3. The area of the earth's surface is about 196,940,000 sq. mi. Of this, 141,486,000 sq. mi. is covered with water. Find the area of the land.

4. A cattle dealer bought some cattle, for which he paid $380. He paid out $67 for feed and care of the cattle. He then sold them for $500. How much was his net profit, that is, the profit after deducting all expenses?

5. A real estate dealer bought a city lot for $1750. He built a house on it that cost $3275 and then sold the property for $6000. Find the amount of his gain or loss.

6. The total area under broom-corn cultivation in the United States in 1899 was 178,584 acres. In 1889 it was 93,425 acres. How much was the increase in the area under cultivation during the decade?

7. The appropriation for the maintenance of the navy for 1907 was $98,773,692, for the military $72,305,270, for pensions $143,746,106. How much was appropriated for these three purposes? How much more were the combined appropriations for the navy and military than for pensions?

8. The total appropriations of the government for 1907 amounted to $701,551,566. Find the appropriations for all purposes other than the navy, military, and pensions.

MULTIPLICATION OF INTEGERS AND DECIMALS

30. 1. Find the sum of a column of four 2's.

2. The sum of a column of four 2's is ——. In this column the addend 2 is repeated 4 times. Four times 2 are ——.

3. Four 2's are 8 may be indicated thus: $\begin{array}{r} 2 \\ \times\ 4 \\ \hline 8 \end{array}$. Here 2 is taken 4 times, or is *multiplied by* 4. The 2 is the addend that is repeated, and the 4 tells the number of times this addend is repeated.

4. The sum $32 taken 4 times may be found by addition, thus:

$$\begin{array}{r} \$\ 32 \\ \$\ 32 \\ \$\ 32 \\ \$\ 32 \\ \hline \$128 \end{array}$$

It may also be found by multiplication, thus:

$$\begin{array}{r} \$32 \\ 4 \\ \hline \$128 \end{array}$$

Since four 2's are 8 and four 3's are 12, four $32's are $128.

5. Find the cost of 3 cows at $43 each by addition; by multiplication. Which method is the shorter?

6. A man paid the following amounts for three horses: $120, $85, and $100. Can the cost of the three horses be found by multiplication? Give a reason for your answer.

7. **Multiplication** is the process of taking one number as many times as there are units in another.

8. The number that is multiplied is called the **multiplicand**; the number by which we multiply is called the **multiplier**; and the result obtained is called the **product**.

$$\begin{array}{rl} \$234, & \text{multiplicand.} \\ 3, & \text{multiplier.} \\ \hline \$702, & \text{product.} \end{array}$$

9. Express $\begin{array}{r} \$234 \\ \times\ 3 \\ \hline \end{array}$ as an exercise in addition.

10. Regard the multiplicand as a repeated addend and the multiplier as the number of times the addend is repeated.

11. The product and the multiplicand are *always* like quantities. Why?

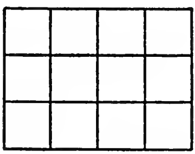
12. The multiplier is *always* an abstract number. It tells how many times the multiplicand is to be taken, or how many times the addend is to be repeated.

13. The sign of multiplication is $\times$. It indicates that the number before it is to be multiplied by the number after it. $\$3 \times 4$ is read \$3 multiplied by 4. The sign ($\times$) is sometimes used in place of the word *times* in such an expression as 2 *times* \$5.

14. Express each of the following in the form of addition: $\$4 \times 6$; 5 lb. $\times$ 3; 4 times 6 yd.; 7 in. $\times$ 8; 9×4.

31. Law of Commutation.

1. In the following diagram there are 3 rows of squares, with 4 squares in each row. Or, there are 4 rows of squares, with 3 squares in each row. There are in all 12 squares. We see that 3 times 4 squares and 4 times 3 squares are the same number of squares.



2. Find the sum of three 4's and of four 3's. Since the sum of four 3's is the same as the sum of three 4's, the product of 3 and 4 is the same, without regard to which is multiplier and which is multiplicand.

3. Show by the addition of columns that the sum of five 6's equals the sum of six 5's. Show by a diagram that 5 times 6 squares equals 6 times 5 squares.

32. Remembering that the multiplicand is the same as the repeated addend, answer the following:

1. Can you multiply \$6 by 3? 3 by \$6? 6 ft. by 3 ft.?
2. Can you find 8 lb. × 2? 2 × 8? 2 ft. × 3 ft.?
3. When the multiplicand is some number of yards, what is the product?
4. Can the multiplier ever be concrete? Why?
5. Which is more, \$6 × 3 or \$3 × 6?

33. Table of Products and Quotients.

For reference only.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

NOTE. If the pupils are not thoroughly familiar with the facts of multiplication and division, a thorough mastery of these facts should precede the attempt to use them in the exercises that follow. For a systematic method of teaching these facts, see Elementary Arithmetic of this series.

34. Oral Exercises.

Supply products for x, and add to each product the numbers above the column, as in *a* (products 12, 16, 35, etc.); adding 2: 14, 18, 37, etc.; adding 6: 18, 22, 41, etc.

a	*b*	*c*	*d*
$\underline{2},\ \underline{6}$	$\underline{3},\ \underline{5}$	$\underline{4},\ \underline{7}$	$\underline{8},\ \underline{9}$
$6 \times 2 = x$	$9 \times 3 = x$	$7 \times 7 = x$	$7 \times 4 = x$
$4 \times 4 = x$	$8 \times 5 = x$	$8 \times 9 = x$	$7 \times 5 = x$
$5 \times 7 = x$	$4 \times 9 = x$	$6 \times 6 = x$	$4 \times 5 = x$
$8 \times 4 = x$	$6 \times 7 = x$	$7 \times 9 = x$	$9 \times 8 = x$
$3 \times 7 = x$	$9 \times 5 = x$	$6 \times 4 = x$	$5 \times 5 = x$
$9 \times 4 = x$	$6 \times 8 = x$	$9 \times 6 = x$	$4 \times 8 = x$
$5 \times 6 = x$	$9 \times 9 = x$	$8 \times 8 = x$	$3 \times 9 = x$
$9 \times 3 = x$	$7 \times 8 = x$	$8 \times 7 = x$	$8 \times 6 = x$

35. Written Exercises.

1. Multiply 25,435 by 304.

MODEL:

```
  25435
    304
 101740
 76305
7732240
```

EXPLANATION:

```
  25435
    304
 101740 =   4 times 25435
7630500 = 300 times 25435
7732240 = 304 times 25435
```

Multiply by 4; multiply by 3 (that is, 300), placing the product of 3 and 5 directly below the 3, or in hundreds' place. Add.

Solve:

2. $435,450 \times 504$
3. $978,689 \times 450$
4. $230,302 \times 800$
5. $967,843 \times 769$
6. $845,397 \times 896$
7. $\$386.50 \times 527$ *
8. $\$868.75 \times 689$
9. $\$768.30 \times 843$
10. $\$364.97 \times 107$
11. $\$426.87 \times 489$

* Point off two places for cents in the answer.

36. Oral Exercises. *

1. How much will 4 sheep cost at $5 each?

MODEL for oral recitation: Since 1 sheep costs $5, 4 sheep will cost 4 times $5, or $20.

2. How much will 4 chairs cost at $3 each?

3. At $2 each, how much will 6 books cost?

4. There are 4 quarts in a gallon. How many quarts are there in 5 gallons?

5. Make and solve ten similar problems in multiplication.

37. Written Exercises.

1. Find the cost of 32 acres of land at $75 an acre.

MODEL: Statement: $\$75 \times 32 = x$

Work: $75, cost of 1 acre.

$$\begin{array}{r} 32 \\ \hline 150 \\ 225 \\ \hline \$2400 \end{array}$$
cost of 32 acres.

2. If a boy attends school 180 days each year for 12 years, how many days will he attend in 12 years?

3. Find the cost of 25 cows at $45 each.

4. If a man earns $1.75 a day, how much will he earn in 24 days?

5. Make and solve ten problems in multiplication similar to those a clerk in a grocery store has to solve.

* Drill should be given upon these and similar problems until the pupils are familiar with the language forms used in the analysis. The written form should be taken up after the oral form has been mastered. Apply this form of analysis (or some suitable form) to similar problems on the succeeding pages of the text. When the form has been mastered, the pupils should be permitted to reply briefly, thus for Prob. 1: Four times five dollars, or twenty dollars.

REVIEW — FARM PROBLEMS

38. **1.** A man bought a farm of 160 acres at $75 an acre. Find the cost of the farm.

2. During the first year he expended the following sums for improvements: repairing fences, $165.80; digging a well, $95; building a carriage house, $640; reshingling the barn, $124.35; draining a marsh, $60. Find the cost of the improvements.

3. The farm was divided into 5 fields of 20 acres each, 3 fields of 10 acres each, 10 acres of orchard, 5 acres for yards and garden, and the rest was timber land. How many acres of timber were on the farm?

4. To stock up the farm, the farmer bought 14 head of cattle at an average of $37 apiece, 5 horses at an average of $120 apiece, 24 sheep at $4.50 apiece, 6 hogs at $5.25 apiece, and 30 chickens at $.35 apiece. Find the cost of all.

5. The following amounts were received from the sale of milk for one year: Jan., $60.80; Feb., $68.17; March, $70.30; April, $71.90; May, $79.25; June, $80; July, $72.35; Aug., $66.10; Sept., $63.28; Oct., $59.37; Nov., $50.40; Dec., $54.30. Find the amount received during the year.

6. The farmer employed one man for 8 months, paying him $35 a month, and another man for 3 months, paying him $38 a month. Find the amount expended in wages.

7. Two of the 20-acre fields were sown in oats, and the yield was 45 bu. to the acre. If oats were worth 34¢ per bushel, find the value of the crop.

8. During the month of April the farmer sold $15.75 worth of eggs. At the same rate, how much would the sale of eggs amount to in one year?

39. Multiplication and Division by 10, 100, etc.

1. Compare the value of 2 in 20 and in 2; in 200 and in 20; in 2000 and in 200.

2. What effect upon the value of a figure has (*a*) moving it one place to the left? (*b*) moving it two places to the left? (*c*) moving it three places to the left?

3. Annexing a cipher to an integer has the effect of moving the digits each one place to the left. This multiplies the number by 10. Annexing two ciphers has what effect upon the places occupied by the digits? State a short method of multiplying an integer by 100; by 1000.

4. Using the short method, multiply each by 10; by 100; by 1000: 6, 47, 390, 20, 475, 8, 600, 72, 25, 64, 640.

5. State how an integer may be multiplied by 10; by 100; by 1000.

6. Moving a figure one place to the right has what effect upon its value? Compare the value of 6 in 60 and in 6; in 600 and in 6; in 6000 and in 6.

7. Dropping the cipher at the right of 60 changes the number to 6. What change does this make in the value of the number? State a short method of dividing an integer ending in a cipher by 10.

8. What change is made in the value of 400 by dropping the two ciphers? State a short method of dividing a number ending in two ciphers by 100.

9. Divide each by 10; by 100: 4500, 700, 400, 3700.

10. An integer that does not end in a cipher may be divided by 10 by placing a decimal point to the left of the right-hand figure, thus: 87 divided by 10 is 8.7. Is this the same as moving each digit one place to the right?

O

11. An integer that does not end in two ciphers may be divided by 100 by placing a decimal point at the left of the figure in tens' place, thus: 475 divided by 100 is 4.75.

12. Give the quotient of each divided by 10; by 100: 325, 560, 4582, 4500, 48, 4, 2, 10, 5, 50.

13. Write integers and divide each by 10; by 100.

+

40. Short Methods. (To be used in subsequent work.)

1. What part of 100 is 25? Compare 25 times a number with 100 times the number.

2. To multiply by 25, multiply by 100 and divide by 4. Multiply $489 by 25.

MODEL:

```
 $489
   25
$12225
```

EXPLANATION: Write as in multiplication. Mentally multiply $489 by 100. Divide the product by 4.

3. Multiply by 25: $680, $1225, 5280 ft., 231 mi., $87.56, $247.82.

4. Multiply 7865 by 369.

MODEL:

```
   7865
    369
  70785
 283140
2902185
```

EXPLANATION: First multiply by 9. As 36 is 4 times 9, multiply 70785 by 4, writing the product as in the model. Add.

5. In multiplying by 84, first multiply by 4; then multiply this product by 2, writing the first figure of the product in tens' place. State how you would multiply by 63; by 126; by 246; by 729; by 279. Illustrate each.

6. Multiply 6840 by 248; by 328; by 648; by 168.

7. Multiply $840 by 287; by 147; by 637; by 639.

41. Multiplication of Decimals.

1. Find the sum of 5.2 mi., 5.2 mi., and 5.2 mi. This sum is the same as the product of 5.2 mi. × 3. How many decimal places are there in this product? Why?

2. Find the sum of 5.2 mi., 5.2 mi., 5.2 mi., 5.2 mi., and 5.2 mi. Multiply 5.2 mi. by 5. How many decimal places are there in the product? Why?

3. Write 6.08 × 3 in the form of addition and find the sum. Multiply 6.08 by 3. How many decimal places are there in the product? Why?

4. Write 6.08 × 5 in the form of addition and find the sum. Multiply 6.08 by 5. Compare the results. How many decimal places are there in the result? Why?

5. State a short method of multiplying an integer by 10. 6.25 may be multiplied by 10 by moving the decimal point one place to the right. 6.25 × 10 is 62.5. Has moving the decimal point one place to the right the same effect as moving the digits one place to the left? Compare 2.2 with 22. Compare 75.25 with 752.5.

6. State a short method of multiplying a decimal by 100. How many places to the right must the decimal point be moved to multiply by 10? by 100? by 1000?

7. Compare 25 with 2.5. Here the digits have been moved one place to the right. Compare 2.5 with .25. Here the digits have been moved another place to the right. This has been done by moving the decimal point one place to the left. Moving the decimal point one place to the left has what effect upon the value of a decimal? upon the value of an integer?

8. State a quick way of dividing a decimal by 10; by 100. Illustrate with integers and with decimals.

42. Oral Exercises.

1. Divide each by 10 and by 100: 450, 25.74, 45, .4, 4.5, 346.2.

2. What is 6 times 4? 1 times 4? $\frac{1}{2}$ of 4? $\frac{1}{10}$ of 4?

3. What is meant by 4×6? 4×1? $4 \times \frac{1}{2}$? $4 \times .1$?

4. $4 \times .1$ is the same as 4 divided by what number? $4 \times .1 = x$. $4 \times .2 = x$. $4 \times .3 = x$.

5. Divide 4 by 100. What is meant by $4 \times .01$? $4 \times .01 = x$. $4 \times .06 = x$.

6. What is meant by $.4 \times 2$? $.4 \times 13$? $.4 \times .1$? $.4 \times .1$ is the same as .4 divided by 10. $.4 \div 10 = x$. $.4 \times .1 = x$. $.4 \times .2 = x$. $.4 \times .5 = x$.

7. Multiply \$2.30 by 10. Multiply \$2.36 by 100. Multiply \$2.30 by .1. Multiply \$24.50 by .01; by .1. Multiply \$45.75 by 10; by .1; by 100; by .01.

8. To multiply by .1 is the same as to divide by 10. What change made in the place of the decimal point in the multiplicand divides it by 10?

43. Multiply each by 10; by .1; by 100; by .01.

1. \$37.50	5. 625 ft.	9. 2240 lb.	13. 3.1416
2. \$2500	6. 1726 yd.	10. 2000 lb.	14. 24 cwt.
3. \$4.525	7. 5280 ft.	11. 625 lb.	15. 4.75 cwt.
4. \$7500	8. 2150.42	12. 630 lb.	16. 20 T.

17. $624 \times .001 = x$. 2000 lb. $\times .001 = x$.

18. Divide by 100: 4632 lb.; 3000 lb.; 2160 mi.; 37.40 mi.; \$234.50; .425 mi.; .03 mi.; 3.1416 ft.; \$60.

19. At \$5 per hundredweight, how much is sugar worth per pound? at \$4.75 per cwt.? at \$4.50 per cwt.?

20. Multiply each in the shortest way: 5280 ft. $\times$ 25; 1728×25; 987,647 by 648; 7,389,675 by 369.

44. Written Exercises.

1. Multiply 6.23 by 4.2.

MODEL: EXPLANATION:

Model	Explanation
6.23	6.23
4.2	4.2
1246	1.246
2492	24.92
26.166	26.166

First multiply 6.23 by .2. This is equivalent to dividing 6.23 by 10 and multiplying the quotient by 2. $6.23 \div 10 = .623$; $.623 \times 2 = 1.246$. Next, multiply 6.23 by 4. $6.23 \times 4 = 24.92$. Add the products.

Notice that in the above the number of decimal places in the product is the same as the sum of the number of decimal places in the multiplicand and multiplier. As this is always true, the following method may be employed:

To multiply decimals, multiply as in integers and point off in the product as many decimal places as there are in both multiplicand and multiplier.

Solve. Estimate each result before multiplying:

2. 59.786×8.97
3. $487.69 \times .479$
4. 53.008×7.086
5. $.69387 \times 6.9075$
6. 13.006×3.1416
7. $.0056 \times 385.07$
8. 7.0758×67.09
9. $.07854 \times 8.0065$
10. $46{,}897 \times 4.008$
11. 785.06×6300

45. Written Exercises.

1. If a train travels at an average rate of 43.5 mi. an hour, how far will it travel in 24 hours?

2. The circumference of a circle is 3.1416 times its diameter. Find the circumference of a circle that is 9.5 in. in diameter.

3. If it costs a boy $.50 a week to keep a pony, how much will it cost to keep it for 1 year (52 wk.)?

BILLS AND ACCOUNTS

46. A Receipted Bill.

LOS ANGELES, CAL., May 31, 1907.

MR. JAMES J. DAVIES, 2217 Vine St.

In account with S. D. JAMES & CO.

May	3	*3 lb. coffee*	$.40	$1	20		
		2 lb. tea	.65	1	30		
		6 bars soap	.05		30		
"	5	*3 cans corn*	.08		24		
		1 doz. lemons	.15		15		
"	6	*10 lb. sugar*	.06		60	$3	79
		Received Payment,					

S. D. JAMES & Co.

1. The person who buys on account is called the **debtor**, and the person who sells on account is called the **creditor**.

2. Each purchase, or payment, is an **item**. How many items of debit are there in this bill? of credit?

3. A bill must show the date of each transaction. When was the above bill made out, or *rendered?*

4. A bill must also name the debtor and the creditor and the several items of debit and credit. Who is the debtor named in the above bill? Who is the creditor?

5. When a bill is paid, the creditor writes "Paid" or "Received payment" on the bill and signs his name below. Has the above bill been paid?

6. Should a person make a practice of keeping receipted bills? Why?

47. Written Exercises.

Make out and receipt the following bills. Supply all necessary data not contained in the problems:

1. Mr. J. S. White, residing at 234 First Street, bought of the grocery firm of Allen and Baker the following: May 25, 1907, 2 lb. tea @ 60¢; 3 lb. coffee @ 45¢; 2 lb. bacon @ 20¢; 1 lb. butter @ 30¢. On May 31, 4 cans tomatoes @ 8¢; 2 doz. eggs @ 18¢; 1 lb. cheese @ 20¢.

2. Mrs. Harry Smith, residing at 1450 Jackson Street, bought of Cole Bros. the following: April 24, 1907, 9 yd. silk @ $1.50; 2 yd. dress lining @ 25¢; 4 spools silk @ 10¢; 1 bolt skirt binding, 15¢; dress trimmings, $1.50. The bill was rendered April 30.

3. Insert your own name as purchaser of the following bill of hardware: 1 garden rake, 45¢; 1 shovel, 60¢; 3 lb. nails @ 6¢; 12 yd. wire netting @ 30¢; 2 lb. staples @ 5¢; 1 lawn mower, $2.50.

4. Dr. C. L. Ward employed a schoolboy to take care of his horses, for which he agreed to pay him $6 per month, with extra pay for additional services. During the month of May the boy mowed the lawn twice, for which he was to receive 50¢ each time; and he also worked 12 hr. in the garden, at 10¢ per hour. At the end of the month Dr. Ward asked the boy to render his bill for services during the month. Make out the above bill, using your own name or the name of some boy in your school as the creditor.

5. Make out a bill for 8 music lessons at $1.50 each.

6. Make out a bill for purchases at a furniture store.

7. Make out a bill for purchases at a meat market.

8. Make out a bill for purchases at a dry goods store.

RECEIPTS

48. 1. Explain the meaning of the following:

OAKLAND, CAL., May 1, 1907.

Received of Mr. C. W. Smith twenty-five dollars ($25) in full for rent of house at 704 Logan Avenue for the month of May, 1907. D. S. STONE.

2. E. M. Day bought a sewing machine of P. Orr for $30. Write a receipt for the payment of this amount.

3. Mr. J. E. Thomas rented a farm of D. R. James for $300 per year. Write out a receipt for the payment of rent for the year beginning March 1, 1905.

4. The manager of an athletic club received $7.50 from the treasurer of the club. Write the receipt.

49. Oral Exercises.

Whenever the partial sum is 10, 20, etc., take the next two numbers together, as in *a*, 10, 19, 26, 30, 39, etc.

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*
6	8	4	6	8	7	7	9	5	4	7	8
2	3	6	7	8	6	5	9	8	9	8	5
3	3	3	9	5	7	8	5	4	7	9	4
8	7	9	8	9	6	5	7	3	8	7	3
4	7	6	3	8	7	7	9	8	5	5	8
5	3	6	8	8	4	8	9	5	7	8	9
4	3	8	9	4	3	5	2	7	6	6	8
7	4	6	3	8	6	7	9	5	7	2	3
4	3	6	8	8	7	5	9	8	8	2	8
5	3	4	9	6	9	8	7	4	6	9	6
4	2	7	7	7	6	8	8	9	8	4	7
6	8	9	3	7	5	9	5	7	8	7	9

DIVISION OF INTEGERS AND DECIMALS

50. Factors and Multiples.

1. 4 times 3 are 12. 4 and 3 are each a *factor* of 12. Name two other factors of 12; two factors of 14; of 20.

2. 2 and 8 are each a factor of ——. 3 is a factor of ——. 5 and —— are each a factor of 10.

3. The **factors** of a number are the integers which, when multiplied, make the number. A number may have several pairs of factors, thus: the pairs of factors of 24 are 4 and 6, 3 and 8, 2 and 12. Some numbers have only a single pair of factors, thus: the factors of 15 are 3 and 5.

4. Name all the pairs of factors of each of the following: 4, 6, 8, 10, 12, 14, 15, 16, 18, 20, 21, 24, 27, 28, 30.

5. As 3 times 5 are 15, 15 is a *multiple* of both 3 and 5. Name a multiple of both 4 and 5; of both 4 and 7.

6. The number obtained by multiplying together two integers is called a **multiple** of either integer. Name a multiple of 4; of both 6 and 5.

7. In multiplication, two factors are given to find their product. In division, the product and one of two factors are given to find the other factor.

51. 1. The process of finding the other factor, when the product and one of two factors are given, is called **division.**

2. Since three 5's are 15, the number of 5's in 15 is 3.

This may be indicated thus: $5\overline{)15}^{\,3}$ Here 5 is the given factor, 15 is the given product, and 3 is the other factor.

3. The given factor is called the **divisor,** the given product is called the **dividend,** and the factor found is called the **quotient.**

52. 1. Name all the multiples of 3 to 30; of 4 to 40; of 5 to 50; of 6 to 60; of 7 to 70; of 8 to 80; of 9 to 90.

2. In each of the following, name the highest multiple:

a. Of 2 in 17, 11, 13, 9, 5, 7, 15, 19, 3, 10, 14, 21, 17, 19.
b. Of 3 in 10, 8, 14, 19, 57, 16, 20, 29, 23, 26, 17, 22.
c. Of 4 in 10, 17, 7, 15, 38, 31, 22, 27, 35, 19, 25, 30.
d. Of 5 in 17, 23, 7, 12, 28, 34, 42, 39, 48, 27, 33, 19.
e. Of 6 in 16, 10, 29, 22, 38, 45, 51, 34, 57, 43, 53, 41.
f. Of 7 in 52, 46, 17, 33, 25, 68, 57, 39, 13, 30, 44, 53.
g. Of 8 in 52, 15, 28, 20, 35, 46, 69, 75, 58, 30, 60, 44.
h. Of 9 in 20, 78, 34, 42, 47, 37, 59, 68, 11, 29, 53, 16.

3. Repeat Ex. 2, naming the highest multiple and the difference between it and the given number, thus for *g*: 48 and 4; 8 and 7; 24 and 4, etc.

4. Repeat Ex. 2, giving the quotients and the remainders, thus for *g*: 6 and 4 over; 1 and 7 over; etc.

The sign of division is $\div$. It indicates that the number before it is to be divided by the number after it. $6 \div 3$ is read 6 *divided by* 3. Division may also be indicated thus: $3\overline{)6}$; or thus: $\frac{6}{3}$.

53. Oral Exercises.

Supply the value of x in each of the following:

a	*b*	*c*	*d*
$24 \div 6 = x$	$58 \div 8 = x$	$67 \div 9 = x$	$76 \div 8 = x$
$30 \div 8 = x$	$76 \div 9 = x$	$69 \div 8 = x$	$61 \div 9 = x$
$42 \div 9 = x$	$40 \div 7 = x$	$60 \div 7 = x$	$45 \div 6 = x$
$18 \div 7 = x$	$23 \div 9 = x$	$52 \div 6 = x$	$39 \div 7 = x$
$25 \div 7 = x$	$70 \div 8 = x$	$43 \div 9 = x$	$26 \div 3 = x$
$36 \div 8 = x$	$52 \div 7 = x$	$59 \div 6 = x$	$22 \div 6 = x$
$51 \div 6 = x$	$28 \div 6 = x$	$39 \div 5 = x$	$50 \div 8 = x$

54. Written Exercises.

Use as successive divisors the numbers above the columns. Solve by short division.

	a	*b*	*c*	*d**
	6, 7, 8	9, 5, 4	3, 8, 7	2, 9, 6
1.	672,458	327,459	836,594	$4387.24
2.	237,400	574,063	683,127	$3058.26
3.	946,305	508,342	427,060	$9576.30
4.	375,268	970,654	738,967	$4256.75
5.	834,008	624,307	520,380	$5687.50
6.	463,925	207,193	946,425	$7495.38
7.	927,384	423,075	315,017	$8250.15

55. Written Exercises.

Use as multipliers the numbers above the columns:

	a	*b*	*c*	*d*
	789	465	305	890
1.	$8796.50	7568.93	975.864	432.501
2.	$4578.69	835.769	586.097	34.2056
3.	$9760.50	58.7964	4376.89	6564.65
4.	$6389.75	95,687.3	605.008	5046.32
5.	$4975.86	2456.78	789.645	265.423
6.	$7869.45	697.583	83.7956	123.456
7.	$3698.70	4309.58	3945.78	45.0635
8.	$8970.56	37.6895	687.905	5643.06
9.	$6875.09	709.608	201.003	326.504
10.	$3204.56	854.076	4567.98	65.6456

* Place a decimal point in the answer above the decimal point in the dividend.

56. Measurement and Partition.

1. All division is either *measurement* or *partition*.

2.

$$\begin{array}{r} 6 \text{ ft.} \\ 4 \\ \hline 24 \text{ ft.} \end{array} \quad \text{when expressed as addition is} \quad \begin{array}{r} 6 \text{ ft.} \\ 6 \text{ ft.} \\ 6 \text{ ft.} \\ 6 \text{ ft.} \\ \hline 24 \text{ ft.} \end{array}$$

3. The quantity 24 ft. may be measured by the quantity 6 ft. The measure 6 ft. is contained in 24 ft. 4 times. This may be indicated thus: $6 \text{ ft.}\overline{)24 \text{ ft.}}$ with quotient 4.

4. The process of finding how many times one number or quantity is contained in another is called **division by measurement.**

5. In Ex. 2 the quantity 6 ft. is taken 4 times to give the quantity 24 ft. Therefore, one fourth of 24 ft. is 6 ft. This may be expressed thus: $4\overline{)24 \text{ ft.}}$ with quotient 6 ft.

6. The process of finding one of the equal parts of a number or quantity is called **division by partition.**

7. In every problem in division you will be required to find either (*a*) how many times some unit of measure is contained in a quantity to be measured, or (*b*) to find a given part of some quantity to be divided.

57. Measurement.

1. The number of 4 cubes in 12 cubes may be found by using the unit 4 cubes as a measure to measure the quantity 12 cubes. 4 cubes, the unit of measure, is contained in 12 cubes, the quantity to be measured, exactly 3 times. This may be indicated thus: $4 \text{ cubes}\overline{)12 \text{ cubes}}$ with quotient 3.

2. Show with objects or by a diagram how many 3 apples there are in 12 apples. Indicate this in the form of division. Write the quotient.

3. Show by a diagram or by actual measurement how many 2 ft. there are in 12 ft. Indicate this in the form of division. Write the quotient.

4. Show with objects or by a diagram what is meant by each: $2 \text{ books})\overline{10 \text{ books}}$; $5 \text{ pencils})\overline{15 \text{ pencils}}$; $4 \text{ boys})\overline{12 \text{ boys}}$; $6 \text{ books})\overline{12 \text{ books}}$; $3 \text{ ft.})\overline{9 \text{ ft.}}$; $5¢)\overline{20¢}$. Write a problem for each.

5. In finding the number of 4 hr. there are in 24 hr., what is the unit of measure? What is the quantity to be measured?

6. Show by using books (*a*) how many 3 books there are in 9 books; (*b*) how many 3 books there are in 10 books; (*c*) how many 3 books there are in 11 books. Indicate each in the form of division, with quotient, and remainder, if any.

7. Show with objects that the number of 4 apples in 10 apples is 2, with 2 apples remaining.

8. Find the number of \$4 there are in \$16; of 3 yd. there are in 15 yd.; of 7 da. there are in 21 da. Indicate each in the form of division.

9. In division by measurement the divisor is always like the dividend. The quotient is always an abstract number, since it tells how many times the unit of measure is contained in the quantity measured.

10. Find by measuring how many times a 1-ft. measure must be applied to measure 4 ft. ($4 \text{ ft.} \div 1 \text{ ft.} = x$); a $\frac{1}{2}$-ft. measure to measure 4 ft. ($4 \text{ ft.} \div \frac{1}{2} \text{ ft.} = x$); a $\frac{1}{4}$-ft. measure to measure 4 ft. ($4 \text{ ft.} \div \frac{1}{4} \text{ ft.} = x$).

58. Oral Exercises — Measurement.

1. At \$4 each, how many desks can be bought for \$12? The unit of measure is \$4 and the quantity to be measured is \$12.

MODEL for oral recitation: Since 1 desk costs \$4, as many desks can be bought for \$12 as there are \$4 in \$12, or 3.

In each of the following, name the unit of measure and the quantity to be measured:

2. At \$3 each, how many chairs can be bought for \$15?

3. At \$4 a pair, how many pairs of shoes can be bought for \$24?

4. If berries cost 6¢ a box, how many boxes of berries can be bought for 30¢?

5. How many yards of ribbon at 8¢ a yard can be bought for 40¢?

6. If a boy earns \$9 a month, in how many months will he earn \$45?

7. Write ten additional problems in measurement.

59. Written Exercises — Measurement.

1. At \$9 a ton, how many tons of hay can be bought for \$216?

2. A man bought sheep at \$6 each. He paid \$96 for all. How many sheep did he buy?

3. A farmer divided a farm containing 160 acres into 10-acre fields. Find the number of fields and the value of each field at \$80 per acre.

4. How many 9's are there in 1728? Is this measurement?

5. Write five problems in division by measurement, and solve each.

60. Partition.

1. One fourth of 12 cubes may be found by dividing 12 cubes into 4 equal groups or parts.

One fourth of 12 cubes is 3 cubes. This may be indicated thus:

$$4\overline{)12 \text{ cubes}}^{\;\;3 \text{ cubes}}$$

2. Show with objects and by diagrams what is meant by one third of 12 objects; by one fourth of 8 objects; by one third of 9 objects. Indicate these in the form of division, and write the quotients.

3. Show with objects and by a diagram what is meant by each of the following:

$4\overline{)8 \text{ apples}}$ $2\overline{)10 \text{ in.}}$ $5\overline{)10 \text{ in.}}$ $3\overline{)15 \text{ ft.}}$ $4\overline{)16 \text{ books.}}$

4. In division by partition the quotient is always a part of the dividend. Therefore, the quotient is always like the dividend, — concrete when the dividend is concrete, and abstract when the dividend is abstract. The divisor is always an abstract number. Why?

61. Oral Exercises — Partition.

1. If 2 chairs cost \$8, what is the cost of 1 chair?

MODEL for oral recitation: If 2 chairs cost \$8, 1 chair will cost one half of \$8, or \$4.

Name in each the quantity to be divided and the number of parts into which it is to be divided:

2. If 2 tables cost \$12, what is the cost of 1 table?

3. If 3 stoves cost \$15, what is the cost of 1 stove?

4. At \$12 a ton, what is the cost of one half ton of hay?

5. Write ten additional problems in partition.

62. 1. Show by a diagram or with objects the meaning of:

$2\overline{)\$6}$ $\quad 4\overline{)12 \text{ da.}}$ $\quad 3\overline{)9 \text{ in.}}$ $\quad \$3\overline{)\$9}$ $\quad \$4\overline{)\$12}$

2. Write a problem for each:

$3\overline{)\$18}$ $\quad 4\overline{)20¢}$ $\quad 2\overline{)10 \text{ yd.}}$

$\$6\overline{)\$12}$ $\quad \$7\overline{)\$21}$ $\quad 5\overline{)10 \text{ yd.}}$

3. $4\overline{)12}$ may be either partition or measurement. State what is meant by $4\overline{)12}$ (*a*) when it is partition; (*b*) when it is measurement.

4. Is the divisor ever concrete in partition? Is the quotient ever concrete in measurement? Give reasons.

5. In division by measurement the quotient is always what kind of a number?

6. When the divisor is a concrete number, is the division partition or measurement?

7. Make ten problems in division, and tell which are partition and which are measurement.

8. Make problems for each of the following. Tell which are partition and which are measurement.

$\$2\overline{)\$10}$ $\quad 2\overline{)\$10}$ $\quad 4\overline{)16 \text{ yd.}}$

$8 \text{ wk.}\overline{)16 \text{ wk.}}$ $\quad 3\overline{)\$15}$ $\quad 5\overline{)25¢}$

63. Oral Exercises.

$\frac{12}{3}$ indicates that 12 is to be divided by 3. Solve each:

1. $\frac{15}{3}$	**6.** $\frac{40}{7}$	**11.** $\frac{40}{6}$	**16.** $\frac{59}{7}$
2. $\frac{45}{8}$	**7.** $\frac{68}{7}$	**12.** $\frac{19}{5}$	**17.** $\frac{51}{8}$
3. $\frac{17}{4}$	**8.** $\frac{54}{9}$	**13.** $\frac{69}{8}$	**18.** $\frac{43}{7}$
4. $\frac{56}{9}$	**9.** $\frac{30}{8}$	**14.** $\frac{37}{4}$	**19.** $\frac{70}{8}$
5. $\frac{80}{9}$	**10.** $\frac{45}{6}$	**15.** $\frac{30}{6}$	**20.** $\frac{14}{5}$

64. Written Exercises.*

1. At $8 each, how many tables can be bought for $128?

MODEL for measurement:

16, number bought for $128
cost of 1 table, $8)$128

2. A man spent $216 in 6 mo. What was the average amount spent each month?

MODEL for partition:

$36, spent in 1 mo.
6) $216, spent in 6 mo.

Tell which of the following are partition and which are measurement, and solve:

3. If a boy saves $5 a month, in how many months will he save $120?

4. How many tons of coal at $7 a ton can a man buy for $161?

5. Five boys agreed to share equally the expenses of a camping trip. The trip cost them $21.70. What was each boy's share?

6. A girl bought 8 yd. of cloth for $2.56. How much did the cloth cost her per yard?

7. A hardware merchant bought some stoves at $9 each. His bill amounted to $198. How many stoves did he buy?

8. If a boy worked 65 problems correctly in 1 school week (5 days), what was the average number worked correctly each day?

9 A dealer bought 6 copies of a certain book. His bill amounted to $7.50. What was the price of the book?

* Give the oral analysis of Probs. 1–9.

65. Ratio or Comparison.

1. How many times must the measure 2 ft. be applied **in** measuring 6 ft.? The number 3 expresses the **ratio, or** relation, of the quantity 6 ft. to the unit 2 ft.

2. In measuring 6 ft. by 2 ft. the quantity to be measured is 6 ft., and the unit of measure is 2 ft. The ratio of 6 ft. to 2 ft. is found by dividing 6 by 2.

3. What is the ratio of 8 ft. to 4 ft.? of 12 ft. to 3 ft.?

4. What is the ratio of 6 da. to 3 da.? of 24 hr. to 6 hr.? of 25¢ to 5¢? of $1 to $.25? of 75¢ to 25¢?

5. What is the ratio of 100 to 50? to 25? to 10? to 20? to 5?

6. In measuring 2 ft. by 4 ft. the unit of measure is 4 ft., and the quantity to be measured is 2 ft. The measure 4 ft. is applied $\frac{1}{2}$ time; that is, one half the measure is applied in measuring 2 ft. The ratio of 2 ft. to 4 ft. is $\frac{1}{2}$.

66. Draw on the blackboard lines the length of the quantities to be measured. Make measures the length of the units of measure to be used in measuring each line. By applying the measure to the line to be measured, determine the ratio of the following:

1. Of 2 ft. to 1 ft.
2. Of 2 ft. to $\frac{1}{2}$ ft.
3. Of 1 ft. to $\frac{1}{2}$ ft.
4. Of 3 ft. to $\frac{1}{3}$ ft.
5. Of $\frac{1}{2}$ ft. to 1 ft.
6. Of $\frac{1}{2}$ ft. to 2 ft.
7. Of $1\frac{1}{2}$ ft. to 3 ft.
8. Of $\frac{1}{6}$ ft. to 1 ft.
9. Of $\frac{2}{3}$ ft. to 2 ft.
10. Of $1\frac{1}{3}$ ft. to 2 ft.
11. Of $\frac{2}{3}$ ft. to $\frac{1}{3}$ ft.
12. Of $\frac{2}{3}$ ft. to $\frac{5}{6}$ ft.

13. What part of 14 da. are 7 da.? What is the ratio of 7 da. to 14 da.? of 14 da. to 7 da.?

14. 3 in. is what part of 6 in.? What is the ratio of 3 in. to 6 in.? The ratio tells the number of times the unit of measure must be applied to measure the given quantity. A 6-in. measure must be applied —— times to measure 3 in. A 3-in. measure must be applied —— times to measure 6 in. The ratio of 6 in. to 3 in. is ——. The ratio of 3 in. to 6 in. is ——.

15. What part of the measure 12 in. must be applied to measure 3 in.? What is the ratio of 3 in. to 12 in.? of 12 in. to 3 in.?

16. The ratio of 3 yd. to some quantity is $\frac{1}{5}$. What is the quantity?

17. The ratio of some quantity to $2 is 4. What is the quantity?

18. If 4 is the ratio of some amount to $20, what is the amount?

19. A $\frac{1}{2}$-ft. measure was used 12 times in measuring the length of a line. How long was the line?

20. Draw a line of such length that a 6-in. measure will be applied $1\frac{1}{2}$ times in measuring it. Prove your work by applying the measure.

21. What is the ratio of 2 to 8? of 6 to 2? of 20 to 5? of 5 to 30? of 8 to 48? of 40 to 8?

22. Two tons of coal will cost what part of the cost of 8 tons? of 6 tons? of 12 tons?

23. 3 yd. of cloth will cost what part of the cost of 9 yd.? of 15 yd.? of 6 yd.? of 12 yd.?

24. If the cost of 24 yd. of cloth is given, how may the cost of 8 yd. be found? of 6 yd.? of 4 yd.? of 12 yd.?

25. If the cost of 6 sheep is $24, what is the cost of 18 sheep? of 12 sheep? of 3 sheep? of 2 sheep?

67. Oral Exercises.

1. If 5 desks cost $20, how much will 7 desks cost?

The quantities 5 desks and 7 desks are measured by the common unit 1 desk. In solving this problem, first find the cost of the unit 1 desk. Next find the cost of the required number of units.

MODEL for oral recitation: If 5 desks cost $20, 1 desk will cost $\frac{1}{5}$ of $20, or $4. Since 1 desk costs $4, 7 desks will cost 7 times $4, or $28.

2. If 4 tons of hay cost $32, how much will 6 tons cost?

3. If 7 tablets cost 35¢, how much will 4 tablets cost?

4. At the rate of 5 for 25¢, how much will 8 spelling-blanks cost?

5. A girl paid 30¢ for 5 yards of ribbon. How much would 8 yards have cost at the same rate?

6. Make ten additional problems similar to the above.

68. Written Exercises.

1. A farmer raised 220 bu. of oats on 4 acres of land. How much at the same rate would a 7-acre field have produced?

2. If a train travels 138 mi. in 3 hr., at the same rate, how far will it travel in 8 hr.?

3. From a farm of 160 acres 8 acres were sold for $500. At this rate, what was the value of the entire farm?

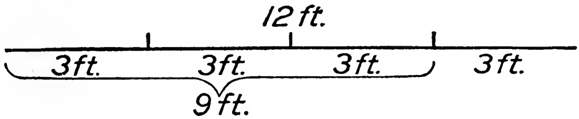
4. 7 men picked 210 boxes of prunes in 1 day. At the same rate, how many boxes would 15 men have picked?

5. The expenses of a family amounted to $328.75 for 5 mo. At the same rate, what would the expenses amount to in 1 yr. (12 mo.)?

69. Oral Exercises.

Finding a part of an amount, when the amount is given:

1. Find $\frac{3}{4}$ of 12 ft.



To find $\frac{1}{4}$ of 12 ft., divide 12 ft. into 4 equal parts. Then $\frac{3}{4}$ of 12 ft. will be 3 of these parts.

2. Show by a diagram that $\frac{1}{3}$ of 12 ft. is 4 ft. and that $\frac{2}{3}$ of 12 ft. are 8 ft; that $\frac{3}{4}$ of 8 yd. are 6 yd.

3. Show with objects that $\frac{3}{5}$ of 10 objects are 6 objects; that $\frac{2}{3}$ of 6 objects are 4 objects.

4. Show by a diagram that if a board is 8 ft. long, $\frac{3}{4}$ of the length of the board is 6 ft.

5. What is $\frac{3}{4}$ of \$12?

MODEL for oral recitation: Since $\frac{1}{4}$ of \$12 is \$3, $\frac{3}{4}$ of \$12 is 3 times \$3, or \$9.

6. What is $\frac{2}{3}$ of \$15? of \$24? of \$30? of \$12?

7. What is $\frac{4}{5}$ of 10 mi.? of 25 mi.? of 35 mi.?

8. What is $\frac{5}{6}$ of 18 lb.? of 30 lb.? of 42 lb.? of 60 lb.?

9. At 20¢ a pound, how much will $\frac{3}{4}$ of a pound of cheese cost?

10. How many months are there in $\frac{3}{4}$ yr.? in $\frac{5}{6}$ yr.?

11. If Fred worked 15 problems and John worked $\frac{2}{3}$ as many, how many did John work?

12. A girl worked 16 problems, and $\frac{3}{4}$ of them were correct. How many of them were correct?

13. How many inches are there in $\frac{2}{3}$ of a foot?

14. At \$8 a ton, how much will $\frac{3}{4}$ of a ton of coal cost?

15. Make ten additional problems similar to the above.

70. Written Problems.

1. There are 2000 pounds in a ton. How many pounds are there in $\frac{3}{4}$ of a ton of hay?

2. There are 320 rods in 1 mile. How many rods are there in $\frac{7}{8}$ of a mile?

3. A girl read a book containing 210 pages. How many pages had she read when she had read $\frac{3}{4}$ of the book?

4. Two boys, Henry and Frank, bought out a newspaper route that cost them $4.50. Frank paid $\frac{2}{3}$ of the cost of the route and Henry paid $\frac{1}{3}$ the cost. How much did each pay?

5. A man had 320 acres. He rented $\frac{3}{4}$ of his land. How many acres did he rent?

6. There are 5280 feet in 1 mile. How many feet are there in $\frac{3}{4}$ of a mile?

71. Dividing by 20, 30, 40, 200, 300, etc.

1. State a short method of dividing a number by 10; by 100.

2. Divide 476 by 40.

MODEL:

$$40\overline{)476}^{\;11.9}$$

First place a decimal point in the quotient above and between 7 and 6. Then divide by 4.

3. State how you would divide a number by 50; by 400; by 4000.

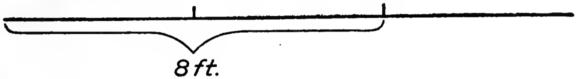
Solve. Before dividing, estimate each quotient:

4. $324 \div 40$	8. $1260 \div 400$	12. $4860 \div 40$
5. $1728 \div 60$	9. $5280 \div 600$	13. $3600 \div 900$
6. $1720 \div 80$	10. $7854 \div 500$	14. $4240 \div 800$
7. $320 \div 20$	11. $6250 \div 500$	15. $1240 \div 30$

72. Oral Exercises.

Finding an amount, when part of the amount is given :

1. When $\frac{2}{3}$ of the length of a board is 8 ft., what is $\frac{1}{3}$ of the length of the board ?



If $\frac{2}{3}$ of the length of a board is 8 ft., $\frac{1}{3}$ of the length of the board is what part of 8 ft.? If $\frac{1}{3}$ of the length of a board is 4 ft., what is $\frac{3}{3}$ of the length of the board?

2. Show by a diagram that if $\frac{3}{5}$ of the length of a line is 6 ft., $\frac{1}{5}$ of the length of the line is 2 ft. If $\frac{1}{5}$ of the length of a line is 2 ft., what is the length of the line?

3. Show by a diagram that if $\frac{3}{4}$ of a line is 6 ft. long, $\frac{1}{4}$ of the line is 2 ft. long and the line is 8 ft. long.

4. Using 12 objects, show that since $\frac{1}{3}$ of 12 objects is 4 objects, $\frac{2}{3}$ of 12 objects are 8 objects.

5. Show that since $\frac{2}{3}$ of 12 objects are 8 objects, $\frac{1}{3}$ of 12 objects is $\frac{1}{2}$ of 8 objects.

6. Draw a diagram to show the length of a room, if $\frac{3}{4}$ of the length of the room is 9 ft.

7. If \$12 is $\frac{3}{4}$ of the cost of a suit of clothes, what is $\frac{1}{4}$ of the cost of the suit? What is the cost of the suit?

MODEL for oral recitation : If \$12 is $\frac{3}{4}$ of the cost of a suit, $\frac{1}{4}$ of the cost of the suit is $\frac{1}{3}$ of \$12, or \$4. Since \$4 is $\frac{1}{4}$ of the cost of a suit, the cost of the suit is 4 times \$4, or \$16.

8. If \$20 is $\frac{4}{5}$ of the cost of a cow, what is the cost of the cow?

9. If $\frac{2}{3}$ of the cost of a book is 40¢, what is the cost of the book?

10. Two boys together bought a baseball. One boy paid \$.60, which was $\frac{2}{3}$ of the cost of the ball. How much did the other boy pay? What was the cost of the ball?

11. A boy spent 90¢, which was $\frac{3}{4}$ of the whole amount of money he had. How much money had he? How much money had he left?

12. Fred weighs 100 lb. This is $\frac{5}{6}$ of George's weight How much does George weigh?

13. Make ten additional problems similar to the above.

73. Oral Exercises.

Two addends whose sum is 10 or less may be taken as a single addend. Exercise *a* below may be added: 13, 23, 30, 47, 54, 62, 70. Add *b–m* in a similar manner:

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*
8	8	8	7	2	3	8	7	3	2	3	2	4
4	6	1	6	5	6	5	4	6	7	3	1	7
4	1	3	2	2	7	3	5	2	7	7	5	2
2	3	2	2	5	6	8	8	9	2	3	6	4
5	2	4	2	2	3	5	5	7	5	5	8	8
8	4	5	5	5	3	3	4	6	2	4	3	3
9	2	2	4	2	6	5	7	2	7	3	4	1
3	3	6	7	5	8	9	4	9	8	3	2	4
4	6	7	8	7	6	5	5	2	7	8	2	5
6	2	2	6	2	3	3	7	6	2	3	7	7
4	3	2	3	5	9	8	9	7	6	3	8	2
5	8	7	7	3	3	3	5	2	2	9	3	3
8	6	9	4	4	6	5	4	6	7	8	7	6

Write ten columns in which two addends whose sum is 10 or less may be taken as a single addend. Add your columns.

Add the columns in Sec. 13.

REVIEW

74. Multiplication.

Name the multiples of 2 to 24; of 3 to 36; of 4 to 48; of 5 to 60; of 6 to 72; of 7 to 84; of 8 to 96; of 9 to 108.

Multiply the numbers in each column by the number at the head of the column, and add to each product the number in parentheses:

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
2 (9)	3 (2)	4 (3)	5 (4)	6 (5)	7 (6)	8 (7)	9 (8)
7	6	5	4	9	8	2	8
3	3	8	2	4	4	6	2
9	2	4	5	7	5	9	9
4	7	2	8	2	7	3	3
8	9	9	3	3	6	4	7
5	8	7	6	8	2	8	4
9	4	6	9	6	9	7	6
2	5	3	7	5	3	5	5
6	7	8	9	7	6	9	7
8	9	6	6	9	8	8	6
6	6	9	8	6	7	6	8
7	8	7	7	8	9	7	9

75. Write the following in the forms of bills, and find the amounts in each. (See p. 42.)

1. Jan. 2, 1907: 3 bars of soap at 6¢ each; 4 lb. of prunes at 8¢ per pound; 85 lb. of potatoes at 2¢ per pound. Jan. 5, 1907: 2 lb. of coffee at 33¢ per pound; 2 lb. of cheese at 18¢ per pound; 3 lb. of tea at 55¢ per pound.

2. Jan. 9, 1907: 7 yd. dress cloth at $1.20 per yard; 1 doz. handkerchiefs at $1.40 per dozen; 3 shirts at $1.75 each. Jan. 10, 1907: 5 pair socks at $.25 each; 1 umbrella at $2.40; 1 pair scissors at $.75.

76. Division.

1. Name the highest multiple of the number at the head of the column in each number in the column, thus for column *a*: 18, 9, 15, etc.

a	*b*	*c*	*d*	*e*	*f*	*g*
3	4	5	6	7	8	9
20	17	38	39	40	30	40
11	10	17	28	25	46	78
16	23	26	11	58	37	31
8	25	14	22	13	22	86
25	31	23	58	32	76	68
14	14	49	46	65	53	23
23	19	33	33	39	60	59
17	27	43	52	52	29	60
28	38	24	16	18	69	51
19	29	39	40	60	18	34
22	34	28	50	34	59	43
13	21	34	55	68	14	70
29	39	63	63	87	97	75
24	26	32	70	48	50	39
32	43	54	57	29	17	110

2. Name the highest multiple of the number at the head of the column in each number in the above columns, and give the difference between the multiple and the number in the column, thus for column *a*; 18 and 2; 9 and 2; 15 and 1, etc.

3. Divide the numbers in each column by the number above the column, giving the quotient and remainder, thus for column *a*: 6 and 2 over; 3 and 2 over, etc.

4. Divide 21,487,249 by 6; 459,738,795 by 8; 59,482,395 by 7; 708,718,907 by 9.

LONG DIVISION

77. When all the steps in division are written, the process is called **long division.** Long division is generally used with divisors of two or more places.

For the purpose of finding the quotient figure, all divisors are classified into two general cases, as follows:

Case I. All divisors in which the second figure* is the same as or less than the first figure, as 66, 65, 832.

Case II. All divisors in which the second figure is greater than the first figure, as 15, 68, 271, 795.

Write in order all numbers from 13 to 100 that come under Case I, as 21, 22; all that come under Case II, as 13, 14, 15.

Case I

78. When the second figure of the divisor is the same as or less than the first figure, a trial quotient figure may be found, as in the following:

Divide 2792 by 65.

Model:

$$\begin{array}{r} 42 \\ 65\overline{)2792} \\ \underline{260} \\ 192 \end{array}$$

It will take three places at the left of 2792 to contain 65 at least once.

Step 1. 6 is contained in 27 four times, with 3 remainder. Is 5 contained in 39 as many as 4 times? Yes. Try 4 as a quotient figure. Place 4 in quotient above 9.

Step 2. Multiply the divisor 65 by 4, writing the product below 279.

Step 3. Subtract 260 from 279.

Step 4. Bring down the next figure in the dividend. The new dividend is 192.

Repeat Step 1. 6 is contained in 19 three times, with 1 remainder. Is 5 contained in 12 as many as 3 times? No. Try 1 less than 3, or 2, as a quotient figure.

Complete the work, writing the remainder as in short division.

* In 65 regard 6 as the first figure and 5 as the second figure.

With divisors of Case I, the trial quotient figure found as in the model is *always* the correct quotient figure when the divisor contains two places, as 42, 87, etc.

It is also the correct quotient figure when the divisor contains three or more places, whenever the second figure of the divisor is *not* contained in its dividend as many times as the first figure is contained in its dividend.

With divisors of Case I, when the first figure of the divisor is contained in its dividend ten times, the correct quotient figure is 9, thus :

$$\begin{array}{r} 9 \\ 532\overline{)52678} \end{array}$$

5 is contained in 52 ten times. Take 9 as the quotient figure. It is unnecessary to test the second figure of the divisor.

79. Written Exercises.

Tell to which case each divisor in the following belongs :

Solve only the exercises in which the divisors are of Case I.

1.	75,679 by 42	13.	15,672 by 29	25.	57,606 by 21
2.	73,496 by 24	14.	71,896 by 83	26.	40,000 by 20
3.	12,500 by 64	15.	83,678 by 95	27.	59,684 by 84
4.	62,847 by 66	16.	79,678 by 88	28.	85,678 by 96
5.	95,438 by 27	17.	53,678 by 61	29.	45,672 by 63
6.	18,245 by 29	18.	29,678 by 54	30.	456,783 by 15
7.	10,000 by 75	19.	38,006 by 47	31.	648,739 by 16
8.	60,000 by 85	20.	47,608 by 96	32.	457,820 by 14
9.	35,640 by 44	21.	64,896 by 88	33.	426,789 by 13
10.	11,045 by 19	22.	52,873 by 68	34.	480,068 by 96
11.	16,712 by 18	23.	49,678 by 79	35.	124,530 by 144
12.	27,672 by 28	24.	68,368 by 72	36.	231,672 by 772

CASE II

80. For the purpose of finding a trial quotient figure that will seldom vary much from the correct quotient figure, divisors of Case II are classified into three groups.

GROUP *a*. When the second figure of the divisor is 7, 8, or 9, as 17, 18, 19; 578, 588, 598, etc.

GROUP *b*. When the first figure is more than 1 and the second figure is 3, 4, 5, or 6, as 23, 24, 35, 46, etc.

GROUP *c*. When the first figure is 1 and the second figure is 3, 4, 5, or 6, *i.e.* 13, 14, 15, 16.

Write the numbers from 13 to 100 that come under Group *a*, Case II; that come under Group *b*, Case II; that come under Group *c*, Case II.

81. Group *a*. With divisors of Group *a*, the trial quotient figure may be found by using as a divisor 1 more than the first figure of the divisor, as in the following:

1. Divide 379,868 by 476.

MODEL:

```
        79
476)379868
     3332
     4666
```

It will take four places to contain 476 at least once. 5 (1 more than 4) is contained in 37 seven times. Try 7 as a quotient figure. Complete the division.

The trial quotient figure found as in the model will sometimes be 1 less than the correct quotient figure.

2. Solve all exercises in Sec. 79 in which the divisors are of Group *a*, Case II.

3. Write and solve five exercises in division, using divisors of Group *a*, Case II, and five exercises using divisors of Case I.

82. Group *b*. With divisors of Group *b*, Case II, the trial quotient figure may be found by using as a divisor 1 more than the first figure of the divisor and adding 1 to the quotient, as in the following:

1. Divide 11,678 by 24.

MODEL:

$$\begin{array}{r} 4 \\ 24\overline{)11678} \\ 96 \\ \hline 207 \end{array}$$

It will take three places to contain 24 at least once. 3 (1 more than 2) is contained in 11 three times. Try 4 as a quotient figure. Complete the division.

The trial quotient figure found as in the model will sometimes vary 1 from the correct quotient figure.

2. Solve all exercises in Sec. 79 in which the divisors are of Group *b*, Case II.

3. Write and solve ten exercises in division, using divisors of Groups *a* and *b*, Case II.

4. Write and solve five exercises in division, using divisors of Case I.

83. Group *c*. With divisors of Group *c*, Case II, the trial quotient figure may be found by using 2 as a divisor, and adding 2 to the quotient, as in the following:

1. Divide 12,678 by 142.

MODEL:

$$142\overline{)12678}$$

It will take four places in the dividend to contain 142 at least once. 2 is contained in 12 six times. Try 8 as a quotient figure. Complete the division.

The trial quotient figure found as in the model will seldom vary more than 1 from the correct quotient figure.

2. Solve all exercises in Sec. 79 in which the divisors are of Group *c*, Case II.

3. Write and solve ten exercises in division, using divisors of Case II.

84. With 1367 as a dividend, find the first trial quotient figure, using each of the following as divisors, and explain how each is found: 32, 16, 327, 48, 59, 24, 375, 698, 156, 426, 276, 149, 137, 161.

85. Written Exercises.

1. Write and solve ten exercises in division, using as divisors numbers of Case I between 100 and 1000.

2. Write and solve ten exercises in division, using as divisors numbers of Groups *a*, *b*, and *c*, Case II, between 100 and 1000.

86. Written Exercises.

1. How many dozen eggs at 18¢ a dozen must be sold to pay for 1 lb. of tea at 60¢, and 1 lb. of coffee at 30¢?

2. A man's yearly salary is $1860. Find his salary per month. What is the amount of his salary per week?

3. If a train travels at an average rate of 45 mi. an hour, in how many hours will it travel 2000 mi.?

4. A ton is 2000 lb. How many pupils of your own weight will it take to weigh 1 T.?

5. A bushel of wheat weighs 60 lb. How many bushels will it take to weigh 1 T.?

6. The monthly rental of an apartment house amounted to $144. The average rental of an apartment was $24. How many apartments were there in the house?

7. The total annual expenditure of the United States government for the year ending June 30, 1905, amounted to $532,122,762.47. What was the average daily expenditure?

8. The number of school children in a certain city is 1640. If the average number of pupils to each room is 40, how many schoolrooms are there in the city?

DIVISION OF DECIMALS

87. 1. How many \$2 are there in \$4? How many 2 tenths are there in 4 tenths? $.4 \div .2 = x$?

2. How many 3 qt. are there in 9 qt.? How many 3 hundredths are there in 9 hundredths?

$$9 \text{ qt.} \div 3 \text{ qt.} = x? \qquad .09 \div .03 = x?$$

3. How many times are 4 yd. contained in 8 yd.? 4 tenths in 8 tenths? 4 hundredths in 8 hundredths? 4 thousandths in 8 thousandths?

4. What is the quotient in each of the following: $4\overline{)8}$; $.4\overline{).8}$; $.04\overline{).08}$; $.004\overline{).008}$? Prove the correctness of your answer by multiplying the divisor by the quotient and comparing it with the dividend.

5. If the divisor contains tenths, tenths of the dividend may give a whole number in the quotient.

$$\begin{array}{r} 3 \\ .4\overline{)1.2} \end{array} \qquad\qquad \begin{array}{r} 3\ 6 \\ .6\overline{)21.6} \end{array}$$

6. If the divisor contains hundredths, hundredths of the dividend may give a whole number in the quotient.

$$\begin{array}{r} 9 \\ .05\overline{).45} \end{array} \qquad\qquad \begin{array}{r} 59 \\ .04\overline{)2.36} \end{array} \qquad\qquad \begin{array}{r} 8\ 54 \\ .04\overline{)34.16} \end{array}$$

Place the decimal point in the quotient above and after the figure in the dividend occupying the same order as the lowest order in the divisor. Divide as in integers.

7. Divide 21.66 by 6.

MODEL: $\begin{array}{r} 3.61 \\ 6\overline{)21.66} \end{array}$ As the lowest order in the divisor is units, place the decimal point in the quotient above and after the figure occupying units' order in the dividend.

Divide as in integers. When the divisor is an integer, the lowest order in the divisor is units, and the decimal point in the quotient is directly above the decimal point in the dividend.

8. Divide 43.38 by .8.

MODEL: $.8\overline{)43.\dot{3}8}$ As the lowest order in the divisor is tenths, place the decimal point above and after the figure occupying tenths' order in the dividend. Divide.

9. Divide 2.4 by .006.

MODEL: $.006\overline{)2.40\dot{0}}$ As the lowest order in the divisor is thousandths, supply two ciphers in the dividend to make the lowest order thousandths. Place the decimal point above and after the figure in the dividend occupying thousandths' order. Divide.

10. With 45.06 as a divisor, the decimal point will be placed in the quotient above and after the figure in the dividend occupying hundredths' order. State where the decimal point should be placed with each of the following as divisors: 5.05; 67; 3.15; 3.1416; .008; 26.1; .0045; 50; .05; 2150.42; 6.

88. Arrange as in the models and fix the decimal points in the quotients:

	a	*b*	*c*	*d*
1.	$.05 \div 2.5$	$.04 \div .002$	$180 \div .006$	$3 \div 4$
2.	$.6 \div .3$	$2.5 \div 50$	$36 \div 750$	$20 \div 50$
3.	$1.44 \div .12$	$.0048 \div .6$	$.27 \div 3$	$1.75 \div .025$
4.	$2.7 \div 9$	$3.6 \div .12$	$.007 \div 3.5$	$.075 \div 2.5$
5.	$.48 \div 8$	$.16 \div 20$	$14 \div .007$	$120 \div .004$
6.	$.1 \div .005$	$2.4 \div 12$	$22.5 \div .15$	$4.8 \div .0012$
7.	$.024 \div .8$	$.012 \div .03$	$2 \div 10$	$.065 \div 3.25$
8.	$3.6 \div .006$	$.005 \div .1$	$45 \div 90$	$64 \div .0008$
9.	$1 \div .045$	$4 \div 4.56$	$100 \div .1$	$.75 \div .6$
10.	$10 \div .01$	$.01 \div 10$	$.001 \div .01$	$101 \div 1.01$

89. 1. Divide 75.51 by 60.4, and carry the result to two decimal places.

MODEL:

$$\begin{array}{r@{}l}
 & \;\;1.25+ \\
60.4 & \overline{)75.51} \\
 & \;\underline{60\ 4} \\
 & \;15\ 11 \\
 & \;\underline{12\ 08} \\
 & \;\;\;3\ 03
\end{array}$$

The quotient, carried to two decimal places, is 1.25. The sign (+) is placed after the quotient to indicate that the division is not exact.

Arrange as in the model, fix the decimal point, estimate the result, then divide. When the division is inexact, carry the quotient to three decimal places.

2. $6.25 \div 25$
3. $.1728 \div .12$
4. $720.405 \div 3.15$
5. $250 \div .75$
6. $1210.605 \div 6.05$
7. $.0045 \div .045$
8. $37.806 \div 8.7$
9. $48.312 \div 3.1416$
10. $1000 \div .0025$
11. $.1224 \div 2.04$
12. $500 \div .005$
13. $512.16 \div 64.02$
14. $68.045 \div 42{,}125$
15. $12.5 \div .0375$
16. $1000 \div .875$
17. $12.75 \div 3.1416$
18. $458{,}766 \div 2150.42$
19. $8.05 \div 40.25$
20. $8790 \div 2150.42$
21. $100 \div .125$

22. Read the decimals in the above exercises.

23. When a number is divided by .4, is the quotient greater or less than the number?

24. When a number is divided by .2, the quotient obtained is how many times the dividend?

25. Estimate the quotient of 12 divided by each: .1, .25, .5, .4.

26. State a short method of dividing by 10; by 100; by 1000; by .1; by .01; by .001.

REVIEW

90. **1.** Solve exercises in Sec. 88.

2. Write five exercises in division of decimals, and solve each.

3. Write five exercises in multiplication of decimals, and solve each.

4. Write five exercises in Case I in long division, and solve each.

5. Write five exercises in Group *a* of Case II in long division, and solve each.

6. Write five exercises in Group *b* of Case II in long division, and solve each.

7. Write five exercises in Group *c* of Case II in long division, and solve each.

8. Write five columns in addition, and add each as indicated in Sec. 73.

91. Add the following:

1.	2.	3.
$786.45	$578.04	$ 16.45
97.08	35.16	8.12
300.90	900.	947.
7.87	80.47	32.76
46.59	570.09	6.58
807.98	98.17	300.
345.56	315.40	97.26
96.	9.98	1.95
4.75	405.56	647.15
400.	58.08	45.
57.09	930.	780.35
815.35	40.76	46.27

92. 1. Give the number of pints in a quart; of quarts in a gallon; of pints in a gallon.

2. How many ounces are there in a pound of sugar? in 5 lb.? How many pounds are there in a ton?

3. There are 2000 lb. in a *short ton*, and 2240 lb. in a *long ton*. A company imported 11,200,000 lb. of coal, paying for it by the long ton. The company sold the coal by the short ton. How many more tons did it sell than it imported?

A long ton (2240 lb.) is used sometimes in weighing coal and in weighing certain materials imported into the United States.

4. In dry measure 2 pints are 1 quart, 8 quarts are 1 peck, and 4 pecks are 1 bushel. How many quarts are there in 1 bu.? in 1 pk. and 3 qt.? in 1 bu. 2 pk.?

5. How many months are there in 1 yr.? in 7 yr.? How many days are there in 1 yr.? in 1 leap year?

6. The depth of the sea is measured in *fathoms*. A fathom is 6 ft. Express 1728 ft. in fathoms.

7. The circumference of a circle is $3\frac{1}{7}$ (3.1416) times its diameter. Show that this is correct by comparing the diameter and the circumference of some circle (top of barrel, pail, stovepipe, etc.).

8. A **denominate number** is a concrete number in which the unit of measure has been established by law or custom, as 4 ft., 12 gal., etc. Such expressions as 10 ft. 6 in., 2 yr. 7 mo. 6 da., etc., are called **compound denominate numbers**.

Tables of denominate numbers are found on pp. 312–319.

9. Mr. Davis's expenses for January, 1907, were $121.45. What were his average daily expenses?

93. 1. The exports for the first ten months of 1906 amounted to \$1,425,184,757, while the exports for the corresponding period in 1905 amounted to \$1,256,924,354. At the same rate of increase, by how much would the exports for 1906 exceed the exports for 1905?

2. The imports for the first ten months of 1906 amounted to \$1,066,462,295, while the imports for the first ten months of 1905 amounted to \$979,917,437. At the same rate of increase, by how much should the imports for 1906 exceed the imports for 1905?

3. The population of Massachusetts was 2,805,346 in 1900. The area of Massachusetts is 8315 sq. mi. Find the average population for each square mile.

4. The area of Texas is 265,780 sq. mi. and of Iowa is 56,025 sq. mi. How many states of the size of Iowa can be made of Texas?

5. Four places, A, B, C, and D, are located on a line running due east and west. B is 16 mi. east of A, C is 12 mi. west of A, and D is 8 mi. west of C. How far apart are B and C? A and D? B and D? (Draw a diagram.)

6. Mr. Wright of Chicago is employed by a wholesale house and receives \$125 per month and necessary expenses while traveling. During the month of January Mr. Wright paid \$59 for railroad fare, \$86 for hotel bills, and \$18.65 for other expenses. How much did the company owe Mr. Wright for the month, including salary?

7. Charles deposited \$2.75 in a savings bank on Oct. 15, and \$3.45 on Oct. 23. He drew out \$4.10 on Oct. 29. He deposited \$4.80 on Dec. 1. How much had he then in the bank?

94. Reading a Railroad Time Table.

SAN FRANCISCO—LOS ANGELES

20 Shore Line Limited	22 The Coaster	18-8 Los Angeles Passenger	10 Sunset Express	Miles From San Francisco	STATIONS	17 San Francisco Passenger	9 Sunset Express	19 Shore Line Limited	21 The Coaster
READ DOWN						READ UP			
8.00	8.30	**3.15**	**8.00**	0	Lv. SAN FRANCISCO Ar.	9.16	10.15	**9.30**	**11.45**
9.25	9.55	**4.45**	**9.30**	51	Lv. SAN JOSE Lv.	7.35	8.45	**8.05**	**10.15**
6.19	**8.15**	4.35	7.30	371	Lv. SANTA BARBARA Lv.	**8.20**	**11.00**	11.15	**12.10**
9.30	**11.45**	8.45	11.00	475	Ar. LOS ANGELES Lv.	**4.00**	**7.30**	8.00	8.30

Light-face figures, A.M.; dark-face, P.M.

95. Answer the following from the above table:

1. How many passenger trains leave San Francisco for Los Angeles each day over this route? How many leave Los Angeles for San Francisco?

2. What is the distance from San Francisco to Los Angeles?

3. What is denoted by the light-face figures? by the dark-face figures?

4. Which is the first train in the morning from San Francisco to Los Angeles? from Los Angeles to San Francisco?

5. How many hours does it take each train to make the run? Which are the fastest trains?

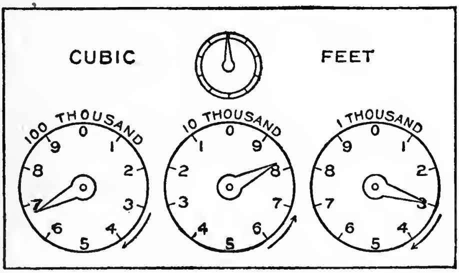
6. Find the average number of miles per hour of train No. 20, of train No. 22, and of the Sunset Express, on the run from San Francisco to Los Angeles, and on the run from Los Angeles to San Francisco.

7. Find the distance from Los Angeles to Santa Barbara; from Los Angeles to San Jose; from San Jose to Santa Barbara.

96. Reading a Meter.

The amount of water, gas, and electricity consumed is usually measured by instruments called **meters.** These instruments are furnished with dials, on which the amounts consumed are indicated in the decimal scale, as shown in the picture.

97. Dials of a Gas Meter.



The unit dial at the top is used for testing the meter.

For every 100 cu. ft. of gas that passes through the meter, the hand on the first (right-hand) dial moves over one of the divisions, as from 0 to 1; for every 1000 cu. ft. consumed, it makes a complete revolution, the hand on the second dial moves over one division, and the hand on the third dial moves over $\frac{1}{10}$ of one division.

Ten revolutions of the hand on any dial produce one revolution of the hand on the dial of the next higher order.

The first dial is now recording 300 cu. ft. How much is the second dial recording? How much are the three dials recording? The dials should be read from left to right as you would read a number, thus: 68,300 cu. ft.

The cost of the gas would be stated for each 1000 cu. ft.

MEASUREMENT OF LENGTH

98. 1. Length and distance are commonly measured in inches, feet, yards, rods, and miles. The yard is the standard unit of length. The other units are derived from it.

2. Draw on the blackboard a line 1 in. long. Draw a line 1 ft. long. Draw a line 1 yd. long. Using a yard stick, test the correctness of your drawings. Practice drawing these lines until you can estimate an inch, a foot, and a yard without much error.

3. Estimate in inches the length and the width of each: your desk top; your book cover; a window pane.

4. Estimate in feet the length and the width of each: your schoolroom; the blackboard; the window; the door.

5. Estimate the length of your room in yards; of your school yard; of the blackboard.

6. A rod is $16\frac{1}{2}$ ft., or $5\frac{1}{2}$ yd. Measure off a rod on the floor of your schoolroom or on the school yard. Estimate the length and width of your school yard in rods.

7. Determine some place that is 1 mi. from your schoolhouse.

99. Table of Linear Measure.

12 inches (in. or ″)	=	1 foot (ft. or ′)
3 ft.	=	1 yard (yd.)
$5\frac{1}{2}$ yd., or $16\frac{1}{2}$ ft.	=	1 rod (rd.)
320 rd., or 5280 ft.	=	1 mile (mi.)

1. How many feet are there in 3 mi.? in 5 mi.?

2. How many rods are there in 2 mi.? in $\frac{1}{4}$ mi.? in $\frac{1}{8}$ mi.?

3. Change to rods: $\frac{1}{2}$ mi., $\frac{1}{4}$ mi., $\frac{1}{8}$ mi.

4. How many inches are there in 1 yd. 6 in.?

5. The lengths of three pieces of blackboard in a schoolroom were measured by the pupils and found to be 18 ft. 6 in., 14 ft. 9 in., and 6 ft. 4 in., respectively. Find the combined length of the three boards.

Model:

18 ft.	6 in.
14 ft.	9 in.
6 ft.	4 in.
39 ft.	7 in.

4 in. and 9 in. and 6 in. are 19 in., or 1 ft. and 7 in. Write 7 in. in the answer as shown in the model, and carry 1 ft. to the column of feet.

6. Find the combined length of the blackboards in your schoolroom.

7. Find the distance around your schoolroom.

8. From 8 ft. 4 in. subtract 4 ft. 10 in.

Model:

8 ft.	4 in.
4 ft.	10 in.
3 ft.	6 in.

As 10 in. are more than 4 in., the sum of 10 in. and the number of inches in the answer is 1 ft. 4 in. Subtract thus: 10 in. and 2 in. are 1 ft.; 2 in. and 4 in. are 6 in. Write 6 in. in the answer as shown in the model. Carry 1 ft. to 4 ft., making 5 ft. 5 ft. and 3 ft. are 8 ft.

9. From a board 9 ft. 6 in. long a carpenter sawed a shelf 3 ft. 10 in. long. How long was the piece of board that was left?

10. From a piece of cloth 4 yd. 8 in. long a woman cut a piece 1 yd. 9 in. long. How long was the piece of cloth that was left?

11. Find how much longer the length of your schoolroom is than its width.

12. On Jan. 1, 1903, a boy's height was 4 ft. 7 in., and on Jan. 1, 1906, it was 5 ft. 2 in. How much taller was he on the second date?

13. How many feet are there in 1 mile? How many yards are there in 1 mile?

MEASUREMENT OF SURFACES

100. 1. The number of square units in any surface is called its **area**.

2. The area of surfaces is commonly measured in square inches, square feet, square yards, square rods, acres, or square miles.

3. Using a ruler, draw on the board a square whose side is 1 foot. This is called a square foot. A **square foot** is a square whose side is 1 foot.

4. Using a ruler, draw upon the board a square whose side is 1 inch. This is called a square inch. A **square inch** is a square whose side is 1 inch.

5. Divide a square foot into square inches. How many square inches are there in 1 square foot?

6. Using a yard stick, draw a square whose side is 1 yard. What is this square called? Divide a square yard into square feet. How many square feet are there in 1 square yard?

7. What is the purpose of having several different units for measuring length and area? In what unit should you express the area of the cover of this book? of the top of your desk? of the surfaces of the walls in your schoolroom?

8. Mark out a square rod on the school yard.

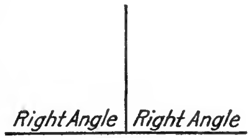
101. Table of Square Measure.

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 sq. ft.	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ sq. yd.	= 1 square rod (sq. rd.)
160 sq. rd.	= 1 acre (A.)
640 A.	= 1 square mile (sq. mi.)

ANGLES

102. 1. The difference in direction of two lines is called an **angle.** The name *angle* is used also to denote the opening between two lines that meet.

2. When two straight lines meet and form two equal angles, the angles are called **right angles**, and the lines are said to be **perpendicular** to each other.



3. The lines that form an angle are called the **sides** of the angle.

4. Angles whose sides are not perpendicular to each other are called **oblique angles.** Oblique angles are either *acute* or *obtuse.*

5. An angle that is less than a right angle is called an **acute angle.**

ACUTE ANGLES.

6. An angle that is greater than a right angle but less than two right angles is called an **obtuse angle.**

OBTUSE ANGLES.

7. Draw a right angle; an obtuse angle; an acute angle.

8. Stand your pencil upon the top of your desk, perpendicular to the desk top. What kind of angles are formed by the pencil and the desk top?

9. Keeping the pencil resting at the same point on the top of the desk, move the top of the pencil to the left. Does the pencil now form any right angles with the desk top? any obtuse angles? any acute angles?

10. Point to surfaces in your schoolroom that meet at right angles. Are there any that meet so as to form obtuse angles? acute angles?

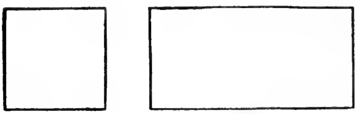
RECTANGLES

103. 1. Lines that extend in the same direction and are everywhere the same distance apart are called **parallel lines.** If parallel lines are extended, will they ever meet?

PARALLEL LINES.

2. Point to two surfaces in the schoolroom that are parallel to each other; that are perpendicular to each other; that meet at right angles. Are there any surfaces in your schoolroom that meet at obtuse or acute angles?

3. A figure having four straight sides and four right angles is called a **rectangle.** Find by drawing figures how many sides a figure must have in order that all its angles may be right angles.



RECTANGLES.

4. A rectangle whose sides are equal is called a **square.**

5. What is a square inch? Draw a square inch; a square foot. What is a square yard? a square mile?

6. All rectangles are either square or oblong. Point to surfaces in the schoolroom that are rectangles.

7. Draw a 2-inch line. Build a square upon it. This is a 2-inch square. Divide it into square inches. How many square inches are there in a 2-inch square?

8. Draw a 3-inch square. Divide it into square inches. How many square inches are there in a 3-inch square?

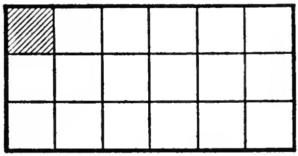
9. Draw a rectangle 2 in. wide and 8 in. long. Divide it into square inches.

104. Area of Rectangles.

1. Draw a rectangle 6 in. long and 1 in. wide. Divide it into square inches.

A rectangle 6 in. long and 1 in. wide contains 6 sq. in.

2. Draw a rectangle 6 in. long and 3 in. wide. Divide it into square inches.



As both dimensions are given in inches, the unit in which the area of the rectangle is to be expressed is 1 sq. in. In a rectangle 6 in. long and 1 in. wide there are 6 times 1 sq. in., or 6 sq. in. Since the given rectangle is 3 in. wide, it contains 3 times 6 sq. in., or 18 sq. in.

Or, a rectangle 3 in. long and 1 in. wide contains 3 times 1 sq. in., or 3 sq. in. Since the given rectangle contains 6 such rectangles, its area is 6 times 3 sq. in., or 18 sq. in.

3. The *number* of square inches in the given rectangle may be found by multiplying 6 by 3. 6 times 3 is 18, the *number* of square inches in the rectangle. Never multiply inches by inches. Why?

105. The following are dimensions of rectangles. State the unit in which the area of each is to be found. Represent 1–4 by drawings. Give the area of each.

1. 8 in. by 5 in.
2. 12 yd. by 9 yd.
3. 8 ft. by 6 ft.
4. 4 ft. by 3 ft.
5. 14 ft. by 10 ft.
6. 20 ft. by 15 ft.
7. 20 rd. by 10 rd.
8. 40 rd. by 40 rd.
9. 6 mi. by $\frac{1}{2}$ mi.
10. 25 ft. by 100 ft.

106. 1. Compare the size of a 2-inch square with 2 square inches; of a 3-inch square with 3 square inches.

2. A 4-inch square is how many times 4 square inches? Compare a 5-inch square with 5 square inches.

3. Draw an inch square. Divide it into 4 equal squares. How long is the side of a square that contains one quarter of a square inch?

4. Draw a square that contains one sixteenth of a square inch.

5. Draw a rectangle containing 18 square inches, making it 6 inches long; 9 inches long.

6. Which is greater, an inch square or a square inch? a half-inch square or one half of a square inch? Show by drawing.

7. How long is the perimeter of a rectangle 6 ft. long and 4 ft. wide? (Perimeter means distance around.)

8. How long is the perimeter of a 9-inch square? of a 4-inch square? of a square inch?

9. How wide is a rectangle that is 8 in. long and contains 8 sq. in.? 16 sq. in.? 24 sq. in.? 32 sq. in.?

10. By what number must 7 sq. in. be multiplied to give 28 sq. in.? If the area of a rectangle is 28 sq. in. and its length is 7 in., how wide is the rectangle?

11. When the area of a rectangle and one dimension are given, how may the other dimension be found? Illustrate with several examples.

12. How many dimensions has a rectangle?

13. A surface that has the same direction throughout, as the surface of a blackboard, a window pane, etc., is called a **plane surface.** The surface of a globe is not a plane surface. Why?

107. In each of the following the area of a rectangle and one dimension are given. Find the other dimension:

1. Area, 20 sq. ft., length, 5 ft.
2. Area, 48 sq. yd., width, 6 yd.
3. Area, 100 sq. in., length, 10 in.
4. Length, 45 ft., area, 900 sq. ft.
5. Width, 50 ft., area, 6500 sq. ft.
6. Area, 1728 sq. in., length, 144 in.

108. 1. Find the area of a garden 10 rd. long and 8 rd. wide.

2. Find the number of acres in a field 40 rd. by 20 rd.

3. At $85 an acre, find the value of a farm 80 rd. by 40 rd.; 160 rd. by 80 rd.

4. A farm containing 80 acres is 80 rd. wide. How long is it?

5. How long is a 10-acre field, if its width is 40 rd.? 20 rd.?

6. How long is a 20-acre field, if its width is 40 rd.? 20 rd.?

109. 1. Find the number of square feet of window space in your schoolroom.

2. Estimate the number of square inches in the cover of this book. Test the correctness of your estimate.

3. Estimate the area of the floor of your schoolroom in square feet. Test the correctness of your estimate.

4. Estimate the number of square rods in your playground. Test the correctness of your estimate.

5. Are there as many as 60 sq. yd. of surface in the ceiling of your schoolroom? Test your answer.

CUBIC MEASURE

110. 1. Describe a rectangle; a square; an oblong. Draw each.

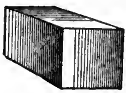
2. How many dimensions has a rectangle? a plane surface?

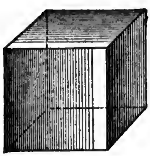
3. How many dimensions has a book? a block? a box?

4. Any object that has length, breadth, and thickness is called a **solid.** Is a book a solid? Name other solids.

5. A solid having six rectangular faces is called a **rectangular solid.** Is a brick a rectangular solid? Name objects which are rectangular solids.

6. A solid having six equal square surfaces is called a **cube.** How many edges has a cube?





RECTANGULAR SOLIDS.

7. A cube whose faces are each a foot square is called a cubic foot. Describe a cubic inch; a cubic yard.

8. How many inch cubes will form a solid 12 in. long, 1 in. wide, and 1 in. thick? 12 in. long, 12 in. wide, and 1 in. thick? 12 in. long, 12 in. wide, and 2 in. thick? 12 in. long, 12 in. wide, and 3 in. thick? 12 in. long, 12 in. wide, and 6 in. thick? 12 in. long, 12 in. wide, and 12 in. thick?

9. What name may be given a solid formed by placing inch cubes 12 deep on a surface 1 ft. square? There are —— cu. in. in 1 cu. ft.

10. How many foot cubes will form a solid 3 ft. long, 3 ft. wide, and 1 ft. thick? 3 ft. long, 3 ft. wide, and 3 ft. thick?

11. How many cubic feet are there in a rectangular solid 8 ft. long, 4 ft. wide, and 4 ft. thick?

As the dimensions are all expressed in feet, the cubic contents will be found in cubic feet. In a rectangular solid having the same base, 8 ft. by 4 ft., but only 1 ft. in height, there are 1 cu. ft. $\times$ 8 $\times$ 4, or 32 cu. ft. Since the given rectangular solid is 4 ft. thick, it contains 4 times 32 cu. ft., or 128 cu. ft. 1 cu. ft. $\times 8 \times 4 \times 4 = 128$ cu. ft.

When all the dimensions of a rectangular solid are expressed in like units, the contents of the solid may be found by multiplying the *number* of units in the length by the *number* of the units in the width and the product by the *number* of units in the thickness, and calling the result cubic units of the given dimension. Thus, the number of cubic feet in a rectangular solid 8 ft. by 4 ft. by 4 ft. is $8 \times 4 \times 4$, or 128.

12. A pile of wood 8 ft. long, 4 ft. wide, and 4 ft. high is called a **cord** of wood. How many cubic feet are there in a cord of wood?

13. The number of cubic units in a solid is called its **volume**.

111. Table of Cubic Measure.

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cu. ft. = 1 cubic yard (cu. yd.)
128 cu. ft. = 1 cord of wood (cd.)

Find the volume of rectangular solids of the following dimensions:

1. $6' \times 4' \times 2'$
2. $10' \times 8' \times 5'$
3. $9'' \times 6'' \times 4''$
4. $12'' \times 9'' \times 8''$
5. $14'' \times 12'' \times 10''$
6. $24' \times 18' \times 16'$

7. Find the number of cubic feet of air in a room 14′ by 12′ by 9′.

8. If each pupil requires 35 cu. ft. of fresh air per minute, how many cubic feet of fresh air per minute will 50 pupils require?

9. Find the number of cubic feet of air in a schoolroom $32' \times 24' \times 12'$.

10. A watering trough is 12 ft. long, 2 ft. wide, and 2 ft. deep. How many gallons will it contain? (231 cu. in. = 1 gal.)

11. Find the number of measured bushels in a bin 6 ft. by 4 ft. by 4 ft. (2150.42 cu. in. = 1 bu.)

12. The length of a rectangular solid is 8 ft., and its width is 5 ft. If its volume is 160 cu. ft., what is its thickness?

13. How many cubic yards of dirt must be removed in excavating a cellar 36 ft. long, 24 ft. wide, and 8 ft. deep? Find the cost of making this excavation at 30¢ per cubic yard.

14. How many cubic yards of dirt must be removed in digging a trench for a sewer, if the trench is 3 ft. wide, 6 ft. deep, and 120 ft. long?

15. A contractor's bid for excavating a basement 60 ft. by 36 ft. and 9 ft. in depth is $216. How much is this per cubic yard?

16. How many cords of wood in a pile 24 ft. long, 4 ft. wide, and 4 ft. high?

17. How many cords of wood in a pile 8 ft. long, 2 ft. wide, and 4 ft. high?

18. At $6 per cord, find the cost of a pile of wood 20 ft. long, 4 ft. wide, and 6 ft. high.

19. State how you would find the capacity of a box car.

DIVISIBILITY OF NUMBERS

112. 1. Count by 2's to 20. Numbers that are exactly divisible by 2 are called **even numbers.** All numbers ending in 0, 2, 4, 6, and 8 are even numbers. Name the even numbers from 40 to 60.

2. Numbers that are not exactly divisible by 2 are called **odd numbers.** Name the odd numbers from 20 to 40.

3. Which of the following are even numbers: 18, 27, 31, 46, 50, 65, 123, 2456?

4. Some numbers are not exactly divisible by any whole number except themselves and 1. Such numbers are called **prime numbers.** Write the prime numbers below 80. Numbers that are exactly divisible by whole numbers other than themselves and 1 are called **composite numbers.**

5. The factors of 15 are 3 and 5. These two numbers when multiplied together give 15. Some numbers have many factors. 2, 3, 4, 6, 8, and 12 are each a factor of 24. By what must each be multiplied to give 24?

A number which when multiplied by another number makes a given number is called a **factor** of the given number.

6. Name the factors of 6; of 12; of 16; of 30; of 36. Has a prime number factors?

7. The **prime factors** of a number are the prime numbers which when multiplied together make the number. The *prime factors* of 24 are 2, 2, 3, 2.

8. Name the prime factors of 18; of 20; of 36.

9. A factor is always an exact measure of a number. What numbers are exact measures of 21? of 30? of 16? of 12? of 48? Is 24 an exact measure of 48?

10. All the prime factors of a number may be found by dividing the number by one of its prime factors, and dividing each quotient in turn by one of its prime factors until the quotient is a prime number. The prime factors of 60 may be found by dividing 60 by 2, and dividing the quotient (30) by 2, and this quotient (15) by 3. The last quotient (5) is a prime number. The prime factors of 60 are the divisors, 2, 2, 3, and the last quotient, 5.

11. Find the prime factors of 48; of 72; of 80.

12. Which of the following are prime numbers: 27, 17, 39, 51, 29, 91, 53, 89, 77, 57?

13. Count by 5's to 35, beginning with 5. Numbers ending in 5 and in 0 are exactly divisible by 5.

14. What numbers are exactly divisible by 10? by 2? by 5?

15. Write a number the sum of whose digits is 3. Is the number exactly divisible by 3? Is 51 exactly divisible by 3?

16. Write a number the sum of whose digits is 6, 9, 12, or some other multiple of 3. Is the number exactly divisible by 3? Show by several illustrations that the following statement is correct:

A number is exactly divisible by 3 *if the sum of its digits is exactly divisible by* 3.

17. Which of the following are exactly divisible by 3: 54, 177, 81, 52, 819, 57, 69, 71, 213, 105, 86, 1612?

18. Write a number the sum of whose digits is divisible by 9.

A number is exactly divisible by 9 *if the sum of its digits is exactly divisible by* 9.

19. Which of the following are exactly divisible by 9: 54, 504, 522, 711, 827, 218, 745, 891, 5375, 457?

20. Which of the following are exactly divisible by 2? by 3? by 5? by 9? by 10? 45, 61, 360, 207, 783, 53, 540, 117, 102, 107, 37, 97, 201, 855, 732, 380, 4320, 105?

21. A number that is exactly divisible by 9 is exactly divisible by 3. Why?

22. All multiples of even numbers are even numbers. Why?

23. The number denoted by the two right-hand figures of 216 is 16. Will 4 exactly divide 16? 200? 216?

24. Write a number of three or more places in which the number denoted by the two right-hand figures is some multiple of 4. Is the number exactly divisible by 4?

A number is exactly divisible by 4 *if the number denoted by its two right-hand figures is exactly divisible by* 4.

25. Which of the following are exactly divisible by 4: 112, 202, 420, 532, 514?

26. Centennial years that are divisible by 400 (1200, 1600, etc.) and other years divisible by 4 are leap years. Which of the following will be leap years: 1910, 1912, 1908, 1926, 1924, 1960, 1990, 2000, 2100?

27. Write 10 numbers that are exactly divisible by 3.

113. The three pairs of factors of 24 are 2 and 12, 3 and 8, 4 and 6. Name all the pairs of factors of each of the following, naming no factor larger than 20:

4	15	25	34	44	56	77	100
6	16	26	35	45	57	80	108
8	18	27	36	48	60	81	110
9	20	28	38	49	64	84	120
10	21	30	39	50	66	90	121
12	22	32	40	51	70	96	132
14	24	33	42	54	72	99	144

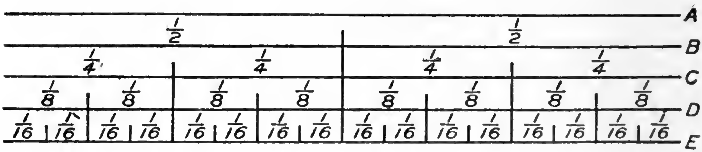
PART II

FRACTIONS

114. **1.** Draw a line 4 inches long. Divide it into two equal parts. What is each part called?

2. If 6 pupils are separated into three equal groups, what part of the pupils will each group contain?

3. Draw a line. Divide it into eight equal parts. What is each part called?



4. *A* represents the line undivided. *B* represents the line as divided into two equal parts. What does *C* represent? *D*? *E*? What name is given to each part of the line in *B*? in *C*? in *D*? in *E*? In each case, how many of the parts does it take to equal the entire line?

5. The length of the line is represented in turn by 1, $\frac{2}{2}$, $\frac{4}{4}$, $\frac{8}{8}$, and $\frac{16}{16}$. $\frac{1}{2}$ of the line $= \frac{x}{4} = \frac{x}{8} = \frac{x}{16}$ of the line.

6. $\frac{3}{4}$ of the line $= \frac{x}{8} = \frac{x}{16}$ of the line.

7. $\frac{1}{2}$ of the line $+$ $\frac{1}{4}$ of the line $= \frac{x}{4}$ of the line. $\frac{1}{4}$ of the line $-$ $\frac{1}{8}$ of the line $= \frac{x}{8}$ of the line.

8. What is the sum of $\frac{1}{2}$ of the line and $\frac{1}{8}$ of the line? What is the difference between $\frac{1}{4}$ of the line and $\frac{1}{2}$ of the line? $\frac{1}{2}$ of the line and $\frac{3}{4}$ of the line?

9. $\frac{3}{8}$ of the line is longer than $\frac{x}{4}$ of the line. $\frac{9}{16}$ of the line is longer than $\frac{x}{8}$ of the line. $\frac{x}{4}$ of the line is longer than $\frac{x}{8}$ of the line.

10. Using 8 objects, show that $\frac{1}{2}$ of 8 objects is the same as $\frac{2}{4}$ of 8 objects, and that $\frac{3}{4}$ of 8 objects is the same as $\frac{6}{8}$ of 8 objects.

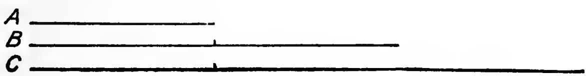
11. Show by dividing circles that $\frac{1}{2}$ of a circle is equal to $\frac{2}{4}$ of a circle; that $\frac{3}{4}$ of a circle is equal to $\frac{6}{8}$ of a circle; that $\frac{1}{2}$ of a circle plus $\frac{1}{8}$ of a circle is equal to $\frac{5}{8}$ of a circle; that $\frac{3}{4}$ of a circle is equal to $\frac{12}{16}$ of a circle.

12. Show by dividing rectangles that $\frac{1}{3}$, $\frac{2}{6}$, $\frac{3}{9}$, $\frac{4}{12}$, and $\frac{5}{15}$ of a rectangle are equivalent parts.

13. Using objects, show that $\frac{1}{2}$ of 12 objects is the same as $\frac{3}{6}$ of 12 objects; that $\frac{1}{3}$ of 12 objects is the same as $\frac{2}{6}$ of 12 objects.

14. Show by folding paper that $\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16}$; that $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$.

115. Ratio.

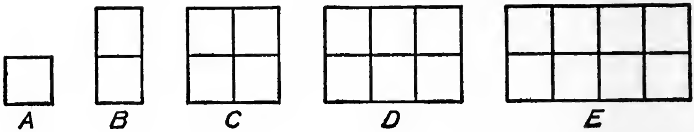


1. Line A is what part of line B? what part of line C?

2. If B is called 1, what is A? C? If C is called 1, what is A? B? If C is called 6, what is A? B?

3. If A is called 3, what is B? C? If A is called $\frac{1}{4}$, what is B? C?

4. The ratio of line A to line B is $\frac{1}{2}$. What is the ratio of line A to line C? of line B to line A? of line B to line C? of line C to line B? of line C to line A?



116. 1. The surface A is what part of the surface B? of C? of D? of E?

2. The ratio of A to B is ——; of A to C is ——; of A to D is ——; of A to E is ——.

3. The surface B is what part of the surface C? of D? of E? The ratio of B to C is ——; of B to D is ——; of B to E is ——.

4. What is the ratio of C to E? If C represents 40 A, what does E represent?

5. What is the ratio of B to A? of C to A? of D to A? of E to A? of C to B? of E to C? of D to B? of E to B?

6. The ratio of C to D is $\frac{4}{6}$, or $\frac{2}{3}$; of D to C is $\frac{6}{4}$, or $\frac{3}{2}$.

7. What is the ratio of D to E? of E to D? of E to C?

8. If A represents 10 acres, what does B represent? C? D? E?

9. If B represents 40 acres, what does A represent? C? D? E?

10. If the cost of the land represented by B is $100, what is the cost of the land represented by A? C? D? E?

11. If the area represented by E is 640 acres, what is the area represented by C? B? A? D?

12. Draw two lines such that the ratio of one to the other is $\frac{1}{6}$; $\frac{1}{4}$; 2; 5.

13. Draw oblongs such that the ratio of one to the other is $\frac{2}{3}$; $\frac{1}{3}$; $\frac{4}{5}$; 3; $\frac{3}{2}$.

117. **1.** The unit of 3 is 1; of 3 da. is 1 da.; of 3 mi. is 1 mi.

2. The unit 1 mi. may be regarded as composed of equal parts, as of 2 half miles, of 4 quarter miles, of 8 eighth miles, etc. If the unit 1 mi. is regarded as composed of 4 equal parts, each part is expressed as $\frac{1}{4}$ mi.; 3 such parts are expressed as $\frac{3}{4}$ mi. A unit may be regarded as composed of 2 or more equal parts.

3. A **fraction** is one or more of the equal parts of a unit, as $\frac{1}{4}$, $\frac{3}{4}$, etc.

4. In the fraction $\frac{3}{4}$, 4 is the **denominator.** It shows the number of equal parts into which the unit has been divided. It *names* the equal parts. 3 is the **numerator.** It shows the number of the equal parts of the unit that have been taken to make the fraction $\frac{3}{4}$. $\frac{3}{4}$ denotes 3 of the 4 equal parts of the unit 1.

5. When a unit is divided into two or more equal parts, each of these parts becomes in turn a unit. Such a unit is called a **fractional unit.** $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., are fractional units. The unit of $\frac{4}{5}$ is $\frac{1}{5}$. What is the unit of each of the following: $\frac{3}{4}$, $\frac{6}{7}$, $\frac{8}{9}$, $\frac{2}{3}$ yd., $\frac{5}{12}$ yr.?

6. Draw a line 1 ft. long. Divide it into 4 equal parts. Show the part that is expressed by $\frac{1}{4}$ ft.; by $\frac{2}{4}$ ft.; by $\frac{3}{4}$ ft. The ratio of 1 part of the line to the whole line is $\frac{1}{4}$. What is the ratio of 2 parts of the line to the whole line? of 3 parts? What is the ratio of the line to 1 part? to 2 parts? to 3 parts?

7. Draw a line 8 in. long. Let it represent 1 mi. Show the part that represents $\frac{3}{8}$ mi.; $\frac{5}{8}$ mi.; $\frac{8}{8}$ mi. Show the part whose ratio to the whole line is $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{8}$, $\frac{3}{4}$. Show the part to which the ratio of the whole line is 2; 8; 4; $\frac{8}{3}$; $\frac{4}{3}$; $\frac{8}{7}$.

118. 1. $\frac{4}{5}$, $\frac{3}{7}$ wk., $\frac{2}{3}$ yd., $\frac{5}{8}$ gal., $\frac{11}{12}$ yr., $\frac{8}{9}$, $\frac{7}{16}$ lb., $\frac{3}{4}$, $\frac{3}{8}$ mi., $\frac{7}{10}$, $\frac{9}{12}$.

a. Read aloud each of the above fractions.

b. Tell into how many parts the unit in each has been divided.

c. Name the unit in which each is expressed.

d. Tell how many of these parts are expressed in each fraction.

e. Read the denominator of each fraction.

f. Read the numerator of each fraction.

g. Draw a line to represent the unit. Mark on this line the parts expressed in each fraction.

2. The numerator and denominator are called the **terms** of the fraction.

3. A fraction whose numerator is less than the denominator is called a **proper fraction**, as $\frac{3}{7}$, $\frac{4}{9}$, etc. Name ten proper fractions.

4. A fraction whose numerator is equal to or greater than the denominator is called an **improper fraction**, as $\frac{3}{3}$, $\frac{5}{4}$, etc. Name ten improper fractions.

5. When a number is composed of an integer and a fraction, it is called a **mixed number**. $6\frac{4}{5}$ is a mixed number. Its value is expressed in two different units. The 6 is expressed in units of *ones;* the $\frac{4}{5}$ is expressed in units of *one fifths.* Name ten mixed numbers.

6. The value of 1, expressed in the fractional unit $\frac{1}{5}$, is $\frac{5}{5}$; of 2 is $\frac{10}{5}$; of 3 is $\frac{x}{5}$; of 4 is $\frac{x}{5}$; of 5 is $\frac{x}{5}$; of 8 is $\frac{x}{5}$.

7. What kind of a number is $5\frac{3}{4}$? In what unit is 5 expressed? In what unit is $\frac{3}{4}$ expressed? The value of $5\frac{3}{4}$ may be expressed in the fractional unit $\frac{1}{4}$. There are $\frac{4}{4}$ in 1. In 5 there are 5 times $\frac{4}{4}$, or $\frac{20}{4}$. $\frac{20}{4}$ and $\frac{3}{4}$ are $\frac{23}{4}$. What kind of a fraction is $\frac{23}{4}$?

REDUCTION

119. Changing Mixed Numbers to Improper Fractions.

Change $4\frac{3}{5}$ to an improper fraction.

MODEL: 5 times 4 is 20; 20 and 3 are 23; write 23 over the denominator, thus: $\frac{23}{5}$.

To change a mixed number to an improper fraction, multiply the integer by the denominator of the fraction, add the numerator, and write the sum over the denominator of the fraction.

120. Oral Exercises.

Change the following to improper fractions: *

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*
1.	$6\frac{1}{2}$	$7\frac{8}{9}$	$8\frac{6}{7}$	$9\frac{7}{8}$	$5\frac{3}{5}$	$4\frac{6}{7}$	$3\frac{4}{9}$	$2\frac{3}{4}$	$1\frac{7}{9}$	$8\frac{4}{9}$	$8\frac{5}{6}$
2.	$9\frac{3}{4}$	$5\frac{5}{9}$	$7\frac{5}{7}$	$8\frac{5}{8}$	$6\frac{5}{6}$	$9\frac{1}{6}$	$4\frac{7}{9}$	$2\frac{5}{9}$	$3\frac{2}{3}$	$3\frac{5}{9}$	$7\frac{7}{8}$
3.	$5\frac{5}{6}$	$5\frac{1}{4}$	$1\frac{8}{9}$	$9\frac{1}{2}$	$2\frac{3}{8}$	$8\frac{4}{5}$	$3\frac{7}{8}$	$7\frac{5}{6}$	$4\frac{7}{8}$	$6\frac{3}{5}$	$5\frac{6}{7}$
4.	$2\frac{5}{8}$	$9\frac{3}{5}$	$7\frac{4}{7}$	$4\frac{5}{6}$	$7\frac{2}{3}$	$5\frac{3}{4}$	$6\frac{3}{4}$	$3\frac{5}{6}$	$8\frac{2}{3}$	$2\frac{3}{7}$	$9\frac{7}{9}$
5.	$2\frac{8}{9}$	$9\frac{5}{7}$	$7\frac{2}{5}$	$6\frac{2}{3}$	$3\frac{7}{9}$	$7\frac{7}{9}$	$6\frac{5}{7}$	$4\frac{4}{9}$	$2\frac{4}{9}$	$7\frac{6}{7}$	$3\frac{4}{5}$
6.	$6\frac{4}{9}$	$2\frac{5}{12}$	$3\frac{8}{9}$	$5\frac{7}{8}$	$9\frac{2}{3}$	$3\frac{7}{12}$	$9\frac{9}{12}$	$7\frac{7}{12}$	$5\frac{4}{12}$	$8\frac{9}{12}$	$6\frac{8}{12}$
7.	$4\frac{7}{12}$	$8\frac{9}{11}$	$9\frac{9}{11}$	$3\frac{5}{7}$	$7\frac{3}{4}$	$6\frac{8}{11}$	$7\frac{9}{11}$	$5\frac{8}{11}$	$7\frac{3}{11}$	$4\frac{6}{11}$	$5\frac{9}{11}$

8. Write ten mixed numbers and change them to improper fractions.

9. Express the value of the following integers in the fractional unit $\frac{1}{4}$: 3, 5, 7, 6, 9, 2, 8, 10, 12.

10. Write ten proper fractions. State what the fractional unit is in each.

11. Change to improper fractions: $3\frac{2}{3}$ yd., $4\frac{3}{4}$ in., $8\frac{1}{2}$ mi.

* This exercise contains practically all the combinations in addition and multiplication. It should be used frequently as a review exercise.

121. Changing Improper Fractions to Whole or Mixed Numbers.

1. What kind of a fraction is $\frac{20}{4}$? What is the unit in which its value is expressed? How many of these fractional units does it take to make the unit 1? How many units of 1 are there in $\frac{20}{4}$? in $\frac{5}{4}$? in $\frac{12}{4}$? in $\frac{13}{4}$? in $\frac{15}{4}$?

2. What does the denominator of a fraction show? Which term of the fraction tells the number of the fractional units it takes to make a unit?

To change an improper fraction to a whole or a mixed number, divide the numerator by the denominator.

122. Oral Exercises.

Change the following to whole or mixed numbers: *

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*
1.	$\frac{55}{8}$	$\frac{71}{9}$	$\frac{62}{7}$	$\frac{79}{8}$	$\frac{28}{5}$	$\frac{34}{7}$	$\frac{31}{9}$	$\frac{11}{4}$	$\frac{16}{9}$	$\frac{76}{9}$	$\frac{53}{6}$
2.	$\frac{39}{4}$	$\frac{54}{9}$	$\frac{54}{7}$	$\frac{69}{8}$	$\frac{41}{6}$	$\frac{55}{6}$	$\frac{43}{9}$	$\frac{23}{9}$	$\frac{11}{3}$	$\frac{32}{9}$	$\frac{63}{8}$
3.	$\frac{35}{6}$	$\frac{21}{4}$	$\frac{17}{9}$	$\frac{19}{2}$	$\frac{19}{8}$	$\frac{44}{5}$	$\frac{31}{8}$	$\frac{47}{6}$	$\frac{39}{8}$	$\frac{33}{5}$	$\frac{41}{7}$
4.	$\frac{21}{8}$	$\frac{48}{5}$	$\frac{53}{7}$	$\frac{29}{6}$	$\frac{23}{3}$	$\frac{23}{4}$	$\frac{27}{4}$	$\frac{23}{6}$	$\frac{26}{3}$	$\frac{17}{7}$	$\frac{88}{9}$
5.	$\frac{26}{9}$	$\frac{68}{7}$	$\frac{37}{5}$	$\frac{20}{3}$	$\frac{34}{9}$	$\frac{70}{9}$	$\frac{47}{7}$	$\frac{40}{9}$	$\frac{22}{9}$	$\frac{55}{7}$	$\frac{19}{5}$
6.	$\frac{58}{9}$	$\frac{29}{12}$	$\frac{35}{9}$	$\frac{47}{8}$	$\frac{29}{3}$	$\frac{43}{12}$	$\frac{115}{12}$	$\frac{91}{12}$	$\frac{64}{12}$	$\frac{105}{12}$	$\frac{80}{12}$
7.	$\frac{43}{12}$	$\frac{97}{11}$	$\frac{108}{11}$	$\frac{26}{7}$	$\frac{31}{7}$	$\frac{74}{11}$	$\frac{86}{11}$	$\frac{63}{11}$	$\frac{80}{11}$	$\frac{50}{11}$	$\frac{64}{11}$

8. Write ten improper fractions and change them to whole or mixed numbers.

9. Write ten mixed numbers and change them to improper fractions.

* This exercise contains nearly all of the facts of division and subtraction. It should be used frequently as a review exercise.

ADDITION AND SUBTRACTION OF FRACTIONS

123. Oral Exercises.

1. What is the sum of 2 books and 3 books and 1 book? Are these quantities expressed in the same unit of measure?

2. Name three quantities that are not expressed in the same unit of measure. Can their sum be found?

3. Are the following fractions expressed in the same unit of measure: $\frac{3}{5}$, $\frac{2}{5}$, $\frac{1}{5}$? Fractions that are expressed in the same unit of measure are said to be **similar fractions.** Similar fractions have the same denominator. Only fractions that are expressed in the same unit of measure can be added.

4. The sum of $\frac{1}{4}$, $\frac{3}{4}$, $\frac{2}{4}$, and $\frac{3}{4}$ is $\frac{9}{4}$, which is equal to $2\frac{1}{4}$.

5. Add $2\frac{2}{3}$ ft., $5\frac{1}{3}$ ft., and $6\frac{1}{3}$ ft.

MODEL:

$$\begin{array}{r} 2\frac{2}{3}\text{ ft.} \\ 5\frac{1}{3}\text{ ft.} \\ 6\frac{1}{3}\text{ ft.} \\ \hline 14\frac{1}{3}\text{ ft.} \end{array}$$

Add the fractions first: $\frac{1}{3}$ ft. and $\frac{1}{3}$ ft. and $\frac{2}{3}$ ft. are $\frac{4}{3}$ ft., which are equal to $1\frac{1}{3}$ ft. Write $\frac{1}{3}$ ft. below the column of fractions, and carry 1 ft. to the column of whole numbers.

6. Add the following:

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
$6\frac{1}{4}$	3	$3\frac{4}{5}$	$4\frac{2}{7}$	$4\frac{1}{2}$	$5\frac{1}{8}$	$6\frac{2}{9}$	$7\frac{4}{10}$	$5\frac{2}{12}$
8	$7\frac{1}{3}$	$9\frac{2}{5}$	$9\frac{4}{7}$	$7\frac{1}{2}$	6	$7\frac{2}{9}$	5	$6\frac{10}{12}$
$3\frac{3}{4}$	$8\frac{2}{3}$	4	$6\frac{5}{7}$	$8\frac{1}{2}$	$8\frac{7}{8}$	$4\frac{7}{9}$	$6\frac{9}{10}$	$8\frac{6}{12}$
$7\frac{1}{4}$	$4\frac{1}{3}$	$7\frac{1}{5}$	2	$9\frac{1}{2}$	$4\frac{5}{8}$	$8\frac{5}{9}$	$4\frac{1}{10}$	$7\frac{7}{12}$
$5\frac{1}{4}$	$9\frac{2}{3}$	$8\frac{3}{5}$	$8\frac{1}{7}$	$6\frac{1}{2}$	$2\frac{1}{8}$	$2\frac{1}{9}$	$3\frac{7}{10}$	$3\frac{5}{12}$
$4\frac{3}{4}$	$6\frac{2}{3}$	$5\frac{4}{5}$	$7\frac{6}{7}$	$5\frac{1}{2}$	$7\frac{3}{8}$	$9\frac{7}{9}$	$8\frac{9}{10}$	$4\frac{11}{12}$
8	$2\frac{1}{3}$	$6\frac{3}{5}$	$9\frac{5}{7}$	$8\frac{1}{2}$	$7\frac{7}{8}$	$6\frac{5}{9}$	$7\frac{3}{10}$	$9\frac{5}{12}$

124. Oral Exercises.

1. $\frac{3}{4}$ ft. $-$ $\frac{1}{4}$ ft. $=$ $\frac{x}{4}$ ft. $\frac{7}{8}$ mi. $-$ $\frac{5}{8}$ mi. $=$ $\frac{x}{8}$ mi. $\frac{11}{12}$ yr. $-$ $\frac{7}{12}$ yr. $=$ $\frac{x}{12}$ yr. $\frac{5}{6}$ da. $-$ $\frac{1}{6}$ da. $=$ $\frac{x}{6}$ da.

2. $6\frac{2}{3}$ ft. $-$ 4 ft. $=$ —— ft. $\$8\frac{4}{5} - \$5 = \$$——. $4\frac{3}{4}$ mi. $-$ 2 mi. $=$ —— mi.

3. Subtract the fractions first and then the whole numbers:

a	*b*	*c*	*d*	*e*
$8\frac{4}{7}$ wk.	$\$9\frac{4}{5}$	$5\frac{2}{3}$ yd.	$5\frac{5}{6}$ yr.	$6\frac{7}{8}$ lb.
$1\frac{3}{7}$ wk.	$\$4\frac{3}{5}$	$3\frac{2}{3}$ yd.	$2\frac{1}{6}$ yr.	$4\frac{5}{8}$ lb.

f	*g*	*h*	*i*	*j*
$8\frac{2}{7}$ wk.	$7\frac{7}{12}$ in.	$\$9\frac{3}{5}$	$24\frac{5}{9}$ yd.	$29\frac{7}{9}$ yd.
5 wk.	$3\frac{5}{12}$ in.	$\$7\frac{1}{5}$	$17\frac{4}{9}$ yd.	$19\frac{5}{9}$ yd.

4. Find the sum of each of the above.

5. Subtract $3\frac{1}{3}$ ft. from 6 ft.

MODEL:

6 ft.
$3\frac{1}{3}$ ft.
$2\frac{2}{3}$ ft.

Since there is no fractional part in the minuend, the sum of the fraction of the subtrahend and the fraction of the difference is 1 ft. $\frac{1}{3}$ ft. and $\frac{2}{3}$ ft. are 1 ft. Carry 1 ft. to 3 ft. and subtract the integers. 4 ft. and 2 ft. are 6 ft.

6. Subtract:

8 hr.	7 da.	$\$9$	$\$6$	5 ft.	8 lb.	9 mi.
$4\frac{3}{4}$ hr.	$2\frac{1}{2}$ da.	$\$3\frac{4}{5}$	$\$2\frac{3}{4}$	$1\frac{2}{3}$ ft.	$3\frac{1}{2}$ lb.	$6\frac{3}{8}$ mi.

7. Subtract:

$7\frac{2}{3}$ yd.	$9\frac{3}{4}$ yr.	8 A.	$6\frac{7}{8}$ mi.	$5\frac{4}{7}$ wk.	6 yd.	$8\frac{3}{4}$ hr.
$2\frac{1}{3}$ yd.	6 yr.	$4\frac{1}{2}$ A.	$3\frac{3}{8}$ mi.	$3\frac{2}{7}$ wk.	$3\frac{2}{3}$ yd.	4 hr.

8. If a boy attended school $3\frac{3}{4}$ da. in a certain week, how many days was he absent?

9. A girl who was taking lessons on the piano practiced as follows during one week: Monday, $1\frac{1}{4}$ hr.; Tuesday morning, 1 hr.; Tuesday afternoon, $\frac{3}{4}$ hr.; Wednesday, $1\frac{3}{4}$ hr.; Thursday, $\frac{3}{4}$ hr.; Friday, $1\frac{1}{4}$ hr.; Saturday morning, $1\frac{3}{4}$ hr.; Saturday afternoon, $\frac{3}{4}$ hr. How many hours did she practice during the week?

10. A boy had 10 mi. to travel. If he traveled $3\frac{3}{4}$ mi. on foot and rode the remainder of the distance, how far did he ride?

125. Oral Exercises.

1. Subtract $6\frac{4}{5}$ from $9\frac{2}{5}$.

MODEL:

$$\begin{array}{r} 9\frac{2}{5} \\ \underline{6\frac{4}{5}} \\ 2\frac{3}{5} \end{array}$$

The sum of $\frac{4}{5}$ and the fraction of the difference is $1\frac{2}{5}$. Find what must be added to $\frac{4}{5}$ to make 1 and add it to $\frac{2}{5}$. $\frac{4}{5}$ and $\frac{1}{5}$ are 1. $\frac{1}{5}$ and $\frac{2}{5}$ are $\frac{3}{5}$. Carry 1 to 6. 7 and 2 are 9.

The numerator of the fraction in the difference may be found by subtracting 4 (the numerator of the fraction in the subtrahend) from 5 (the denominator of the fraction in the minuend), and adding 2 (the numerator of the fraction in the minuend). Explain why this method will give the correct result. Use this method in subtracting.

2. Subtract without the use of a pencil:

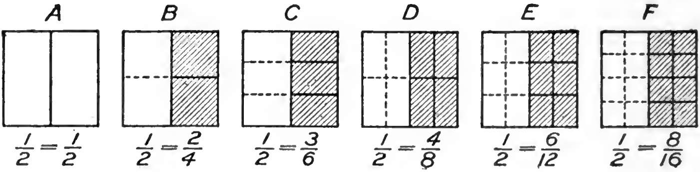
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
$5\frac{3}{8}$	$9\frac{3}{7}$	$7\frac{2}{5}$	$8\frac{7}{9}$	$9\frac{11}{17}$	$7\frac{9}{13}$	$8\frac{15}{19}$	$6\frac{15}{17}$	$9\frac{3}{5}$
$2\frac{5}{8}$	$6\frac{6}{7}$	$3\frac{4}{5}$	$3\frac{8}{9}$	$4\frac{15}{17}$	$5\frac{11}{13}$	$3\frac{17}{19}$	$4\frac{16}{17}$	$7\frac{4}{5}$
$6\frac{5}{11}$	$8\frac{1}{5}$	$9\frac{3}{8}$	$7\frac{11}{15}$	$9\frac{1}{8}$	$8\frac{7}{19}$	$9\frac{7}{10}$	$6\frac{5}{12}$	$8\frac{11}{16}$
$3\frac{9}{11}$	$5\frac{4}{5}$	$6\frac{7}{8}$	$3\frac{13}{15}$	$1\frac{3}{8}$	$2\frac{11}{19}$	$4\frac{9}{10}$	$4\frac{7}{12}$	$3\frac{15}{16}$

3. Find the sum of each of the above exercises.

4. A dressmaker had two pieces of cloth containing $8\frac{3}{4}$ yd. and $6\frac{3}{4}$ yd., respectively. She used $10\frac{1}{4}$ yd. in making a dress. How much cloth was left?

REDUCTION

126. Changing to Higher and Lower Terms.



1. If A represents a unit divided into 2 equal parts, what does B represent? C? D? E? F?

2. The fractional unit of B is $\frac{1}{4}$; what is the fractional unit of C? D? E? F? What part of the fractional unit of A is the fractional unit of B? C? D? E? F? How many of the fractional units of B does it take to make one of the fractional units of A? How many of C? of D? of E? of F?

3. The fractional unit $\frac{1}{8}$ is what part of the fractional unit $\frac{1}{4}$? $\frac{1}{4} = \frac{x}{8}$. $\frac{1}{8}$ is what part of $\frac{1}{2}$?

4. The denominator of the fractional unit $\frac{1}{16}$ is 2 times the denominator of the fractional unit $\frac{1}{8}$. It shows that the unit has been divided into twice as many equal parts. It will therefore take 2 of the fractional units *sixteenths* to make one of the fractional units *eighths*. $\frac{x}{16} = \frac{6}{8}$.

5. The fractions $\frac{1}{2}$, $\frac{2}{4}$, $\frac{4}{8}$, $\frac{8}{16}$ are the same in *value*. They differ in *form*. Changing the form of a fraction without changing its value is called **reduction**.

6. The fraction $\frac{4}{8}$ is equal to the fraction $\frac{8}{16}$. Compare their numerators. 8 is —— times 4. Compare their denominators. 16 is —— times 8. How may $\frac{8}{16}$ be derived from $\frac{4}{8}$? How may $\frac{4}{8}$ be derived from $\frac{8}{16}$?

7. Compare in a similar way the terms of the fractions $\frac{3}{6}$ and $\frac{6}{12}$; $\frac{1}{2}$ and $\frac{5}{10}$; $\frac{2}{3}$ and $\frac{8}{12}$. What effect upon the value of a fraction has multiplying both terms by the same number?

8. A fraction is an indicated division. $\frac{3}{6}$ is the same as $6\overline{)3}$. The denominator of the fraction is the divisor, and the numerator is the dividend. What effect upon the quotient has multiplying both the *dividend* and the *divisor* by the same number? Is multiplying both the numerator and the denominator of a fraction by the same number the same as multiplying both the dividend and the divisor by the same number? $\frac{3}{4}=\frac{6}{8}=\frac{x}{12}=\frac{x}{16}$.

9. The fraction $\frac{10}{16}$ is equal to the fraction $\frac{5}{8}$. Compare their numerators. 5 is what part of 10? Compare their denominators. 8 is what part of 16? Compare in a similar way $\frac{9}{12}$ with $\frac{3}{4}$; $\frac{6}{10}$ with $\frac{3}{5}$. What effect upon the value of a fraction has dividing both terms by the same number? $\frac{8}{16}=\frac{2}{8}=\frac{x}{4}=\frac{x}{2}$.

10. What effect upon the quotient has dividing both dividend and divisor by the same number? Is dividing both numerator and denominator of a fraction by the same number the same as dividing both dividend and divisor by the same number?

Multiplying or dividing both terms of a fraction by the same number does not alter the value of the fraction.

11. Change the form of the following without changing their value: $\frac{2}{3}$, $\frac{8}{12}$, $\frac{5}{7}$, $\frac{12}{16}$, $\frac{7}{9}$, $\frac{10}{25}$, $\frac{20}{30}$, $\frac{9}{15}$.

12. By what must the terms of the fraction $\frac{3}{5}$ be multiplied to reduce the fraction to 10ths? to 15ths? to 20ths?

13. How many 12ths are there in 1? in $\frac{1}{4}$? in $\frac{3}{4}$? in $\frac{4}{4}$?

127. Written Exercises.

1. Change $\frac{2}{3}$ to 12ths. As the denominator 12 is 4 times the denominator of $\frac{2}{3}$, the numerator of the required fraction must be 4 times the numerator of $\frac{2}{3}$.

MODEL: $\frac{2}{3}=\frac{x}{12}$. 3 is contained in 12 four times. 4 times 2 is 8. $\frac{2}{3}=\frac{8}{12}$.

Another method. $1=\frac{12}{12}$; $\frac{1}{3}=\frac{1}{3}$ of $\frac{12}{12}$, or $\frac{4}{12}$; $\frac{2}{3}=2$ times $\frac{4}{12}$, or $\frac{8}{12}$.

2. State how you would find the number that 3 must be multiplied by to change $\frac{3}{4}$ to 20ths.

3. Change to 12ths: $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{1}{3}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{6}$, $\frac{6}{6}$.

4. Change to 18ths: $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{6}$, $\frac{4}{9}$, $\frac{6}{6}$, $\frac{2}{9}$, $\frac{1}{6}$, $\frac{7}{9}$, $\frac{8}{9}$.

5. Change to 24ths: $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{12}$, $\frac{8}{8}$, $\frac{6}{12}$.

6. Change to 20ths: $\frac{1}{2}$, $\frac{2}{5}$, $\frac{4}{5}$, $\frac{8}{10}$, $\frac{3}{4}$, $\frac{10}{10}$, $\frac{1}{4}$, $\frac{2}{10}$.

7. Change to 36ths: $\frac{5}{6}$, $\frac{7}{9}$, $\frac{3}{4}$, $\frac{4}{4}$, $\frac{9}{6}$, $\frac{15}{18}$, $\frac{8}{12}$, $\frac{2}{3}$, $\frac{1}{9}$.

8. Change to 30ths: $\frac{9}{15}$, $\frac{8}{10}$, $\frac{5}{6}$, $\frac{3}{5}$, $\frac{2}{2}$, $\frac{1}{3}$, $\frac{14}{15}$, $\frac{2}{5}$, $\frac{6}{10}$.

9. Write eight fractions and change them to 48ths.

10. When the terms of a fraction have been made larger by reduction, the fraction is said to have been reduced to **higher terms**.

128. Oral Exercises.

1. Express each in a different form without changing the value: $\frac{6}{2}$, $3\frac{4}{5}$, $\frac{12}{20}$, $\frac{1}{3}$, 2, $2\frac{1}{3}$, $\frac{7}{3}$, $\frac{3}{5}$, $7\frac{1}{8}$, $\frac{12}{4}$, $\frac{20}{20}$, 1.

2. Find the sum of $3\frac{1}{4}$, $2\frac{3}{4}$, and 7.

3. Find the difference:

$9\frac{3}{7}$	$18\frac{1}{11}$	$25\frac{4}{5}$	$44\frac{1}{4}$
$3\frac{6}{7}$	$7\frac{9}{11}$	$17\frac{3}{5}$	$18\frac{3}{4}$

4. Show that $\frac{1}{3}$ of 2 yd. is equal to $\frac{2}{3}$ of 1 yd.; that $\frac{1}{4}$ of 8 ft. is the same as $\frac{8}{4}$ of 1 ft.

129. Oral Exercises.

1. Show by a diagram that $\frac{2}{3}$ of a line is equivalent to $\frac{4}{6}$ of the line; that $\frac{1}{2}$ ft. $= \frac{3}{6}$ ft.; that $\frac{2}{3}$ ft. $+ \frac{1}{2}$ ft. $= \frac{7}{6}$ ft., or $1\frac{1}{6}$ ft.

2. Show with objects that $\frac{3}{4}$ of 12 objects $= \frac{9}{12}$ of 12 objects; that $\frac{1}{6}$ of 12 objects $= \frac{2}{12}$ of 12 objects; that $\frac{3}{4}$ of 12 objects $+ \frac{1}{6}$ of 12 objects $= \frac{9}{12}$ of 12 objects $+ \frac{2}{12}$ of 12 objects, or $\frac{11}{12}$ of 12 objects.

3. Why is it necessary to change $\frac{3}{4}$ and $\frac{1}{6}$ to 12ths before finding their sum?

4. Show by a diagram that the difference between $\frac{2}{3}$ ft. and $\frac{1}{4}$ ft. is $\frac{5}{12}$ ft.

5. If $\frac{2}{3}$ of a group of objects contains 6 objects, show the number of objects in $\frac{1}{3}$ of the group.

6. Show with objects that if $\frac{2}{3}$ of a group of objects is 6 objects, the whole group contains 9 objects.

7. Show by a diagram that if $\frac{2}{5}$ of the length of a line is 4 ft., the entire length of the line is 10 ft.

8. If 6 objects represent $\frac{2}{5}$ of the number of books on a certain shelf, represent by objects $\frac{1}{5}$ of the number of books on the shelf. Represent all the books.

9. What is the least number of boys that may be separated either into groups of 3 boys or of 4 boys?

10. What is the least number of equal parts into which a rectangle can be divided so that either $\frac{1}{3}$ or $\frac{1}{4}$ of the rectangle may be shown?

11. What is the least number of girls that can be separated into groups containing as many girls as are indicated in the denominator of any one of the following: $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$?

130. Common Factors.

1. The exact measures of 12 ft. are 1 ft., 2 ft., 3 ft., 4 ft., and 6 ft. Name the exact measures of 18 ft. Which are exact measures of both 12 ft. and 18 ft.? 1 ft., 2 ft., 3 ft., and 6 ft. are *common measures* of 12 ft. and 18 ft.

2. A number that is a factor of two or more numbers is called a **common factor,** or a **common measure,** of the numbers.

3. Name the common factors of 18 and 24. Name the *largest* common factor of 18 and 24.

4. The **greatest common factor** of two or more numbers is the largest number that will exactly divide each of them. This is also called the *greatest common measure,* or the *greatest common divisor,* of the numbers.

5. Draw a line 12 in. long and another line 18 in. long. What is the longest measure that can be applied without a remainder in measuring both of these lines? What is the greatest common measure of 12 ft. and 18 ft.?

6. Show with objects or by a diagram all the common measures of 8 objects and 12 objects.

131. Name the exact measures of :

1. 10 ft.	6. 30 mi.	11. 50 rd.	16. $80
2. 16 gal.	7. 36 yd.	12. 27 pt.	17. $90
3. 20 da.	8. 40 yr.	13. 28 da.	18. $60
4. 18 hr.	9. 48 lb.	14. 64 ft.	19. $75
5. 24 in.	10. 45 qt.	15. 72 mi.	20. $84

132. Name the greatest common factor of :

1. 6 and 8	4. 12 and 18	7. 12 and 48	10. 20 and 60
2. 8 and 12	5. 24 and 30	8. 15 and 30	11. 18 and 36
3. 9 and 12	6. 24 and 36	9. 30 and 36	12. 14 and 28

133. Oral Exercises.

1. The fractional unit $\frac{1}{8}$ must be repeated how many times to equal the fractional unit $\frac{1}{2}$? The fractional unit $\frac{1}{24}$ must be repeated how many times to equal the fractional unit $\frac{1}{12}$?

2. By what must the terms of the fraction $\frac{12}{24}$ be divided to give the fraction $\frac{6}{12}$? $\frac{4}{8}$? $\frac{3}{6}$? $\frac{2}{4}$? $\frac{1}{2}$?

3. When the terms of a fraction have been made smaller by reduction, the fraction is said to have been reduced to **lower terms.** A fraction is in its **lowest terms** when the terms have no common factor.

4. The fraction $\frac{10}{15}$ is not in its lowest terms, as both terms are exactly divisible by 5. Which of the following are in their lowest terms: $\frac{2}{4}$, $\frac{3}{5}$, $\frac{4}{6}$, $\frac{7}{8}$, $\frac{10}{12}$, $\frac{14}{21}$, $\frac{17}{20}$, $\frac{45}{60}$?

5. Dividing both terms of a fraction by a common factor is called **canceling** the common factor.

In reducing a fraction to its lowest terms, cancel in turn the largest factors that are seen to be common to both terms. Canceling the greatest common factor of both terms reduces the fraction to its lowest terms.

134. Oral Exercises.

Reduce to lowest terms:

1. $\frac{2}{12}$, $\frac{3}{12}$, $\frac{4}{12}$, $\frac{6}{12}$, $\frac{9}{12}$, $\frac{10}{12}$
2. $\frac{2}{24}$, $\frac{3}{24}$, $\frac{4}{24}$, $\frac{6}{24}$, $\frac{8}{24}$, $\frac{9}{24}$
3. $\frac{10}{24}$, $\frac{12}{24}$, $\frac{15}{24}$, $\frac{18}{24}$, $\frac{20}{24}$, $\frac{22}{24}$
4. $\frac{3}{30}$, $\frac{4}{30}$, $\frac{5}{30}$, $\frac{6}{30}$, $\frac{8}{30}$, $\frac{9}{30}$
5. $\frac{10}{30}$, $\frac{12}{30}$, $\frac{15}{30}$, $\frac{18}{30}$, $\frac{20}{30}$, $\frac{24}{30}$
6. $\frac{25}{30}$, $\frac{4}{36}$, $\frac{6}{36}$, $\frac{9}{36}$, $\frac{12}{36}$, $\frac{18}{36}$
7. $\frac{24}{36}$, $\frac{30}{36}$, $\frac{3}{48}$, $\frac{4}{48}$, $\frac{6}{48}$, $\frac{8}{48}$
8. $\frac{10}{48}$, $\frac{12}{48}$, $\frac{15}{48}$, $\frac{16}{48}$, $\frac{18}{48}$, $\frac{20}{48}$
9. $\frac{24}{48}$, $\frac{30}{48}$, $\frac{32}{48}$, $\frac{36}{48}$, $\frac{40}{48}$, $\frac{42}{48}$
10. $\frac{8}{72}$, $\frac{9}{72}$, $\frac{16}{72}$, $\frac{18}{72}$, $\frac{24}{72}$, $\frac{27}{72}$
11. $\frac{32}{72}$, $\frac{36}{72}$, $\frac{40}{72}$, $\frac{45}{72}$, $\frac{48}{72}$, $\frac{54}{72}$
12. $\frac{63}{72}$, $\frac{56}{72}$, $\frac{14}{49}$, $\frac{28}{49}$, $\frac{35}{49}$, $\frac{21}{49}$
13. $\frac{16}{64}$, $\frac{24}{60}$, $\frac{21}{35}$, $\frac{32}{56}$, $\frac{42}{60}$, $\frac{45}{60}$
14. $\frac{45}{81}$, $\frac{54}{63}$, $\frac{40}{55}$, $\frac{50}{75}$, $\frac{63}{81}$, $\frac{72}{108}$
15. $\frac{26}{39}$, $\frac{72}{144}$, $\frac{45}{75}$, $\frac{21}{91}$, $\frac{24}{28}$, $\frac{20}{50}$
16. $\frac{18}{54}$, $\frac{17}{51}$, $\frac{19}{57}$, $\frac{21}{63}$, $\frac{84}{144}$, $\frac{60}{108}$

135. Multiples.

1. 2 is a factor of 4, 6, 8, 10, etc. Each of these numbers is a *multiple* of 2. Name the multiples of 3 to 27.

2. A number that is exactly divisible by a given number is called a **multiple** of the given number. Name a multiple of 6; of 8; of 7.

3. Write the multiples of 3 to 27 and of 4 to 36. Which of the numbers written are multiples of both 3 and 4? These numbers are *common multiples* of 3 and 4. Which is the least multiple common to 3 and 4?

4. A number that is a multiple of each of two or more numbers is called a **common multiple**, and the least number that is a common multiple of each of two or more numbers is called the **least common multiple** of the numbers.

5. Write all the multiples of 4 to 36 and of 6 to 54. Name the multiples common to 4 and 6. Which of these is the least common multiple of 4 and 6?

6. Name the least common multiple of 3 and 4; of 2, 3, and 4. Since 4 is a multiple of 2, the least common multiple of 2, 3, and 4 is the same as the least common multiple of 3 and 4. The least common multiple of 2, 3, 4, and 6 is the same as the least common multiple of 4 and 6. Why?

7. Write four numbers such that the least common multiple of the numbers is the same as the least common multiple of some two of the numbers.

8. In finding the least common multiple of 2, 3, 4, and 9, which numbers need not be considered, and why?

9. Find the least common multiple of 3, 5, and 7. Show by several illustrations that the least common multiple of two or more prime numbers is their product.

136. Name the least common multiple of:

1. 4 and 5
2. 4 and 6
3. 3 and 4
4. 6 and 8
5. 5 and 7
6. 4 and 8
7. 8 and 12
8. 6 and 9
9. 4 and 10
10. 5 and 10
11. 10 and 4
12. 5 and 8
13. 10 and 12
14. 12 and 9
15. 3 and 7
16. 2, 3, and 6
17. 3, 4, and 8
18. 4, 5, and 15

137. 1. Find the least common multiple of 8, 10, 18, and 48.

MODEL:

$\cancel{8}$
$\cancel{10}$ 5
$\cancel{18}$ 3
48

As 48 is a multiple of 8, cancel 8. As the factor 2 is common to 10 and to 48, cancel this factor of 10, leaving the factor 5. As 6 is a factor common to 18 and 48, cancel this factor of 18, leaving the factor 3. $5 \times 3 \times 48$, or 720, is the least common multiple of 8, 10, 18, and 48.

2. In finding the least common multiple of 3, 4, 6, 9, and 12, which numbers may be canceled? From which number may a factor be canceled?

3. In finding the least common multiple of 4, 12, 7, and 35, which number may be canceled because it is a factor of 12? Which may be canceled because it is a factor of 35?

138. Find by inspection the least common multiple of:

1. 4, 5, 8, 24
2. 3, 12, 15, 30
3. 5, 8, 25, 40
4. 2, 6, 15, 45
5. 7, 21, 49, 84
6. 9, 15, 36, 60
7. 10, 15, 25, 40
8. 36, 48, 60, 72
9. 12, 18, 24, 36
10. 3, 5, 30, 45
11. 4, 18, 27, 72
12. 8, 12, 15, 60
13. 7, 35, 45, 90, 70
14. 5, 14, 42, 60
15. 6, 7, 8, 9, 84
16. 20, 24, 30, 100
17. 4, 9, 20, 54
18. 6, 15, 24, 36

ADDITION AND SUBTRACTION

139. Written Exercises.

1. Change to 12ths and add $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.

MODEL:

$$\begin{aligned}\tfrac{2}{3} &= \tfrac{8}{12}\\ \tfrac{3}{4} &= \tfrac{9}{12}\\ \tfrac{5}{6} &= \tfrac{10}{12}\\ \hline \tfrac{27}{12} &= 2\tfrac{3}{12} = 2\tfrac{1}{4}\end{aligned}$$

2.	3.	4.	5.	6.	7.
$\frac{1}{6}$	$\frac{5}{6}$	$\frac{3}{6}$	$\frac{11}{12}$	$\frac{2}{6}$	$\frac{9}{12}$
$\frac{1}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{5}{6}$
$\frac{1}{2}$	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{1}{3}$	$\frac{9}{12}$	$\frac{3}{4}$
$\frac{2}{3}$	$\frac{1}{2}$.	$\frac{2}{3}$	$\frac{4}{6}$	$\frac{1}{3}$	$\frac{2}{3}$

8. Add the fractions in Exs. 3–8, Sec. 127.

9. Change to 24ths and add $3\frac{5}{8}$, $4\frac{1}{6}$, $6\frac{11}{12}$, $7\frac{2}{3}$.

MODEL:

$$\begin{aligned}3\tfrac{5}{8} &= 3\tfrac{15}{24}\\ 4\tfrac{1}{6} &= 4\tfrac{4}{24}\\ 6\tfrac{11}{12} &= 6\tfrac{22}{24}\\ 7\tfrac{2}{3} &= 7\tfrac{16}{24}\\ \hline & 22\tfrac{3}{8}\end{aligned}$$

The sum of the fractions is $\frac{57}{24}$, which reduces to $2\frac{3}{8}$. Write $\frac{3}{8}$ as the fractional part of the answer. Carry 2 to the column of integers.

10. Change to 12ths and add $4\frac{5}{6}$, $3\frac{3}{4}$, $4\frac{2}{3}$, $5\frac{1}{2}$, $6\frac{5}{12}$.
11. Change to 18ths and add $8\frac{2}{3}$, $7\frac{8}{9}$, $9\frac{1}{6}$, $6\frac{1}{2}$, $5\frac{17}{18}$.
12. Change to 24ths and add $7\frac{5}{8}$, $6\frac{7}{12}$, $9\frac{1}{3}$, $8\frac{5}{6}$, $4\frac{1}{4}$.
13. Change to 36ths and add $3\frac{11}{12}$, $7\frac{13}{18}$, $8\frac{7}{9}$, $5\frac{1}{6}$, $9\frac{3}{4}$.
14. Change to 48ths and add $7\frac{1}{8}$, $9\frac{5}{6}$, $3\frac{1}{12}$, $5\frac{19}{24}$, $8\frac{5}{48}$.
15. Change to 72ds and add $8\frac{1}{9}$, $9\frac{3}{8}$, $7\frac{5}{36}$, $18\frac{7}{12}$, $3\frac{2}{3}$.
16. Reduce to lowest terms: $\frac{45}{63}$, $\frac{28}{42}$, $\frac{18}{48}$, $\frac{20}{72}$.
17. Change to improper fractions: $7\frac{8}{9}$, $9\frac{6}{7}$, $8\frac{5}{6}$, $7\frac{3}{4}$.
18. Change to mixed numbers: $\frac{51}{6}$, $\frac{70}{9}$, $\frac{41}{7}$, $\frac{37}{8}$.
19. Add $4\frac{3}{7}$, $6\frac{1}{7}$, $3\frac{5}{7}$, $8\frac{6}{7}$, $9\frac{2}{7}$, $7\frac{4}{7}$.
20. Change 4 to 12ths; 3 to 18ths; 5 to 20ths.
21. Write ten fractions and reduce them to lower terms.

140. Oral Exercises.

1. What is the least common multiple of 2, 3, and 4? of 3, 5, and 6? of 4, 5, and 6? of 4, 6, and 8? of 3, 6, and 9? of 5, 8, and 12?

2. Can you add the following fractions without first reducing them: $\frac{3}{7}$, $\frac{4}{7}$, $\frac{2}{7}$? Are they expressed in the same fractional unit?

3. Can you add the following fractions without first reducing them: $\frac{2}{7}$, $\frac{3}{4}$, and $\frac{1}{2}$? Are they expressed in the same fractional unit? Only fractions that are expressed in the same fractional unit can be added.

4. What is the unit of measure in $\frac{2}{3}$ ft.? $\frac{5}{6}$ ft.? $\frac{7}{12}$ ft.? These fractions may be expressed in the same unit of measure, $\frac{1}{12}$ ft. $\frac{2}{3}$ ft. $= \frac{x}{12}$ ft. $\frac{5}{6}$ ft. $= \frac{x}{12}$ ft.

5. Can the following be expressed in the same unit of measure: $\frac{1}{3}$ ft., $\frac{3}{4}$ da., and $\frac{1}{2}$ gal.? Can the following: $\frac{1}{3}$ ft., $\frac{5}{6}$ ft., and $\frac{7}{12}$ ft.?

6. What is the least common multiple of the denominators of the fractions $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{8}$? The least common multiple of the denominators of two or more fractions is called their **least common denominator.**

141. Reduce the fractions to fractions having the least common denominator, and add:

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
$8\frac{4}{5}$	$9\frac{1}{6}$	$4\frac{7}{9}$	$3\frac{3}{10}$	$4\frac{11}{12}$	$3\frac{7}{9}$	$5\frac{8}{9}$	$4\frac{7}{15}$	$6\frac{3}{7}$
$6\frac{3}{4}$	$8\frac{1}{4}$	$6\frac{5}{6}$	$4\frac{2}{5}$	$7\frac{7}{8}$	$8\frac{3}{4}$	$8\frac{7}{12}$	$2\frac{11}{12}$	$8\frac{2}{5}$
$9\frac{1}{2}$	$5\frac{3}{8}$	$7\frac{5}{12}$	$9\frac{1}{6}$	$6\frac{5}{6}$	$6\frac{17}{18}$	$4\frac{7}{8}$	$9\frac{1}{6}$	$4\frac{7}{10}$
$4\frac{7}{10}$	$6\frac{11}{12}$	$8\frac{1}{4}$	$6\frac{1}{2}$	$9\frac{1}{4}$	$7\frac{1}{3}$	$5\frac{1}{4}$	$7\frac{3}{5}$	$6\frac{11}{14}$
$7\frac{1}{5}$	$7\frac{2}{3}$	$5\frac{1}{3}$	$7\frac{2}{3}$	$5\frac{7}{24}$	$6\frac{1}{6}$	$7\frac{1}{6}$	$8\frac{3}{4}$	$9\frac{1}{2}$

142. Written Exercises.*

1. From $5\frac{3}{4}$ subtract $3\frac{2}{3}$.

MODEL: $\begin{array}{r} 5\frac{3}{4} = 5\frac{9}{12} \\ 3\frac{2}{3} = 3\frac{8}{12} \\ \hline 2\frac{1}{12} \end{array}$

Reduce the fractions to fractions having the least common denominator, and subtract.

Subtract:

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
2.	$6\frac{4}{5}$	$8\frac{3}{4}$	$9\frac{8}{9}$	$7\frac{5}{6}$	$5\frac{6}{7}$	$12\frac{4}{5}$	$34\frac{15}{16}$	$56\frac{14}{15}$
	$4\frac{2}{3}$	$5\frac{4}{7}$	$6\frac{5}{6}$	$3\frac{6}{11}$	$2\frac{7}{20}$	$9\frac{3}{4}$	$23\frac{7}{8}$	$43\frac{5}{6}$
3.	$9\frac{2}{3}$	$8\frac{1}{4}$	$6\frac{2}{5}$	$8\frac{1}{4}$	$7\frac{2}{3}$	$6\frac{1}{3}$	$5\frac{2}{3}$	$6\frac{3}{4}$
	$6\frac{4}{5}$	$4\frac{2}{3}$	$4\frac{3}{4}$	$5\frac{3}{4}$	$5\frac{1}{2}$	$4\frac{1}{4}$	$2\frac{3}{4}$	$4\frac{1}{3}$
4.	$5\frac{1}{2}$	$6\frac{3}{8}$	$7\frac{5}{12}$	$9\frac{4}{5}$	$18\frac{2}{7}$	$19\frac{2}{5}$	$12\frac{1}{4}$	$15\frac{1}{5}$
	$4\frac{5}{6}$	$4\frac{1}{12}$	$2\frac{3}{4}$	$4\frac{1}{5}$	$7\frac{2}{3}$	$9\frac{3}{4}$	$8\frac{1}{12}$	$8\frac{3}{4}$
5.	$21\frac{2}{5}$	$30\frac{2}{3}$	20	$26\frac{4}{5}$	$43\frac{2}{3}$	16	$18\frac{3}{4}$	$29\frac{3}{4}$
	$10\frac{2}{5}$	9	$9\frac{2}{3}$	$4\frac{2}{5}$	7	$10\frac{8}{9}$	$8\frac{1}{2}$	$9\frac{4}{5}$
6.	$87\frac{8}{21}$	$79\frac{4}{17}$	$90\frac{6}{13}$	$58\frac{7}{12}$	$74\frac{5}{8}$	20	20	$4\frac{2}{7}$
	$64\frac{5}{7}$	$50\frac{8}{51}$	$37\frac{5}{39}$	$19\frac{7}{11}$	$16\frac{11}{64}$	$13\frac{15}{17}$	$9\frac{10}{11}$	$3\frac{6}{7}$

143. 1. Add each exercise in Sec. 142.

2. The lengths of the blackboards in a certain schoolroom are $12\frac{1}{3}$ ft., $14\frac{1}{2}$ ft., $8\frac{3}{4}$ ft., and $6\frac{1}{4}$ ft., respectively. Find the combined length of the four blackboards.

3. Find the difference in the weight of two turkeys, if one of the turkeys weighs $22\frac{1}{2}$ lb. and the other $17\frac{3}{4}$ lb.

* See Sec. 125.

144. Review Sec. 134.

Reduce to lowest terms:

1. $\frac{51}{60}, \frac{54}{81}, \frac{72}{96}, \frac{49}{84}, \frac{36}{60}, \frac{32}{96}$
2. $\frac{20}{100}, \frac{25}{100}, \frac{30}{100}, \frac{40}{100}, \frac{50}{100}$
3. $\frac{70}{100}, \frac{80}{100}, \frac{90}{100}, \frac{15}{100}, \frac{45}{100}$
4. $\frac{20}{125}, \frac{25}{521}, \frac{50}{125}, \frac{75}{125}, \frac{100}{125}$
5. $\frac{48}{86}, \frac{48}{144}, \frac{60}{144}, \frac{84}{144}, \frac{96}{144}$
6. $\frac{30}{72}, \frac{40}{50}, \frac{25}{75}, \frac{15}{80}, \frac{15}{90}, \frac{16}{80}$
7. $\frac{100}{150}, \frac{75}{150}, \frac{25}{150}, \frac{50}{200}, \frac{40}{200}$
8. $\frac{75}{200}, \frac{72}{144}, \frac{120}{216}, \frac{5}{100}, \frac{10}{50}$

145. Review Sec. 120.

Reduce to improper fractions:

1. $14\frac{2}{3}, 30\frac{1}{4}, 16\frac{1}{2}, 33\frac{1}{3}, 66\frac{2}{3}$
2. $16\frac{2}{3}, 37\frac{3}{8}, 11\frac{1}{9}, 9\frac{1}{11}, 28\frac{4}{7}$
3. $38\frac{5}{12}, 5\frac{5}{9}, 16\frac{3}{8}, 25\frac{5}{6}, 67\frac{1}{2}$
4. $17\frac{3}{4}$ mi., $5\frac{1}{2}$ yd., $8\frac{3}{7}$ wk.
5. $23\frac{1}{3}$ ft., $8\frac{3}{4}$ lb., $9\frac{6}{7}$ wk.
6. $6\frac{5}{12}$ yd., $4\frac{1}{8}$ mi., $8\frac{11}{16}$ A.

146. Review Sec. 122.

Reduce to integers or mixed numbers:

1. $\frac{100}{7}, \frac{100}{6}, \frac{100}{8}, \frac{100}{9}, \frac{100}{5}$
2. $\frac{100}{3}, \frac{100}{2}, \frac{100}{10}, \frac{100}{12}, \frac{100}{20}$
3. $\frac{100}{40}, \frac{100}{50}, \frac{100}{60}, \frac{100}{75}, \frac{100}{80}$
4. $\frac{17}{10}, \frac{121}{4}, \frac{20}{3}, \frac{50}{4}, \frac{75}{2}, \frac{25}{4},$
5. $\frac{16}{3}$ yd., $\frac{20}{8}$ gal., $\frac{24}{7}$ wk.
6. $\$\frac{13}{4}, \$\frac{18}{2}, \$\frac{20}{4}, \$\frac{30}{5}, \$\frac{50}{8}$
7. $\frac{15}{4}$ da., $\frac{31}{8}$ mi., $\frac{22}{8}$ gal.
8. $\frac{21}{4}$ T., $\frac{17}{2}$ A., $\frac{50}{8}$ bu.

147. Add:

a	*b*	*c*	*d*	*e*	*f*	*g*
$43\frac{3}{4}$	$64\frac{1}{4}$	$18\frac{1}{2}$	$77\frac{3}{4}$	$19\frac{1}{2}$	$39\frac{1}{4}$	$56\frac{3}{5}$
$69\frac{1}{2}$	$58\frac{2}{3}$	$79\frac{1}{6}$	$24\frac{5}{9}$	$67\frac{1}{8}$	$34\frac{2}{3}$	$76\frac{1}{3}$
$73\frac{3}{5}$	$74\frac{4}{5}$	$83\frac{1}{5}$	$66\frac{1}{2}$	$53\frac{2}{5}$	$65\frac{1}{5}$	$84\frac{3}{4}$
$87\frac{1}{10}$	$56\frac{1}{2}$	$95\frac{2}{3}$	$73\frac{1}{3}$	$49\frac{3}{4}$	$47\frac{3}{10}$	$59\frac{5}{6}$

148. Subtract:

	a	*b*	*c*	*d*	*e*	*f*	*g*
1.	$190\frac{5}{7}$	$265\frac{3}{8}$	$398\frac{3}{7}$	$443\frac{5}{12}$	$178\frac{2}{3}$	$296\frac{1}{2}$	$467\frac{2}{5}$
	$137\frac{2}{6}$	$124\frac{3}{4}$	$154\frac{2}{5}$	$217\frac{3}{8}$	$25\frac{3}{4}$	$180\frac{5}{8}$	$337\frac{5}{6}$

2. Add each of the above exercises.

149. Review Sec. 137.

Give the least common multiple of:

1. 2, 3, 4	8. 4, 5, 8	15. 7, 8, 9
2. 3, 4, 5	9. 3, 4, 7	16. 3, 4, 6, 8, 12
3. 4, 5, 6	10. 6, 7, 8	17. 5, 7, 8, 12, 4
4. 2, 3, 5	11. 3, 7, 9	18. 3, 5, 6, 8, 15
5. 3, 4, 7	12. 4, 7, 9	19. 7, 9, 12, 14, 21
6. 4, 6, 8	13. 6, 8, 12	20. 8, 10, 12, 15, 20
7. 5, 7, 8	14. 5, 8, 12	21. 4, 6, 10, 14, 20

150. When the least common multiple of the denominators cannot be found readily by inspection, use the following method:

1. Find the least common denominator: $\frac{5}{24}$, $\frac{3}{50}$, $\frac{7}{72}$, $\frac{17}{80}$.

MODEL:

$$\begin{array}{r|rrrr} 2 & \cancel{24} & 50 & 72 & 80 \\ \hline 2 & & 25 & 36 & 40 \\ \hline 2 & & 25 & 18 & 20 \\ \hline 5 & & 25 & 9 & 10 \\ \hline & & 5 & 9 & 2 \end{array}$$

Find the least common multiple of 24, 50, 72, 80. Cancel 24 as it is a factor of 72. Select a prime number that is a factor of two or more of the remaining numbers. Divide the multiples of this number by the number used as a divisor, and write the quotients and the numbers that are not exactly divisible as shown in the model. Continue the division until no two numbers brought down have a common factor. The product of the several divisors and numbers remaining is the least common multiple of the denominators.

l. c. m. $= 2 \times 2 \times 2 \times 5 \times 5 \times 9 \times 2 = 3600$.

Find the least common multiple of the same numbers by the method explained in Sec. 137.

MULTIPLICATION AND DIVISION OF FRACTIONS

151. Multiplying a Fraction by an Integer.

1. In the fraction $\frac{2}{5}$, which term tells the number of equal parts into which the unit has been divided? How many of the equal parts are expressed in the fraction? Write a fraction expressing twice as many equal parts.

2. Draw a diagram to show what part of a mile is expressed in $\frac{6}{8}$ mi. Show the part that represents $\frac{3}{8}$ mi. Compare the part $\frac{3}{8}$ mi. with the part $\frac{6}{8}$ mi.

3. What is the sum of $\frac{3}{8}$, $\frac{3}{8}$, and $\frac{3}{8}$? 3 times $\frac{3}{8} = \frac{x}{8}$.

4. Write $\frac{2}{9}$ four times as an addend and find the sum. State how the sum was found.

5. If $\frac{3}{5}$ is written five times as an addend, what is the sum? If $\frac{3}{5}$ is multiplied by 5, what is the product?

6. State how a fraction may be multiplied by a whole number. Compare the results thus obtained with the results obtained by addition.

7. Multiply. Reduce all products to their simplest forms: $\frac{3}{4}$ by 5; $\frac{4}{5}$ by 3; $\frac{5}{6}$ by 6; $\frac{3}{8}$ by 7; $\frac{3}{4}$ yd. by 4.

8. If $\frac{2}{9}$ is multiplied by 3 by multiplying the numerator by 3, the result will not be in its lowest terms. Why? $\frac{2}{9}$ may be multiplied by 3 by dividing the denominator 9 by 3. $\frac{2}{9} \times 3 = \frac{2}{3}$.

9. Dividing the denominator of a fraction by a whole number has what effect upon the value of the fraction?

10. Multiply $\frac{7}{8}$ by 12.

As the factor 4 is common to both 12 and 8, it is canceled before multiplying. Canceling 4 in 12 leaves 3; canceling 4 in 8 leaves 2. 3 times $\frac{7}{2}$ $= \frac{21}{2} = 10\frac{1}{2}$.

152. Oral Exercises.

Solve each in the shortest way:

	a	*b*	*c*	*d*	*e*
1.	$\frac{8}{9}\times 5$	$\frac{11}{15}\times 5$	$\frac{21}{40}\times 10$	$\frac{13}{18}\times 9$	$\frac{18}{35}\times 5$
2.	$\frac{7}{8}\times 4$	$\frac{9}{10}\times 3$	$\frac{32}{35}\times 7$	$\frac{41}{48}\times 8$	$\frac{11}{12}\times 12$
3.	$\frac{5}{6}\times 3$	$\frac{8}{21}\times 7$	$\frac{17}{19}\times 8$	$\frac{15}{17}\times 3$	$\frac{13}{30}\times 6$
4.	$\frac{3}{5}\times 5$	$\frac{12}{13}\times 4$	$\frac{17}{24}\times 6$	$\frac{25}{56}\times 7$	$\frac{24}{27}\times 4$
5.	$\frac{4}{5}\times 20$	$\frac{8}{9}\times 12$	$\frac{13}{16}\times 24$	$\frac{3}{4}\times 16$	$\frac{35}{72}\times 48$
6.	$\frac{7}{8}\times 24$	$\frac{5}{14}\times 7$	$\frac{5}{18}\times 30$	$\frac{6}{7}\times 42$	$\frac{54}{65}\times 3$
7.	$\frac{11}{12}\times 18$	$\frac{6}{7}\times 5$	$\frac{5}{17}\times 8$	$\frac{27}{48}\times 16$	$\frac{18}{35}\times 28$
8.	$\frac{14}{15}\times 10$	$\frac{4}{15}\times 5$	$\frac{23}{25}\times 75$	$\frac{15}{16}\times 24$	$\frac{12}{25}\times 50$
9.	$\frac{7}{18}\times 36$	$\frac{17}{21}\times 28$	$\frac{14}{35}\times 7$	$\frac{3}{25}\times 8$	$\frac{31}{51}\times 17$

153. Written Exercises.

1. Multiply $43\frac{3}{4}$ by 8.

MODEL:

$$\begin{array}{r} 43\frac{3}{4} \\ \underline{8} \\ 6 \\ \underline{344} \\ 350 \end{array}$$

First, multiply $\frac{3}{4}$ by 8. Next, multiply 43 by 8. Add the products.

Solve. Perform the cancellation and reductions without the use of a pencil:

	a	*b*	*c*	*d*	*e*
2.	$47\frac{3}{5}\times 5$	$64\frac{5}{6}\times 7$	$82\frac{3}{7}\times 14$	$749\frac{11}{15}\times 5$	$708\frac{5}{6}\times 30$
3.	$68\frac{4}{7}\times 7$	$74\frac{3}{8}\times 9$	$65\frac{2}{5}\times 15$	$896\frac{13}{14}\times 7$	$580\frac{21}{25}\times 15$
4.	$96\frac{3}{4}\times 3$	$59\frac{2}{3}\times 6$	$94\frac{3}{4}\times 12$	$780\frac{14}{15}\times 3$	$496\frac{11}{36}\times 48$
5.	$78\frac{2}{3}\times 9$	$76\frac{4}{9}\times 9$	$70\frac{5}{6}\times 18$	$973\frac{2}{3}\times 9$	$573\frac{8}{9}\times 25$
6.	$56\frac{3}{4}\times 9$	$38\frac{1}{2}\times 8$	$27\frac{5}{8}\times 24$	$587\frac{3}{4}\times 2$	$609\frac{5}{7}\times 42$

7. Write ten mixed numbers and multiply them by integers.

154. Multiplying an Integer by a Fraction.

1. What is the meaning of 4 ft. $\times$ 2? of 4 ft. $\times$ 1? of 4 ft. $\times \frac{1}{2}$? Name the multiplicand and the multiplier in each, and tell what each shows.

2. 4 ft. $\times \frac{1}{2}$ is the same as $\frac{1}{2}$ of 4 ft. How may $\frac{1}{2}$ of a number be found? How may $\frac{1}{3}$ of a number be found? When you know what $\frac{1}{3}$ of a number is, how can you find $\frac{2}{3}$ of the number?

3. Show with objects what is meant by $\frac{2}{3}$ of 9 things; of 12 things; of 6 things.

4. $\frac{5}{6}$ of 24 yd. means 5 of the 6 equal parts of 24 yd. Draw a line to represent 24 yd. Divide it into 6 equal parts. Show the part that represents $\frac{5}{6}$ of 24 yd.

5. Show by a diagram what is meant by $\frac{3}{4}$ of 12 in.; by $\frac{2}{3}$ of 1 mi.; by $\frac{2}{3}$ of 6 mi.; by $\frac{4}{5}$ of 10 mi.

6. How many thirds of 18 are equivalent to 18? Are $\frac{2}{3}$ of 18 more or less than 18? If 18 is multiplied by $\frac{5}{6}$, will the answer be more or less than 18? Why?

7. Read each of the following, name the multiplicand and the multiplier in each, and tell what each shows: \$20 $\times \frac{4}{5}$; 16 yr. $\times \frac{3}{4}$; 25 mi. $\times \frac{2}{5}$; 18 mo. $\times \frac{2}{9}$; 24 lb. $\times \frac{5}{6}$.

8. Compare $\frac{3}{4}$ of \$20 with $\frac{1}{4}$ of 3 times \$20. Compare $\frac{4}{5}$ of 25 mi. with $\frac{1}{5}$ of 4 times 25 mi.

9. $\frac{2}{3}$ of 8 ft. is the same as $\frac{1}{3}$ of —— ft. $\frac{2}{7}$ of \$5 is the same as $\frac{1}{7}$ of \$——.

10. Show by a diagram that $\frac{3}{4}$ of 1 yd. is the same as $\frac{1}{4}$ of 3 yd.

11. 5 divided by 7 may be indicated $7\overline{)5}$, or $\frac{5}{7}$. Indicate $\frac{1}{4}$ of 3; $\frac{1}{3}$ of 2 ft.; $\frac{1}{6}$ of 5 mi.; $\frac{2}{3}$ of 5 mi.

12. The products of $\frac{4}{9} \times 18$ and of $18 \times \frac{4}{9}$ are the same.

155. Written Exercises.

1. Multiply 36 by $\frac{11}{24}$.

MODEL: $\overset{3}{\cancel{36}} \times \frac{11}{\underset{2}{\cancel{24}}} = \frac{33}{2} = 16\frac{1}{2}$.

12 is a factor common to 36 and 24. Cancel the common factor. 3 times 11 is 33; $\frac{33}{2} = 16\frac{1}{2}$.

Solve:

	a	*b*	*c*	*d*	*e*
2.	$30 \times \frac{4}{5}$	$8 \times \frac{15}{16}$	$25 \times \frac{3}{5}$	$144 \times \frac{3}{4}$	$100 \times \frac{2}{3}$
3.	$48 \times \frac{11}{20}$	$7 \times \frac{3}{4}$	$30 \times \frac{5}{12}$	$54 \times \frac{4}{9}$	$100 \times \frac{3}{4}$
4.	$36 \times \frac{7}{12}$	$8 \times \frac{5}{12}$	$27 \times \frac{17}{36}$	$60 \times \frac{5}{12}$	$100 \times \frac{4}{5}$
5.	$21 \times \frac{3}{8}$	$9 \times \frac{17}{18}$	$45 \times \frac{8}{9}$	$16 \times \frac{17}{24}$	$100 \times \frac{3}{8}$
6.	$4 \times \frac{3}{5}$	$6 \times \frac{5}{7}$	$72 \times \frac{51}{84}$	$36 \times \frac{4}{5}$	$100 \times \frac{2}{7}$

156. Written Exercises.

1. Multiply 845 by $4\frac{3}{7}$.

MODEL:

$$\begin{array}{rl} 845 & \\ 4\frac{3}{7} & \\ \hline 120\frac{5}{7} & (\frac{1}{7} \text{ of } 845) \\ 3 & \\ \hline 362\frac{1}{7} & (\frac{3}{7} \text{ of } 845) \\ 3380 & (4 \text{ times } 845) \\ \hline 3742\frac{1}{7} & \end{array}$$

$\frac{1}{7}$ of 845 is $120\frac{5}{7}$; $\frac{3}{7}$ of 845 are 3 times $120\frac{5}{7}$, or $362\frac{1}{7}$. 4 times 845 are 3380. Add the products.

Multiply:

	a	*b*	*c*	*d*
2.	$60 \times 8\frac{4}{5}$	$64 \times 45\frac{2}{3}$	$827 \times 47\frac{2}{3}$	$801 \times 84\frac{5}{9}$
3.	$36 \times 9\frac{3}{4}$	$81 \times 47\frac{7}{10}$	$459 \times 75\frac{5}{9}$	$153 \times 46\frac{3}{4}$
4.	$55 \times 8\frac{9}{10}$	$72 \times 67\frac{3}{7}$	$693 \times 68\frac{2}{3}$	$360 \times 48\frac{7}{8}$
5.	$27 \times 6\frac{7}{9}$	$96 \times 87\frac{3}{5}$	$745 \times 47\frac{4}{5}$	$578 \times 96\frac{5}{6}$
6.	$33 \times 4\frac{3}{8}$	$48 \times 54\frac{11}{12}$	$584 \times 37\frac{5}{8}$	$609 \times 24\frac{3}{7}$
7.	$45 \times 8\frac{3}{7}$	$75 \times 49\frac{11}{15}$	$144 \times 35\frac{11}{12}$	$586 \times 27\frac{5}{9}$

157. Dividing a Fraction by an Integer.

1. Divide 12 by 3. Find $\frac{1}{3}$ of 12. $\frac{1}{3}$ of a number may be found by dividing the number by 3. State how $\frac{1}{4}$ of a number may be found.

2. Draw a line 12 inches long. Show $\frac{9}{12}$ of the line. Show $\frac{1}{3}$ of $\frac{9}{12}$ of the line. Divide $\frac{9}{12}$ of the line into 3 equal parts. How does each of these parts compare with $\frac{1}{3}$ of $\frac{9}{12}$ of the line? $\frac{1}{3}$ of $\frac{9}{12}$ is the same as $\frac{9}{12} \div 3$.

3. What is $\frac{1}{4}$ of $\frac{4}{5}$? of $\frac{8}{9}$? of $\frac{12}{17}$? of $\frac{20}{21}$?

4. State how a fraction may be divided by an integer, when the numerator is exactly divisible by the integer.

5. Solve: $\frac{9}{10} \div 3$; $\frac{14}{15} \div 7$; $\frac{12}{19} \div 6$; $\frac{24}{25} \div 8$; $\frac{15}{17} \div 5$.

6. Draw a line 12 in. long. Divide it into 12 equal parts. Each part is $\frac{1}{12}$ of the whole line. Divide each part into 3 equal parts. Each of these smaller parts is what part of the entire line? To divide $\frac{8}{12}$ of the line into 3 equal parts, each of the 8 parts must be divided into 3 equal parts and $\frac{1}{3}$ of these taken. Show that $\frac{1}{3}$ of $\frac{8}{12}$ of the line is $\frac{8}{36}$ of the line. This result may be found by multiplying the denominator of $\frac{8}{12}$ by 3.

7. Divide $\frac{12}{13}$ by 4 by dividing the numerator by 4; by multiplying the denominator by 4. Compare the results.

158. Oral Exercises.

Divide, using the shortest method for each:

1. $\frac{9}{10}$ by 3	5. $\frac{15}{17}$ by 2	9. $\frac{3}{5}$ T. by 2
2. $\frac{20}{21}$ by 10	6. $\frac{18}{27}$ by 6	10. \$$\frac{4}{5}$ by 7
3. $\frac{7}{8}$ by 6	7. $\frac{14}{17}$ by 7	11. $1\frac{3}{8}$ mi. by 5
4. $\frac{24}{25}$ by 8	8. $2\frac{2}{3}$ ft. by 3	12. $\frac{2}{7}$ da. by 3

159. Written Exercises.

1. Divide $65\frac{3}{4}$ by 8.

MODEL: $\begin{array}{r} 8\frac{7}{32} \\ 8\overline{)65\frac{3}{4}} \end{array}$ 8 is contained in 65 eight times, with 1 over; $\frac{1}{8}$ of $1\frac{3}{4}$ is $\frac{1}{8}$ of $\frac{7}{4}$, or $\frac{7}{32}$.

	a	*b*	*c*
2.	$37\frac{1}{3} \div 7$	$325\frac{4}{5} \div 4$	$132\frac{1}{5} \div 5$
3.	$62\frac{3}{5} \div 9$	$423\frac{3}{7} \div 6$	$836\frac{3}{8} \div 4$
4.	$87\frac{4}{7} \div 8$	$756\frac{1}{3} \div 8$	$456\frac{1}{5} \div 2$
5.	$46\frac{3}{7} \div 9$	$637\frac{2}{9} \div 7$	$387\frac{3}{4} \div 3$
6.	$90\frac{1}{2} \div 8$	$436\frac{1}{5} \div 9$	$726\frac{8}{9} \div 14$

7. Divide $645\frac{3}{8}$ by 24; $1645\frac{5}{6}$ by 38; $195\frac{7}{9}$ by 27.

8. Divide $347\frac{3}{4}$ by 46; $73\frac{2}{3}$ by 14; $472\frac{1}{2}$ by 65.

160. Written Exercises.

1. A carpenter sawed a board 9 ft. 4 in. ($9\frac{1}{3}$ ft.) long into 4 equal parts to make shelves for a bookcase. How long was each shelf?

Or, divide 9 ft. 4 in. by 4 thus: $\begin{array}{r} \text{2 ft. 4 in.} \\ 4\overline{)\text{9 ft. 4 in.}} \end{array}$ $\frac{1}{4}$ of 8 ft. is 2 ft.; $\frac{1}{4}$ of 16 in. (1 ft. 4 in.) is 4 in. Ans.: 2 ft. 4 in.

2. The perimeter of a square flower bed is 14 ft. 8 in. ($14\frac{2}{3}$ ft.). How long is each side?

3. If a train travels at an average rate of 45 mi. per hour, how far will it travel in 4 hr. 45 min. ($4\frac{3}{4}$ hr.)?

4. At 5¢ per pound, how much will $11\frac{3}{4}$ lb. of sugar cost?

5. In a magazine of 160 pages, 45 pages were devoted to advertisements. What part of the magazine was devoted to advertisements?

6. Two boys caught 8 fish. The combined weight of the fish was $10\frac{1}{2}$ lb. What was their average weight?

161. Multiplying a Fraction by a Fraction.

1. Show by a diagram or with objects what is meant by $\frac{2}{3}$ of 6 ft.; by $\frac{2}{3}$ of 1 ft.; by $\frac{2}{3}$ of $\frac{1}{2}$ ft. $\frac{2}{3}$ of $\frac{1}{2}$ ft. is what part of 1 ft.?

2. Draw a line and divide it into 5 equal parts. What is each part called? Show $\frac{1}{4}$ of one of these parts. $\frac{1}{4}$ of $\frac{1}{5}$ of the line is what part of the line?

3. Draw a line and divide it into 4 equal parts. Show $\frac{1}{5}$ of one of these parts. $\frac{1}{5}$ of $\frac{1}{4}$ of a line is what part of the line?

4. Draw a line and divide it into 3 equal parts. Show $\frac{1}{3}$ of one of these parts. Show $\frac{2}{3}$ of one of these parts. $\frac{1}{3}$ of $\frac{1}{3}$ of the line is what part of the entire line?

5. How much is $\frac{1}{2}$ of 6 da.? $\frac{1}{2}$ of 6 sevenths?

6. How much is $\frac{1}{3}$ of $\frac{9}{10}$? of $\frac{6}{11}$? of $\frac{12}{13}$?

7. When you know what $\frac{1}{3}$ of a number is, how can you find $\frac{2}{3}$ of the number? When you know what $\frac{1}{4}$ of a fraction is, how can you find $\frac{3}{4}$ of the fraction?

8. State how you would find $\frac{1}{4}$ of $\frac{12}{13}$; $\frac{1}{3}$ of $\frac{15}{17}$; $\frac{1}{6}$ of $\frac{12}{13}$.

9. State how you would find $\frac{1}{3}$ of $\frac{6}{7}$; $\frac{2}{3}$ of $\frac{6}{7}$; $\frac{1}{4}$ of $\frac{8}{9}$; $\frac{3}{4}$ of $\frac{8}{9}$.

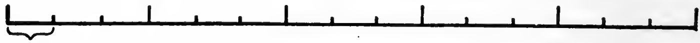
10. Divide rectangles to show $\frac{2}{3}$ of $\frac{3}{4}$; $\frac{3}{4}$ of $\frac{4}{5}$; $\frac{2}{3}$ of $\frac{6}{7}$; $\frac{5}{6}$ of $\frac{10}{12}$.

11. Show by a diagram that $\frac{1}{4}$ of $\frac{1}{3}$ is $\frac{1}{12}$. Since $\frac{1}{4}$ of $\frac{1}{3}$ is $\frac{1}{12}$, $\frac{3}{4}$ of $\frac{1}{3}$ is how many times $\frac{1}{12}$?

12. Divide rectangles to show that $\frac{1}{4}$ of $\frac{1}{3}$ is equivalent to $\frac{1}{3}$ of $\frac{1}{4}$.

13. Divide rectangles to show $\frac{3}{4}$ of $\frac{1}{2}$ and $\frac{1}{2}$ of $\frac{3}{4}$; $\frac{2}{3}$ of $\frac{3}{4}$ and $\frac{3}{4}$ of $\frac{2}{3}$; $\frac{3}{8}$ of $\frac{1}{2}$ and $\frac{1}{2}$ of $\frac{3}{8}$.

14. Draw a line. Show $\frac{1}{5}$ of the line. Show $\frac{2}{5}$ of the line. Show $\frac{1}{3}$ of $\frac{1}{5}$ of the line. What part of the line is $\frac{1}{3}$ of $\frac{1}{5}$ of the line? Show $\frac{1}{3}$ of $\frac{2}{5}$ of the line. $\frac{1}{3}$ of $\frac{2}{5}$ of the line is what part of the line? Show $\frac{2}{3}$ of $\frac{2}{5}$ of the line. $\frac{2}{3}$ of $\frac{2}{5}$ of the line is what part of the line?



$\frac{1}{3}$ of $\frac{1}{5}$ of the line $= \frac{1}{15}$ of the line.

$\frac{2}{5}$ of the line $= \frac{6}{15}$ of the line.
$\frac{1}{3}$ of $\frac{2}{5}$ of the line $= \frac{2}{15}$ of the line. $\frac{1}{3}$ of $\frac{2}{5} = \frac{2}{15}$.

$\frac{2}{3}$ of $\frac{2}{5}$ of the line $= \frac{4}{15}$ of the line. $\frac{2}{3}$ of $\frac{2}{5} = \frac{4}{15}$.

15. Draw a line. Show $\frac{1}{5}$ of the line. Show $\frac{2}{5}$ of the line. Show $\frac{1}{4}$ of $\frac{1}{5}$ of the line. $\frac{1}{4}$ of $\frac{1}{5}$ of the line is what part of the line? Show $\frac{1}{4}$ of $\frac{2}{5}$ of the line. $\frac{1}{4}$ of $\frac{2}{5}$ of the line is what part of the line? Show $\frac{3}{4}$ of $\frac{2}{5}$ of the line. $\frac{3}{4}$ of $\frac{2}{5}$ of the line is what part of the line?

16. Finding $\frac{1}{2}$ of a number is the same as multiplying the number by $\frac{1}{2}$. Finding $\frac{2}{5}$ of a number is the same as multiplying the number by $\frac{2}{5}$. Examine the illustrations under Ex. 14, and tell how $\frac{1}{3}$ of $\frac{1}{5}$ of a number is found; $\frac{1}{3}$ of $\frac{2}{5}$ of a number; $\frac{2}{3}$ of $\frac{2}{5}$ of a number.

17. Since $\frac{1}{3}$ of $\frac{1}{5}$ is $\frac{1}{15}$, $\frac{1}{3}$ of $\frac{2}{5}$ is $\frac{2}{15}$, and $\frac{2}{3}$ of $\frac{2}{5}$ is $\frac{4}{15}$. In each case the product of the numerators is the numerator of the answer, and the product of the denominators is the denominator of the answer.

To multiply a fraction by a fraction, multiply the numerators for the numerator of the product and the denominators for the denominator of the product. Before multiplying, cancel factors common to both terms.

162. Written Exercises.

1. Multiply $\frac{24}{25}$ by $\frac{5}{8}$.

MODEL:

$$\frac{\overset{3}{\cancel{24}}}{\underset{5}{\cancel{25}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{8}}} = \frac{3}{5}.$$

8 is a common factor of 24 and 8. 5 is a common factor of 5 and 25. Cancel these common factors and multiply.

Solve. Before multiplying, cancel common factors:

	a	*b*	*c*	*d*	*e*
2.	$\frac{8}{9} \times \frac{2}{3}$	$\frac{21}{25} \times \frac{3}{7}$	$\frac{48}{69} \times \frac{13}{24}$	$\frac{15}{4} \times \frac{7}{8}$	$\frac{45}{59} \times \frac{13}{15}$
3.	$\frac{7}{8} \times \frac{4}{5}$	$\frac{15}{22} \times \frac{11}{12}$	$\frac{5}{7} \times \frac{2}{3}$	$\frac{7}{2} \times \frac{9}{5}$	$\frac{64}{72} \times \frac{16}{18}$
4.	$\frac{3}{4} \times \frac{2}{3}$	$\frac{18}{25} \times \frac{5}{9}$	$\frac{8}{9} \times \frac{4}{5}$	$\frac{7}{8} \times \frac{10}{21}$	$\frac{100}{3} \times \frac{9}{10}$
5.	$\frac{6}{7} \times \frac{3}{8}$	$\frac{13}{17} \times \frac{3}{4}$	$\frac{11}{2} \times \frac{2}{3}$	$\frac{4}{5} \times \frac{11}{2}$	$\frac{144}{53} \times \frac{2}{5}$

163. Written Exercises.

Change the mixed numbers to improper fractions and solve:

	a	*b*	*c*	*d*
1.	$3\frac{4}{5} \times \frac{5}{7}$	$\frac{6}{7} \times 2\frac{1}{3}$	$2\frac{3}{5} \times 4\frac{4}{7}$	$1\frac{3}{8} \times 3\frac{1}{8}$
2.	$4\frac{1}{2} \times \frac{5}{6}$	$\frac{5}{8} \times 6\frac{3}{4}$	$6\frac{2}{3} \times 4\frac{5}{9}$	$4\frac{1}{3} \times 2\frac{1}{2}$
3.	$3\frac{2}{3} \times \frac{4}{7}$	$\frac{2}{9} \times 4\frac{2}{5}$	$7\frac{2}{3} \times 4\frac{3}{8}$	$6\frac{8}{9} \times 4\frac{7}{8}$
4.	$5\frac{1}{2} \times \frac{8}{9}$	$\frac{8}{13} \times 5\frac{1}{5}$	$9\frac{5}{6} \times 6\frac{5}{9}$	$3\frac{4}{5} \times 2\frac{1}{4}$

164. Written Exercises.

Review Secs. 152, 153, and 156.

1. Multiply $45\frac{2}{3}$ by $\frac{5}{6}$.

MODEL:

$$\begin{array}{rl} 45\frac{2}{3} & \\ \frac{5}{6} & \\ \hline 7\frac{11}{18} & (\frac{1}{6} \text{ of } 45\frac{2}{3}) \\ 5 & \\ \hline 38\frac{1}{18} & (\frac{5}{6} \text{ of } 45\frac{2}{3}) \end{array}$$

$\frac{1}{6}$ of $45\frac{2}{3}$ is $7\frac{11}{18}$; $\frac{5}{6}$ of $45\frac{2}{3} = 5$ times $7\frac{11}{18}$, or $38\frac{1}{18}$.

Solve without reducing the mixed numbers to improper fractions :

	a	*b*	*c*	*d*	*e*
2.	$34\frac{3}{4} \times \frac{3}{5}$	$84\frac{4}{7} \times \frac{3}{7}$	$546\frac{3}{4} \times \frac{5}{6}$	$654\frac{3}{8} \times \frac{4}{7}$	$840\frac{5}{6} \times \frac{4}{7}$
3.	$18\frac{4}{5} \times \frac{5}{6}$	$55\frac{2}{3} \times \frac{5}{6}$	$385\frac{2}{3} \times \frac{3}{5}$	$235\frac{5}{6} \times \frac{3}{4}$	$468\frac{3}{4} \times \frac{2}{3}$
4.	$72\frac{6}{7} \times \frac{7}{8}$	$40\frac{1}{2} \times \frac{3}{5}$	$463\frac{1}{3} \times \frac{3}{7}$	$900\frac{9}{10} \times \frac{2}{3}$	$479\frac{7}{8} \times \frac{3}{5}$
5.	$48\frac{5}{9} \times \frac{2}{3}$	$38\frac{7}{8} \times \frac{2}{9}$	$847\frac{4}{9} \times \frac{2}{3}$	$783\frac{3}{4} \times \frac{4}{5}$	$673\frac{2}{3} \times \frac{3}{8}$
6.	$96\frac{6}{7} \times \frac{5}{8}$	$94\frac{1}{3} \times \frac{7}{8}$	$170\frac{8}{11} \times \frac{4}{5}$	$680\frac{7}{10} \times \frac{3}{10}$	$574\frac{4}{5} \times \frac{5}{9}$
7.	$48\frac{1}{3} \times \frac{4}{9}$	$63\frac{7}{8} \times \frac{7}{9}$	$431\frac{1}{6} \times \frac{7}{9}$	$598\frac{1}{7} \times \frac{7}{8}$	$650\frac{1}{2} \times \frac{7}{8}$

165. Written Exercises.

1. Multiply $349\frac{2}{3}$ by $3\frac{3}{5}$.

MODEL: First multiply $349\frac{2}{3}$ by $\frac{3}{5}$. Next multiply $349\frac{2}{3}$ by 3. Add the products.

$$\begin{array}{rl}
349\frac{2}{3} & \\
3\frac{3}{5} & \\
\hline
69\frac{14}{15} & (\tfrac{1}{5} \text{ times } 349\tfrac{2}{3}) \\
3 & \\
\hline
209\frac{4}{5} & (\tfrac{3}{5} \text{ times } 349\tfrac{2}{3}) \\
1049 & (3 \text{ times } 349\tfrac{2}{3}) \\
\hline
1258\frac{4}{5} &
\end{array}$$

Solve without reducing the mixed numbers to improper fractions :

	a	*b*	*c*	*d*
2.	$645\frac{2}{5} \times 6\frac{3}{4}$	$584\frac{5}{12} \times 4\frac{3}{4}$	$963\frac{7}{8} \times 7\frac{7}{9}$	$642\frac{3}{4} \times 7\frac{1}{3}$
3.	$867\frac{4}{7} \times 5\frac{4}{5}$	$982\frac{2}{7} \times 8\frac{3}{5}$	$333\frac{1}{3} \times 6\frac{2}{3}$	$789\frac{9}{10} \times 7\frac{2}{3}$
4.	$694\frac{3}{8} \times 7\frac{1}{4}$	$648\frac{5}{9} \times 5\frac{1}{4}$	$781\frac{1}{8} \times 9\frac{3}{5}$	$537\frac{3}{13} \times 6\frac{1}{2}$
5.	$748\frac{7}{9} \times 5\frac{3}{4}$	$457\frac{3}{5} \times 7\frac{4}{9}$	$450\frac{5}{9} \times 7\frac{2}{3}$	$521\frac{1}{3} \times 2\frac{3}{4}$
6.	$384\frac{2}{3} \times 4\frac{3}{4}$	$926\frac{1}{8} \times 2\frac{3}{4}$	$467\frac{1}{3} \times 9\frac{1}{2}$	$830\frac{4}{5} \times 4\frac{4}{7}$
7.	$412\frac{3}{8} \times 6\frac{2}{3}$	$726\frac{4}{9} \times 8\frac{7}{8}$	$940\frac{4}{7} \times 3\frac{2}{5}$	$590\frac{3}{11} \times 7\frac{1}{2}$
8.	$240\frac{4}{5} \times 4\frac{3}{7}$	$948\frac{3}{5} \times 7\frac{4}{11}$	$640\frac{4}{9} \times 7\frac{3}{8}$	$810\frac{3}{4} \times 3\frac{3}{7}$

166. Review Questions.

1. What is a proper fraction? an improper fraction? a mixed number?

2. Write 5 proper fractions; 5 improper fractions; 5 mixed numbers.

3. What is a fractional unit? How many fractional units are expressed in $\frac{5}{8}$?

4. What is meant by a factor of a number? Illustrate.

5. What is meant by a multiple of a number? Illustrate.

6. When is a fraction said to be in its lowest terms? Write five fractions that are in their lowest terms.

7. How may a fraction be reduced to its lowest terms? Write five fractions and reduce them to their lowest terms.

8. How may an improper fraction be reduced to a whole or a mixed number? Write five improper fractions and reduce them to whole or mixed numbers.

9. How may a mixed number be changed to an improper fraction? Write five mixed numbers and change them to improper fractions.

10. What effect upon the value of a fraction has multiplying or dividing both terms of the fraction by the same number? Illustrate.

11. What is cancellation? Illustrate.

12. State two ways in which a fraction may be multiplied by an integer. Illustrate. When a proper fraction is multiplied by an integer, is the product greater or less than the multiplicand? Why? Is the product greater or less than the multiplier? Why? Illustrate.

13. When an integer is multiplied by a proper fraction, is the product greater or less than the multiplicand? than the multiplier? Why? Illustrate.

167. Dividing by a Fraction.

1. Draw a line 4 ft. long. Make a measure $\frac{1}{2}$ ft. long. Apply this measure to the line. How many times must the measure $\frac{1}{2}$ ft. be applied to measure a 4-ft. line?

2. After finding how many times the measure $\frac{1}{2}$ ft. must be applied to measure 1 ft., how may you find, without performing the actual measurement, how many times the measure must be applied to measure a 4-ft. line?

3. Repeat Exs. 1 and 2 above, using a $\frac{1}{3}$-ft. measure to measure a 6-ft. line.

4. As the measure $\frac{1}{2}$ ft. must be applied 2 times to measure 1 ft., to measure any given number of feet it must be applied as many times 2 as the number of feet to be measured. To measure 24 ft., it must be applied 24 times 2, or 48 times. How many times must the measure $\frac{1}{3}$ ft. be applied to measure 6 ft.? 10 ft.? 12 ft.?

5. To measure a line $\frac{1}{4}$ ft. long, the measure $\frac{1}{2}$ ft. must be applied $\frac{1}{4}$ times 2, or $\frac{1}{2}$ times. That is, one half of the measure must be applied.

6. The expression $\frac{1}{4}$ ft. $\div \frac{1}{2}$ ft. indicates that a line $\frac{1}{4}$ ft. in length is to be measured by a measure $\frac{1}{2}$ ft. in length. What is meant by each of the expressions: 6 ft. $\div \frac{1}{2}$ ft.? $\frac{1}{2}$ ft. $\div \frac{1}{2}$ ft.? $\frac{1}{12}$ ft. $\div \frac{1}{2}$ ft.?

7. Draw a line $\frac{1}{2}$ ft. long. Determine how many times each of the following measures must be applied to measure it: $\frac{1}{4}$ ft., $\frac{1}{6}$ ft., $\frac{1}{12}$ ft.

8. The measure $\frac{2}{3}$ ft. must be applied how many times to measure a 1-ft. line? $1\frac{1}{2} = \frac{3}{2}$. It must be applied $\frac{3}{2}$ times. How many times must it be applied to measure a 4-ft. line? a 6-ft. line? a 15-ft. line? What is meant by the expression 18 ft. $\div \frac{2}{3}$ ft.?

9. If a $\frac{2}{3}$-ft. measure must be applied $\frac{3}{2}$ times to measure a 1-ft. line, to measure a $\frac{3}{4}$-ft. line it must be applied $\frac{3}{4}$ times $\frac{3}{2}$, or $\frac{9}{8}$ times.

10. To measure a 1-ft. line, a 2-ft. measure must be applied $\frac{1}{2}$ time. That is, one half of the measure must be applied. How many times must the measure 3 ft. be applied to measure a 1-ft. line?

11. Determine how many times the measure in each must be applied to measure 1 ft., and solve each: 5 ft. $\div \frac{1}{3}$ ft.; 3 ft. $\div \frac{1}{4}$ ft.; $\frac{1}{2}$ ft. $\div$ 2 ft.; $\frac{1}{3}$ ft. $\div$ 3 ft.; 4 ft. $\div \frac{1}{6}$ ft.

12. To measure a 1-ft. line, the measure $\frac{1}{2}$ ft. must be used 2 times. 2 is the *reciprocal* of $\frac{1}{2}$. To measure a 1-ft. line, the measure 2 ft. must applied $\frac{1}{2}$ times. $\frac{1}{2}$ is the reciprocal of 2.

13. The reciprocal of 4 is $\frac{1}{4}$; of 3 is $\frac{1}{3}$; of $\frac{1}{3}$ is 3; of $\frac{2}{3}$ is $\frac{3}{2}$; of $\frac{4}{5}$ is $\frac{5}{4}$. If $\frac{4}{5}$ is used as a measure to measure 1, the quotient is $\frac{5}{4}$. Multiply $\frac{4}{5}$ by $\frac{5}{4}$. The product is 1.

14. When the product of two numbers is 1, the numbers are said to be **reciprocals** of each other.

15. The reciprocal of the number used as divisor shows the number of times the divisor is contained in a unit, thus: The divisor $\frac{1}{3}$ is contained in 1 three times. The divisor $\frac{5}{6}$ is contained in 1 $\frac{6}{5}$ times. Hence the following rule:

To divide by a fraction, multiply the reciprocal of the divisor by the dividend.

16. What is the reciprocal of each: $\frac{3}{5}$? $\frac{4}{7}$? $\frac{6}{7}$? $\frac{1}{10}$? $\frac{1}{5}$? $2\frac{1}{2}$? $\frac{5}{2}$? $\frac{4}{3}$? $\frac{11}{12}$? $\frac{1}{6}$? $1\frac{1}{7}$? $\frac{1}{20}$?

17. Compare the terms of a fraction with the terms of the reciprocal of the fraction. When the terms of a fraction are interchanged, the fraction is said to be **inverted**.

168. Written Exercises.

1. Divide $6\frac{3}{4}$ by $\frac{5}{6}$.

MODEL: $6\frac{3}{4} \div \frac{5}{6} = \frac{27}{\underset{2}{\cancel{4}}} \times \frac{\overset{3}{\cancel{6}}}{5} = \frac{81}{10} = 8\frac{1}{10}$

Solve:

2. $3\frac{3}{4}$ yd. $\div \frac{2}{3}$ yd.
3. $5\frac{1}{2}$ yd. $\div \frac{4}{7}$ yd.
4. $6\frac{4}{7}$ wk. $\div \frac{3}{7}$ wk.
5. $8\frac{2}{3}$ yd. $\div \frac{3}{4}$ yd.
6. $\$7\frac{3}{5} \div \$\frac{3}{4}$
7. $3\frac{5}{12}$ in. $\div \frac{2}{3}$ in.
8. $8\frac{4}{9} \div 4\frac{3}{4}$
9. $7\frac{1}{3} \div 5\frac{3}{7}$
10. $6\frac{5}{8} \div 6\frac{3}{5}$
11. $4\frac{2}{9} \div 6\frac{2}{3}$
12. $5\frac{2}{5} \div 2\frac{1}{10}$
13. $7\frac{3}{8} \div 8\frac{1}{4}$
14. $\frac{6}{7} \div \frac{4}{5}$
15. $\frac{9}{10} \div \frac{4}{15}$
16. $\frac{8}{13} \div \frac{1}{2}$
17. $\frac{4}{21} \div \frac{2}{7}$
18. $\frac{12}{13} \div \frac{6}{7}$
19. $3\frac{1}{4} \div 2\frac{1}{2}$
20. $8\frac{11}{15} \div 9\frac{6}{7}$
21. $7\frac{17}{18} \div 4\frac{9}{10}$
22. $16\frac{2}{3} \div 14\frac{2}{7}$
23. $6\frac{3}{8} \div 12\frac{3}{5}$
24. $7\frac{1}{3} \div 6\frac{21}{25}$
25. $24\frac{3}{4} \div 8$
26. Divide 100 by $33\frac{1}{3}$; by $66\frac{2}{3}$; by $37\frac{1}{2}$; by $87\frac{1}{2}$.
27. Divide $\frac{12}{13}$ by 4; $\frac{12}{13}$ by 5; $316\frac{7}{8}$ by 8; $435\frac{4}{5}$ by 27.
28. Multiply $635\frac{2}{3}$ by 8; $315\frac{4}{5}$ by 8; $80\frac{4}{7}$ by 9.
29. Add $8\frac{3}{4}$, $4\frac{4}{5}$, $3\frac{1}{2}$, $6\frac{5}{12}$, $8\frac{7}{15}$.
30. Take $32\frac{4}{7}$ from $96\frac{2}{9}$. From $80\frac{3}{8}$ take $19\frac{4}{5}$.
31. Divide 8.125 by .04; $80.40 by .05; 725 by 1.25.
32. Multiply 3.1416 by $4\frac{3}{4}$; .7854 by $6\frac{1}{4}$; 2150.42 by $60\frac{3}{4}$.

169. Oral Exercises.

1. Divide each by 100: $43, 3.14, 60.75, .9, 2000.

2. Add $\frac{1}{2}$ and $\frac{1}{3}$; $\frac{1}{3}$ and $\frac{1}{4}$; $\frac{1}{4}$ and $\frac{1}{5}$. State a short method of getting the sum of two fractions whose numerators are 1 and whose denominators are prime to each other.

170. **1.** How many strips of carpet 1 yd. wide will it take to cover a room 7 yd. wide?

2. How many strips of carpet $\frac{3}{4}$ yd. wide will it take to cover a room 9 yd. wide? 6 yd. wide? 3 yd. wide?

3. Draw a diagram of a room 24 ft. long and 18 ft. wide. Show on the diagram the number of strips of carpet $\frac{3}{4}$ yd. wide that are needed to cover the floor, the strips running lengthwise of the room.

4. What is the length in yards of each strip (Prob. 3)? How many yards of carpet are needed to cover the room, making no allowance for matching the strips?

5. At 75¢ per yard, how much will it cost for carpet for a room 28 ft. long and 18 ft. wide, the carpet being 27 in. wide, the strips running lengthwise of the room?

6. How many ribbons each $\frac{2}{3}$ yd. long can be made of 8 yd. of ribbon? of 12 yd.? of 18 yd.? of 6 yd.?

7. At 5¢ a pound, how many pounds of sugar can be bought for 40¢?

8. How many pounds of sugar can be bought for \$5, at $4\frac{1}{4}$¢ a pound? at $4\frac{3}{4}$¢ a pound? at $5\frac{1}{4}$¢ a pound?

9. How many strips of matting 42 in. ($1\frac{1}{6}$ yd.) wide will it take to cover a room 21 ft. (7 yd.) wide?

10. If a certain lamp consumes $\frac{1}{2}$ pt. of oil each evening, how long will a gallon of oil last? 5 gal.?

11. What part of 1 yd. is 1 ft.? 30 in.? 32 in.? 27 in.?

171. **1.** How many times must $2\frac{2}{5}$ T. be written as an addend so the sum of the column will be 24 T.?

2. If a boy earns \$$\frac{3}{4}$ a day, in how many days will he earn \$15?

3. 1 yd. of cloth will cost how many times the cost of $\frac{3}{4}$ yd.?

4. If a dealer charges \$6 for $\frac{3}{4}$ T. of coal, what is the price of the coal per ton?

5. Find the cost of 1 yd. of lace if $2\frac{1}{2}$ yd. cost 75¢.

6. At \$$1\frac{1}{3}$ per yard, what will be the cost of $8\frac{1}{4}$ yd. of silk?

7. Find the area of a rectangle $8\frac{1}{2}$ in. by $6\frac{3}{4}$ in.; of a square whose side is $2\frac{2}{3}$ ft.

8. How many pounds of meat at $12\frac{1}{2}$¢ a pound can be bought for 75¢?

9. If $3\frac{1}{3}$ lb. of coffee are sold for \$1, how many pounds can be bought for \$6? for \$3? for \$9? for \$12?

10. If 1 lb. of tea costs \$$\frac{3}{5}$, how many pounds can be bought for \$$2\frac{2}{5}$? for \$6? for \$12?

11. A tailor used $2\frac{3}{4}$ yd. of cloth for each pair of trousers. How many pairs can be made from 22 yd.?

12. Change to feet: 6 in.; 9 in.; 10 in.; 3 in.; 4 in.

13. Change to inches: $\frac{3}{4}$ ft.; $\frac{2}{3}$ ft.; $\frac{5}{6}$ ft.; $\frac{1}{6}$ ft.; $\frac{1}{12}$ ft.

14. Change to months: $\frac{1}{4}$ yr.; $\frac{1}{6}$ yr.; $\frac{2}{3}$ yr.; $\frac{5}{6}$ yr.; $\frac{3}{4}$ yr.

15. The atmosphere presses equally in all directions with a pressure of about 15 lb. to the square inch. Find the pressure on the top of your desk.

16. If the circumference of the wheel of a bicycle is $6\frac{1}{4}$ ft., how many times will the wheel turn in going 1 mi.?

17. The cost of laying a concrete sidewalk at $11\frac{1}{2}$¢ per square foot was \$34.50. Find the area of the sidewalk. If the walk was 6 ft. wide, how long was it?

172. Review Exercises.

1. Find the perimeter of a rectangle 8 ft. 6 in. wide and 12 ft. 9 in. long. Find its area.

2. Find the number of square feet of blackboard surface in the schoolroom.

3. The diameter of a cylindrical tank is 6.5 ft. Find its circumference. (circum. = diam. $\times$ $3\frac{1}{7}$.)

4. A farmer asked his two boys, George and Frank, to figure out the number of posts necessary to build a fence 28 rd. long, the posts to be placed $\frac{1}{2}$ rd. apart. George said it would take 56 posts, and Frank said it would take 57. Was either boy's answer correct?

5. The farmer (Prob. 4) asked the boys to find the number of posts necessary to build a fence around a garden 6 rd. by 8 rd., the posts to be placed $\frac{1}{2}$ rd. apart. Both boys said it would take 57 posts. Was the answer correct?

6. If a man had 60 sheep and sold $\frac{5}{6}$ of them, how many did he sell? How many did he have left?

7. There are 640 acres in 1 square mile. How many are there in $\frac{3}{4}$ of a square mile?

8. If $\frac{3}{5}$ of the distance between two cities is 15 mi., how far apart are the cities?

9. Mary's age is 12 years. She is $\frac{3}{4}$ as old as Ethel. How old is Ethel?

10. The cost of 15 T. of hay was \$112.50. What was the cost per ton?

11. At 75¢ a yard, how many yards of cloth can be bought for \$15?

12. Find the value of the potato crop on 50 acres, if the average yield is 125 sacks to the acre and the potatoes are worth $1.20 per sack.

13. Draw three dials, as in Sec. 97, and fix the hands to read 84,500 cu. ft.; 32,800 cu. ft.; 47,000 cu. ft.

14. Is the height of any mountain given in your geography text? If so, express the height of some mountains in miles.

15. Draw a line 20 in. long. Test it with a ruler.

16. If the height of your schoolroom is 12 ft. 6 in., find the distance from the highest point on your desk to the ceiling.

17. Find the area of your desk top.

18. Find the area of a blackboard in your schoolroom.

19. Draw a diagram to show the ratio of 2 in. to 4 in.; of 4 in. to 6 in.; of 8 in. to 12 in.

20. What number expresses the ratio of 3 in. to 6 in.? of 6 in. to 3 in.? of 8 in. to 4 in.? of 4 in. to 12 in.?

21. Express the value of $5\frac{2}{5}$ in the fractional unit $\frac{1}{5}$; of $4\frac{3}{7}$ in the fractional unit $\frac{1}{7}$; of $8\frac{5}{6}$ in the fractional unit $\frac{1}{6}$.

22. From a piece of cloth containing $22\frac{1}{4}$ yd. a tailor used $17\frac{3}{4}$ yd. How many yards remained in the piece?

23. How much heavier is Mary than Ethel, if Mary weighs $101\frac{1}{4}$ lb. and Ethel weighs $92\frac{3}{4}$ lb.? Find the combined weight of the two girls.

24. What effect upon the value of a fraction has multiplying or dividing both terms by the same number? Illustrate, using $\frac{4}{12}$.

25. Show by a diagram that $\frac{2}{3}$ of 3 ft. is the same as $\frac{1}{3}$ of 2 times 3 ft.

173. Finding what Fraction One Number is of Another.

1. What fraction of a dollar is $33\frac{1}{3}$¢?

MODEL: $33\frac{1}{3}$¢ is $\frac{33\frac{1}{3}}{100}$ of a dollar. Performing the indicated division: $33\frac{1}{3} \div 100$, or $\frac{100}{3} \times \frac{1}{100} = \frac{1}{3}$.

To find what fraction one number is of another, take the number denoting the part for the numerator and the number denoting the whole for the denominator. Express the result in its simplest form.

What fraction of:

2. 8 is 5?
3. 15 is 10?
4. 20 is 24?
5. 16 is 10?
6. 36 is 30?
7. 100 is 25?
8. 100 is 150?
9. 100 is 75?
10. 100 is 60?
11. 100 is 125?
12. 100 is 175?
13. 100 is 120?
14. 100 is 40?
15. 100 is $87\frac{1}{2}$?
16. 100 is $12\frac{1}{2}$?
17. 100 is $37\frac{1}{2}$?
18. 100 is $137\frac{1}{2}$?
19. 100 is $133\frac{1}{3}$?
20. 100 is $66\frac{2}{3}$?
21. 100 is $166\frac{2}{3}$?
22. 100 is $112\frac{1}{2}$?

23. What fraction of a dollar is 75¢? 50¢? 25¢? $12\frac{1}{2}$¢? 20¢? 60¢? $37\frac{1}{2}$¢? $33\frac{1}{3}$¢? $66\frac{2}{3}$¢? $87\frac{1}{2}$¢? $62\frac{1}{2}$¢? 80¢? 70¢? 40¢? 150¢? 125¢? 175¢? $14\frac{2}{7}$¢ $16\frac{2}{3}$¢? $112\frac{1}{2}$¢? 140¢? $6\frac{1}{4}$¢?

24. If a train travels at an average rate of 40 mi. per hour, in what part of an hour will the train travel 15 mi.? 25 mi.? 60 mi.? 75 mi.? 100 mi.?

25. If a 20-acre field produced $600 worth of wheat in a certain year, at the same rate what part of this amount would a 10-acre field have produced? a 30-acre field? a 15-acre field? a 25-acre field? a 60-acre field?

DRAWING TO A SCALE

174. 1. Lines and surfaces are frequently represented by drawings. As most lines and surfaces are too large to be drawn full size, they have to be drawn on a reduced **scale.**

2. By letting $\frac{1}{4}$ in. represent 1 ft., a line 8 ft. long may be represented thus:

8 ft.

Scale $\frac{1}{4}$ in. = 1 ft.

3. Using the scale $\frac{1}{4}$ in. = 1 ft., represent a line 12 ft. long; 5 ft. long; 20 ft. long; 16 ft. long.

4. Using the scale $\frac{1}{2}$ in. = 1 ft., represent a square whose side is 4 ft.; 6 ft.; 12 ft.

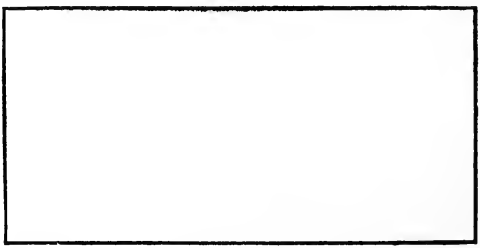
5. Using the scale $\frac{1}{4}$ in. = 1 yd., represent a line 12 yd. in length and a line 8 yd. in length.

6. Using a convenient scale, represent a rectangle 12 ft. by 10 ft.; a garden 16 ft. by 12 ft.

7. Using the scale $\frac{1}{8}$ in. = 1 ft., represent a school garden 24 ft. by 16 ft. Find the area of this garden.

8. Using a convenient scale, represent a sidewalk 6 ft. wide and 48 ft. long. Find the area of this walk.

9. Below is a diagram of a school yard, drawn to the scale $\frac{1}{8}$ in. = 20 ft. Find the dimensions of the yard. Find its area.



175. 1. Find the scale to which each of the following lines has been drawn:

A 60 ft.

B 90 ft.

C 150 ft.

D 45 ft.

E 120 yd.

2. Find the dimensions of the floor of your schoolroom. Using a convenient scale, draw a floor plan of your schoolroom.

3. Find the dimensions of the school grounds. Using a convenient scale, draw a plan of the school grounds. Show the ground plan of the schoolhouse properly located and in correct proportions.

4. The relative areas of the several oceans are represented by the lines below, as follows: *A*, Arctic; *B*, Antarctic; *C*, Indian; *D*, Atlantic; *E*, Pacific. The numbers above the lines indicate the number of millions of square miles in each.

A 4

B 7.5

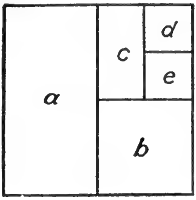
C 28

D 34

E 71

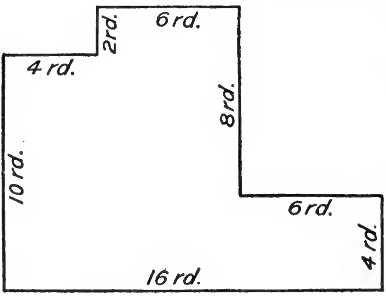
5. Using a convenient scale, represent the relative population of Asia, Europe, Africa, North America, South America.

176. 1. This figure represents a section of land. Find the dimensions of each division; the number of acres in each; the cost of the section; and of each division at $45 per acre. (Section = 1 sq. mi. = 640 A.)



2. A field containing 10 A. is 40 rd. long. How wide is it? Draw it to a scale.

3. This figure represents a garden. What is the scale? Draw the plan of this garden on a scale twice as large as the figure, viz. $\frac{1}{4}$ in. = 1 rd. Divide the garden into three rectangles. Find the area of each rectangle. Find the area of the garden; the perimeter. Find the cost of fencing this garden at $1.75 per rod.

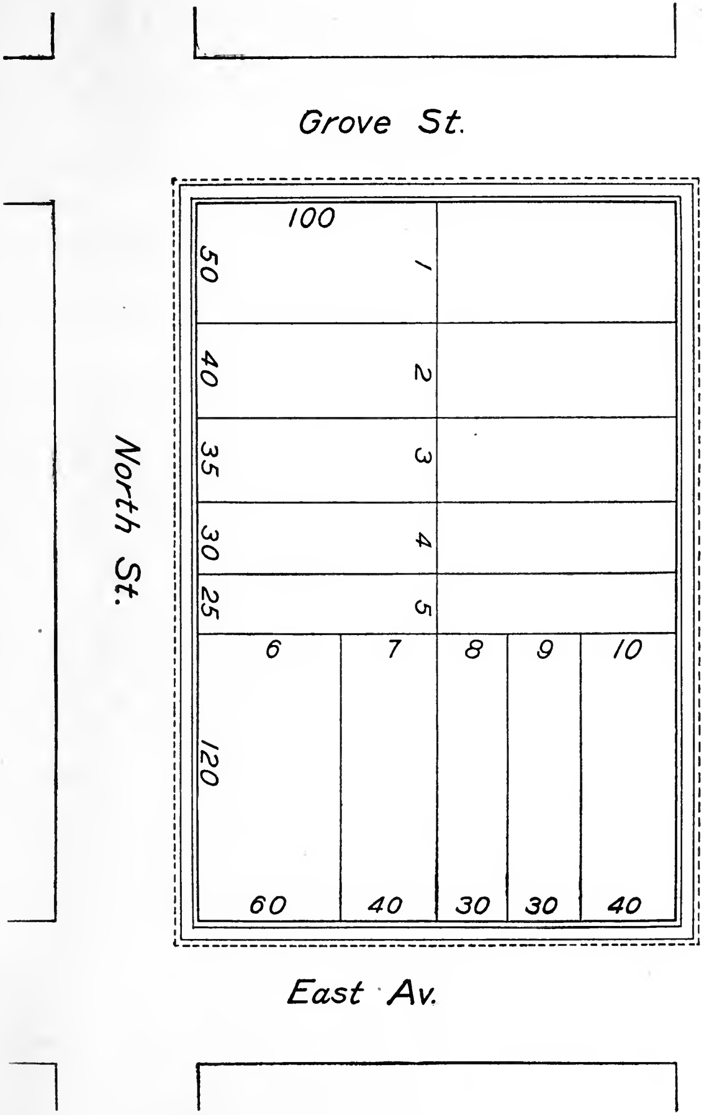


4. Draw to a scale the side of your schoolroom and locate the openings. Mark the dimensions on your drawing.

5. Draw lines to represent the population of New York City, Chicago, Philadelphia, London, and Paris, using the same scale for each.

177. City Lots.

1. The figure on p. 135 is a diagram of a city block. The dimensions are expressed in feet. Using your ruler, determine the scale used in making this diagram. Find the width of East Avenue.


Grove St.
North St.
East Av.
100
50
40
35
30
25
120
1
2
3
4
5
6
7
8
9
10
60
40
30
30
40

2. Find the length and the width of the block.

3. Find the area of each of the lots.

4. Lot 7 was sold at \$35 per front foot. Find the selling price of the lot.

5. At the same price per front foot, what is the value of each of the other lots?

6. At \$45 per front foot, what is the value of Lot 6 (60 ft. front)?

7. Lot 1 was sold for \$2250. How much was this per front foot?

8. Lot 10 was sold for \$1600. How much was this per front foot?

9. The selling price of Lot 5 was \$750. How much was this a front foot?

10. From its location in the block, which should be worth the more per front foot, Lot 7 or Lot 10? Lot 4 or Lot 5?

11. Find the cost of laying a 6-ft. cement sidewalk in front of Lot 9 at 12¢ per square foot.

12. Find the cost of laying a 6-ft. cement sidewalk in front of Lot 2 at 13¢ per square foot.

13. Mr. Thomas bought Lot 8 at \$35 per front foot. He sold it for \$1200. Did he gain or lose, and how much?

14. Mr. Brown paid \$35 per front foot for Lots 3 and 4. He sold both lots for \$2500. Did he gain or lose, and how much?

15. Mr. Newton paid \$35 per front foot for Lot 7. He had a 6-ft. cement sidewalk laid in front of the lot, costing 12¢ per square foot. He afterward sold the lot for \$1500. Did he gain or lose, and how much?

REVIEW

178. Written Exercises.

1. Add 34.125, 4.36, 180.006, .67, 3.1416, 10.07.

2. Solve: $326.87 - 83.65$; $9.82785 - 4.003$; $346.85 - 184$.

3. Multiply: 32.064 by .045; \$465.73 by .08; \$2456 by .06.

4. Divide: 13.046 by 1.8; \$43.78 by .06; \$120.78 by 1.06.

5. Write five improper fractions, and change each to a whole or a mixed number.

6. Write five mixed numbers, and change each to an improper fraction.

7. Write five proper fractions that are not in their lowest terms, and change each to lowest terms.

8. Add $6\frac{5}{6}$, $4\frac{3}{5}$, $7\frac{1}{8}$, $2\frac{1}{4}$, $14\frac{3}{10}$.

9. From $28\frac{4}{11}$ subtract $9\frac{9}{11}$. From $834\frac{3}{5}$ take $186\frac{7}{9}$.

10. Multiply $8\frac{2}{5}$ by $6\frac{8}{9}$; $683\frac{4}{5}$ by $3\frac{4}{7}$; $31\frac{1}{4}$ by 18.

11. Divide $654\frac{3}{8}$ by 9; $195\frac{1}{3}$ by 7; 200 by $3\frac{1}{4}$.

12. Divide $\frac{3}{4}$ by $\frac{4}{5}$; $6\frac{2}{3}$ by $7\frac{1}{2}$; $8\frac{7}{9}$ by $7\frac{8}{9}$.

13. Cancel and simplify: $\dfrac{12 \times 14 \times 21 \times 9}{16 \times 20 \times 15 \times 30}$.

14. Reduce to lowest terms: $\frac{51}{69}$, $\frac{72}{102}$, $\frac{112}{128}$, $\frac{111}{201}$, $\frac{42}{222}$.

15. Reduce to lowest terms: $\dfrac{75}{100}$, $\dfrac{66\frac{2}{3}}{100}$, $\dfrac{33\frac{1}{3}}{100}$, $\dfrac{40}{100}$, $\dfrac{12\frac{1}{2}}{100}$, $\dfrac{87\frac{1}{2}}{100}$, $\dfrac{37\frac{1}{2}}{100}$.

179. Oral Exercises.

Solve:

1. $12 \times \frac{1}{4}$
2. $24 \times \frac{1}{8}$
3. $15 \times \frac{1}{5}$
4. $10 \times \frac{3}{5}$
5. $18 \times \frac{5}{6}$
6. $14 \times \frac{3}{7}$
7. $30 \times \frac{5}{6}$
8. $16 \times \frac{5}{8}$
9. $27 \times \frac{4}{9}$
10. $\frac{3}{5}$ of $25
11. $\frac{7}{8}$ of $40
12. $\frac{2}{7}$ of 21 ft.
13. $\frac{3}{8}$ of 64 mi.
14. $\frac{4}{9}$ of 36 in.
15. $\frac{2}{3}$ of 18 in.
16. $\frac{6}{7}$ of 35 yd.
17. $\frac{3}{8}$ of 24 lb.
18. $\frac{2}{9}$ of 45 lb.
19. $\frac{5}{7}$ of 21 da.
20. $\frac{7}{12}$ of 84 yd.
21. $\frac{7}{8}$ of 48 mo.
22. $\frac{13}{15}$ of 30 da.
23. $\frac{8}{5}$ of $20
24. $\frac{3}{2}$ of $120
25. $\frac{7}{3}$ of $60
26. $\frac{3}{7}$ of $70
27. $\frac{6}{5}$ of $100

180. Oral Exercises.

Find the quotient of:

1. $\frac{8}{9} \div 4$
2. $\frac{12}{17} \div 6$
3. $\frac{14}{15} \div 7$
4. $\frac{10}{11} \div 5$
5. $\frac{2}{5} \div 6$
6. $\frac{5}{6} \div 10$
7. $\frac{2}{3} \div 2$
8. $\frac{4}{7} \div 8$
9. $1\frac{8}{11} \div 4$
10. $6\frac{5}{9} \div 3$
11. $3\frac{10}{11} \div 20$
12. $1\frac{5}{6} \div 10$
13. $2\frac{1}{2} \div 5$
14. $3\frac{3}{5} \div 8$
15. $4\frac{3}{4} \div 4$
16. $9\frac{4}{7} \div 4$

181. Oral Exercises.

Find the product of:

1. $\frac{1}{2}$ of $\frac{4}{5}$
2. $\frac{1}{3}$ of $\frac{4}{5}$
3. $\frac{2}{3}$ of $\frac{6}{7}$
4. $\frac{3}{5}$ of $\frac{1}{2}$
5. $\frac{3}{4}$ of $\frac{8}{11}$
6. $\frac{5}{7}$ of $\frac{1}{8}$
7. $\frac{6}{5}$ of $\frac{2}{5}$
8. $\frac{2}{5}$ of $\frac{5}{8}$
9. $\frac{3}{8}$ of $\frac{3}{5}$
10. $\frac{7}{10}$ of $\frac{10}{11}$
11. $\frac{3}{4}$ of $\frac{8}{9}$
12. $\frac{3}{4}$ of $\frac{6}{7}$
13. $\frac{6}{7} \times \frac{1}{6}$
14. $\frac{3}{4} \times \frac{2}{3}$
15. $\frac{2}{7} \times \frac{4}{5}$
16. $\frac{5}{6} \times \frac{8}{3}$
17. $\frac{3}{7} \times \frac{1}{8}$
18. $\frac{7}{8} \times \frac{16}{21}$

182. Oral Exercises.

1. If $\frac{3}{5}$ of a ton of coal costs \$6, what is the cost of a ton?

2. If $\frac{3}{8}$ of the cost of a farm is \$2400, what is the cost of the farm? What is $\frac{5}{8}$ of the cost of the farm?

3. If $\frac{2}{3}$ of the cost of a carriage is \$80, what is the cost of the carriage? What is $\frac{1}{2}$ of the cost of the carriage?

4. A farmer sold $\frac{4}{5}$ of his crop of oats for \$160. At the same rate, how much was the entire crop worth? $\frac{1}{4}$ of the crop?

5. Some men entered into partnership. One man contributed \$800, which was $\frac{2}{5}$ of the capital invested. How much capital was invested? How much was contributed by one of the partners who furnished $\frac{1}{4}$ of the capital?

6. A man sold $\frac{3}{8}$ of his land for \$1200. At this rate, what was the value of all his land?

7. A poultry dealer sold 80 turkeys and then had $\frac{1}{6}$ of his stock left. What part of his stock of turkeys did he sell? How many turkeys had he at first?

8. After spending \$18 for an overcoat, a man had \$6 left. What part of his money did he spend? What part of his money did he have left?

9. After traveling 24 miles, a man still had $\frac{2}{3}$ of his journey to travel. Find the length of the entire journey.

10. Mary had $\frac{5}{4}$ as much money as Ethel. If Mary had 60 ¢, how much did Ethel have?

11. If George has $\frac{4}{3}$ as many books as Walter, and George has 12 books, how many books has Walter?

12. \$20 is $\frac{4}{5}$ of —— $\frac{4}{5}$ of \$20 = ——

13. \$35 is $\frac{5}{7}$ of —— $\frac{5}{7}$ of \$35 = ——

14. After increasing his farm by buying $\frac{1}{4}$ as many acres as his farm contained, a farmer owned 120 acres. How many acres did he own before making the purchase?

15. Ethel weighs $\frac{1}{6}$ more than Edna. Ethel's weight is 105 lb. What is Edna's weight?

16. Thomas solved $\frac{3}{8}$ more problems than Henry. He solved 6 more problems than Henry. How many problems did Henry solve? How many did Thomas solve?

17. \$80 is $1\frac{3}{5}$ ($\frac{8}{5}$) times what amount?

18. \$120 is $2\frac{1}{2}$ times what amount?

19. \$200 is $\frac{4}{3}$ of x. \$60 is $1\frac{1}{5}$ of x.

20. What amount less $\frac{1}{5}$ of itself equals \$100?

21. What amount less $\frac{2}{5}$ of itself equals \$60?

22. \$1200 is $2\frac{1}{2}$ times x. $\frac{4}{5}$ of some amount is \$160. What is the amount?

23. After gaining $\frac{2}{5}$ of his capital, a merchant had \$14,000. Find the amount of his capital at first.

24. After buying 3 books, a girl had 8 books. The number of books bought was what part of the number she previously had?

25. \$80 is $\frac{2}{5}$ of ——. \$120 is $\frac{5}{4}$ of ——. \$90 is $\frac{2}{3}$ of ——.

26. \$60 is $1\frac{1}{2}$ times ——. \$150 is $1\frac{1}{4}$ times ——.

27. \$6 ÷ \$.75. \$9 ÷ \$1.50. \$50 ÷ \$1.25.

183. Oral Exercises.

1. If $\frac{1}{2}$ of the cost of a pair of skates is 60 ¢, the cost of the pair of skates is how many times 60 ¢?

2. If $\frac{1}{4}$ of the cost of a desk is \$3, the cost of the desk is how many times \$3?

3. Compare $\frac{3}{4}$ with $\frac{4}{4}$. Show by a diagram that $\frac{4}{4}$ is $1\frac{1}{4}$ times $\frac{3}{4}$, or $\frac{4}{3}$ times $\frac{3}{4}$.

4. If $\frac{3}{4}$ of the cost of a table is \$9, the cost of the table is how many times \$9?

5. $\frac{2}{3}$ of the cost of a clock is \$8. In finding the cost of the clock we may find $\frac{1}{3}$ of its cost, and then $\frac{3}{3}$ of its cost. Show that multiplying \$8 by $\frac{3}{2}$ is the same as finding first $\frac{1}{3}$ of the cost, and then $\frac{3}{3}$ of its cost.

6. If $\frac{3}{5}$ of the value of a horse is \$60, what is its value?

7. If 30 sacks of oats is $\frac{5}{6}$ of the yield per acre, what is the yield per acre? Find the answer in two ways.

8. A man sold $\frac{3}{4}$ of his crop of apples for \$120. At the same rate, what was the value of his entire crop?

9. If $\frac{3}{4}$ of a ton of coal costs \$6, what is the cost per ton? Are these two solutions identical in character?

A.

$$\begin{array}{l} 3\overline{)\$6}, \\ \overline{\$2}, \text{ cost of } \tfrac{1}{4} \text{ ton.} \\ \underline{\quad 4} \\ \$8, \text{ cost of 1 ton.} \end{array}$$

B.

$$\overset{\$2}{\cancel{\$6}} \times \frac{4}{\underset{1}{\cancel{3}}} = \$8$$

10. If $\frac{3}{4}$ of the cost of a farm is \$6000, what is the cost of the farm? What is $\frac{1}{2}$ the cost of the farm?

11. After selling $\frac{1}{3}$ of his sheep, a farmer had 60 sheep left. How many had he at first?

12. A boy sold 16 papers, which was $\frac{2}{3}$ of all he had. How many papers had he at first?

13. After solving 8 problems, a girl had $\frac{3}{5}$ of her problems yet to solve. How many problems had she to solve at first?

14. By selling an article for 45¢, a merchant gained $\frac{1}{8}$ of the cost. Find the cost of the article.

15. By selling a horse for \$90, a man lost $\frac{1}{10}$ of its cost. For what part of the cost did he sell the horse? Find the cost of the horse.

16. By selling a book for 60¢, a boy lost $\frac{2}{5}$ of its cost. For what part of its cost did he sell the book? Find the cost of the book.

17. Two boys bought a sled in partnership, one paying $\frac{2}{5}$ of its cost and the other paying $\frac{3}{5}$ of its cost. The boy who paid $\frac{2}{5}$ of its cost paid 70¢. Find the cost of the sled.

18. After having his salary increased by $\frac{1}{4}$, a boy received \$20 a month. What was his salary before it was raised?

19. A dealer advertised second-hand books at $\frac{2}{3}$ of their ordinary price. At what price does he sell a book that costs 60¢ when new? What is the price when new of a book which he sells for 90¢?

184. Written Exercises.

Find the whole when the part is given:

1. 112 is $\frac{3}{4}$	**7.** 160 A. is $\frac{1}{4}$	**13.** $3\frac{7}{8}$ mi. is $\frac{1}{3}$
2. \$20 is $\frac{4}{5}$	**8.** 320 rd. is $\frac{4}{5}$	**14.** $4\frac{1}{2}$ gal. is $\frac{3}{10}$
3. 75 mi. is $\frac{5}{6}$	**9.** \$42.50 is $\frac{5}{9}$	**15.** 81 ft. is $\frac{9}{11}$
4. $\frac{3}{8}$ is $\frac{3}{4}$	**10.** 36 yd. is $\frac{4}{5}$	**16.** \$1.20 is $\frac{3}{5}$
5. $\frac{3}{4}$ T. is $\frac{3}{8}$	**11.** 90 ft. is $\frac{5}{7}$	**17.** $3\frac{1}{7}$ is $\frac{2}{11}$
6. \$6400 is $\frac{5}{7}$	**12.** 5280 is $\frac{8}{9}$	**18.** 144 is $\frac{6}{7}$

185. Oral Exercises.

1. Find the whole amount when $\frac{3}{4}$ of the amount is \$60; is \$18; is \$54; is \$90; is \$240; is \$1500.

2. Find the whole amount when \$36 is $\frac{3}{4}$ of the amount; $\frac{4}{5}$ of the amount; $\frac{6}{11}$ of the amount; $\frac{9}{10}$ of the amount; $\frac{2}{3}$ of the amount.

3. Find the whole amount when \$120 is $1\frac{1}{4}$ times the amount; $1\frac{1}{5}$ times the amount; $1\frac{1}{2}$ times the amount; $1\frac{1}{3}$ times the amount; $1\frac{2}{3}$ times the amount.

4. Find the whole amount when \$240 is $\frac{2}{3}$ of the amount; $\frac{3}{4}$ of the amount; $\frac{5}{4}$ of the amount; $\frac{8}{5}$ of the amount; $\frac{5}{3}$ of the amount; $\frac{6}{7}$ of the amount.

5. Find the whole amount when \$600 is $\frac{75}{100}$ of the amount; $\frac{125}{100}$ of the amount; $\frac{120}{100}$ of the amount; $\frac{40}{100}$ of the amount; $\frac{25}{100}$ of the amount; $\frac{150}{100}$ of the amount.

6. A boy walked 2 blocks, which was $\frac{1}{5}$ of the distance from his home to the schoolhouse. How many blocks must he walk in going to and coming from school each day, if he goes home for lunch?

7. Charles weighs $\frac{1}{10}$ less than Albert. The difference in their weight is 11 lb. How much does each weigh?

8. Margaret wrote 7 more words than Emma, which was $\frac{1}{5}$ more words than Emma wrote. How many words did each write? How many did both together write?

9. A collector charged $\frac{1}{4}$ of the amount of a certain bill for collecting it. Find the amount of the bill, if the creditor received \$24.

10. After selling 60 acres, a farmer had $\frac{5}{6}$ as much land left. How many acres had he before making the sale?

11. \$80 is $\frac{4}{5}$ of ——. 90 mi. is $\frac{3}{4}$ of ——.

REVIEW

186. Written Exercises.

1. $47\frac{2}{5} - 14\frac{3}{4}$
2. $93\frac{4}{5} - 52\frac{3}{4}$
3. $48\frac{2}{3} \times 84\frac{5}{6}$
4. $9\frac{3}{4} - 4\frac{1}{2}$
5. $324\frac{2}{5} \times \frac{3}{8}$
6. $453\frac{3}{7} \div 5$
7. $526\frac{2}{5} \div \frac{3}{7}$
8. $736\frac{3}{4} \times 5\frac{2}{3}$
9. $87 - 66\frac{5}{8}$
10. $47\frac{3}{8} + 62\frac{2}{3}$
11. $19\frac{3}{7} \times 38\frac{5}{6}$
12. $2\frac{3}{8} \times 6\frac{3}{5}$
13. $96\frac{7}{8} \times 7\frac{7}{9}$
14. $30\frac{1}{4} \times 45\frac{2}{3}$
15. $897\frac{3}{8} \div 6$
16. $730\frac{1}{9} \times \frac{2}{3}$
17. $2\frac{3}{4} \times 3\frac{7}{8}$
18. $4\frac{7}{9} \div \frac{5}{6}$
19. $3\frac{4}{7} - 2\frac{8}{9}$
20. $7\frac{5}{6} + 8\frac{1}{4}$
21. $\frac{4}{5} = \frac{x}{20}$
22. $8\frac{7}{9} = \frac{x}{9}$
23. $\frac{2}{3} \times \frac{2}{3} \times \frac{3}{4}$
24. $3\frac{1}{4} \times 4\frac{2}{5}$

187. Written Exercises.

1. Find the value of $\frac{3}{4}$ of a farm of 160 A. at \$85 per acre.

2. A man sold $\frac{2}{3}$ of his farm for \$4800. At the same rate, what was the value of his entire farm?

3. If $3\frac{3}{4}$ yd. of cloth cost \$2.25, what is the cost per yard?

4. Find the cost of $8\frac{3}{4}$ yd. of silk at \$1.14 per yard.

5. Find the cost of a roast of lamb weighing $4\frac{3}{4}$ lb. at 16¢ per pound.

6. A turkey weighing $9\frac{1}{2}$ lb. was bought for \$1.90. Find the price paid per pound.

7. Express in cents and find the sum of the following: $\$\frac{1}{2}$, $\$\frac{3}{4}$, $\$\frac{1}{4}$, $\$\frac{1}{3}$, $\$\frac{2}{3}$, $\$\frac{1}{5}$, $\$\frac{2}{5}$, $\$\frac{3}{5}$, $\$\frac{4}{5}$, $\$\frac{1}{8}$, $\$\frac{3}{8}$, $\$\frac{5}{8}$, $\$\frac{7}{8}$.

8. Write ten improper fractions and change them to whole or mixed numbers.

9. Write ten mixed numbers and change them to improper fractions.

NUMBER RELATIONS

188. Oral Exercises.

Express all fractional parts in their lowest terms.

1. What part of 10 is 5? of 6 is 3? of 8 is 2? of 12 is 4? of 20 is 5? of 30 is 6?

2. What is the ratio of 5 to 15? of 6 to 12? of 8 to 24? of 9 to 81? of 7 to 56? of 20 to 4? of 28 to 7? of 42 to 6?

3. What part of 4 is 3? of 8 is 5? of 9 is 7? of 11 is 3? of 10 is 6?

4. What is the ratio of 3 to 5? of 5 to 3? of 3 to 11? of 11 to 3? of 7 to 9? of 9 to 7?

5. What is the ratio of 6 sacks of oats to 18 sacks of oats? 6 T. of coal will cost what part of the cost of 18 T.? 18 T. will cost how many times the cost of 6 T.?

6. If 5 sacks of flour cost $7.50, how much will 10 sacks cost?

7. If a boy earns $3 in 4 da., how much will he earn in 16 da.?

8. If a boy rides at the rate of 7 mi. in 2 hr., how far at the same rate will he ride in 6 hr.?

9. If 12 pads cost $.60, how much will 36 pads cost?

10. If 3 T. of coal cost $24, how much will 9 T. cost?

11. What number expresses the ratio of 4 lb. to 8 lb.? of 5 lb. to 20 lb.? of 15 yd. to 5 yd.? of 20 A. to 4 A.? of $20 to $30? of $24 to $36? of 18 bu. to 24 bu.? of 36 ft. to 24 ft.? of 48 mi. to 36 mi.? of $25 to $50?

12. What fraction expresses the ratio of 5¢ to 25¢? of 10¢ to 50¢? of 20¢ to 100¢? of 25¢ to 100¢? of 25¢ to 75¢? of 10¢ to 40¢? of 5¢ to 45¢? of 20¢ to 50¢?

13. What fraction expresses the ratio of 3 qt. to 4 qt.? of 5 mi. to 8 mi.? of 4 lb. to 6 lb.? of 8 bu. to 12 bu.? of 12 yd. to 9 yd.? of 20 mi. to 15 mi.? of 25¢ to 40¢? of 18 yr. to 12 yr.? of 6 mo. to 9 mo.? of 9 mo. to 12 mo.?

14. What part of $1 is 25¢? 50¢? 75¢? 40¢? 60¢? 70¢? 80¢? 90¢? 5¢? 10¢? 20¢? 30¢?

15. What part of $1 is $12\frac{1}{2}$¢? $37\frac{1}{2}$¢? $62\frac{1}{2}$¢? $87\frac{1}{2}$¢? $33\frac{1}{3}$¢? $66\frac{2}{3}$¢? $16\frac{2}{3}$¢? $83\frac{1}{3}$¢? $14\frac{2}{7}$¢? $8\frac{1}{3}$¢?

16. What part of 1 ft. is 2 in.? 3 in.? 4 in.? 5 in.? 6 in.? 7 in.? 8 in.? 9 in.? 10 in.? 11 in.?

17. What part of 1 yd. is 2 in.? 3 in.? 4 in.? 6 in.? 8 in.? 9 in.? 12 in.? 18 in.? 20 in.? 24 in.? 30 in.? 13 in.? 21 in.?

18. What part of 1 lb. is $\frac{1}{2}$ oz.? 4 oz.? 8 oz.? 12 oz.? 7 oz.?

19. What part of 1 yr. is 2 mo.? 3 mo.? 4 mo.? 5 mo.? 6 mo.? 7 mo.? 8 mo.? 9 mo.? 10 mo.? 11 mo.?

20. What part of 1 da. is 2 hr.? 3 hr.? 4 hr.? 5 hr.? 6 hr.? 8 hr.? 10 hr.? 12 hr.? 15 hr.? 16 hr.? 18 hr.? 20 hr.?

21. What part of 1 hr. is 5 min.? 10 min.? 15 min.? 20 min.? 25 min.? 30 min.? 35 min.? 40 min.? 45 min.? 50 min.? 55 min.? 17 min.?

22. What part of 1 mi. is 10 rd.? 20 rd.? 40 rd.? 80 rd.? 60 rd.? 90 rd.?

23. What part of 1 T. is 1000 lb.? 500 lb.? 250 lb.? 200 lb.? 100 lb.?

24. What part of 1 section of land (1 sq. mi., or 640 A.) is 320 A.? 160 A.? 80 A.? 40 A.? 20 A.?

189. Aliquot Parts.

1. Name several amounts that are exactly contained in $36.

2. How many times is each of the following contained in $1: 50¢? 25¢? $12\frac{1}{2}$¢? 10¢? 20¢? 5¢? 4¢? 2¢?

3. A part of a number or a quantity that will divide it without a remainder is called an **aliquot part.** Name several aliquot parts of 100.

4. What part of $1 is each: 50¢? 25¢? 10¢? 20¢? 5¢? $12\frac{1}{2}$¢? $33\frac{1}{3}$¢? $16\frac{2}{3}$¢? $11\frac{1}{9}$¢? $6\frac{1}{4}$¢? 2¢? 4¢? $14\frac{2}{7}$¢?

5. If 40 sheep can be bought for $100, how many sheep can be bought for $20 ($\frac{1}{5}$ of $100)? for $25? for $12.50? for $10? for $5?

6. If 100 sacks of potatoes cost $80, how much will 25 sacks cost? 50 sacks? 10 sacks? 5 sacks? 20 sacks?

7. How much will 30 yards of cloth cost at $1 a yard? at 25¢ a yard? at $12\frac{1}{2}$¢ a yard? at 20¢ a yard? at $16\frac{2}{3}$¢ a yard? at $33\frac{1}{3}$¢ a yard? at $8\frac{1}{3}$¢ a yard?

8. From the cost of any number of articles at $1 each how may the cost of the same number of articles at 25¢ each be found? at 50¢ each? at 20¢ each? at $12\frac{1}{2}$¢ each? at $33\frac{1}{3}$¢ each?

190. Memorize the following fractional parts of 1:

$.50 = \frac{1}{2}$	$.20 = \frac{1}{5}$	$.12\frac{1}{2} = \frac{1}{8}$	$.40 = \frac{2}{5}$
$.25 = \frac{1}{4}$	$.33\frac{1}{3} = \frac{1}{3}$	$.37\frac{1}{2} = \frac{3}{8}$	$.60 = \frac{3}{5}$
$.10 = \frac{1}{10}$	$.66\frac{2}{3} = \frac{2}{3}$	$.62\frac{1}{2} = \frac{5}{8}$	$.16\frac{2}{3} = \frac{1}{6}$
$.75 = \frac{3}{4}$	$.05 = \frac{1}{20}$	$.87\frac{1}{2} = \frac{7}{8}$	$.14\frac{2}{7} = \frac{1}{7}$

From the above table construct a table showing the same fractional parts of $1; of 100; of $100; of 1000.

191. Oral Exercises.

From the cost of 120 articles at \$1 each find the cost:

1. At 50 ¢ each.
2. At 25 ¢ each.
3. At 75 ¢ each.
4. At $12\frac{1}{2}$¢ each.
5. At $33\frac{1}{3}$¢ each.
6. At $66\frac{2}{3}$¢ each.
7. At 20 ¢ each.
8. At $37\frac{1}{2}$¢ each.
9. At $62\frac{1}{2}$¢ each.
10. At 40 ¢ each.
11. At 60 ¢ each.
12. At 80 ¢ each.

192. Written Exercises.

Solve each by the shortest method.

1. Find the cost of 24 yd. of cloth at $37\frac{1}{2}$¢ per yard.

SUGGESTION: At \$1 per yard the cloth would cost \$24.

2. Find the cost of 24 yd. of cloth at $87\frac{1}{2}$¢ per yard.

SUGGESTION: $87\frac{1}{2}$¢ per yard is $\frac{1}{8}$ less than \$1 per yard.

3. Find the cost of 24 yd. of cloth at $66\frac{2}{3}$¢ per yard.

4. Find the cost of 16 articles at \$25 each; at \$250 each ($\frac{1}{4}$ of \$1000); at \$125 each ($\frac{1}{8}$ of \$1000); at \$75 each ($\frac{1}{4}$ less than \$100); at \$37.50 each.

SUGGESTION: At \$100 each the 16 articles would cost \$1600.

5. How many articles can be bought for \$48 at \$1 each? at 25¢ each? at $33\frac{1}{3}$¢ each? at $66\frac{2}{3}$¢ each? at $12\frac{1}{2}$¢ each? at 20¢ each? at $37\frac{1}{2}$¢ each?

193. Short Methods.

Solve each, using the shortest method:

1. $\$40 \times 25$
2. $\$120 \times 25$
3. $\$80 \times 250$
4. 60 mi. $\times 33\frac{1}{3}$
5. $\$2040 \times .12\frac{1}{2}$
6. 640 A. $\times 37\frac{1}{2}$
7. 240 mi. $\times .125$
8. 36 ft. $\times 125$
9. 400 lb. $\times .625$
10. $\$8.60 \times 75$
11. $\$5.60 \times 750$
12. $\$4.64 \times 12.5$

194. 1. Divide by 25: 12; \$16; 640 A.

To divide by 25, *divide by* 100 *and multiply the quotient by* 4.

2. State a short method of dividing a number by 250; by 50; by $33\frac{1}{3}$; by $66\frac{2}{3}$; by $37\frac{1}{2}$; by $12\frac{1}{2}$; by 375; by 75; by 125; by .25; by .125; by 12.5; by 2.5; by 62.5; by 625; by 500.

Divide:

3. \$400 by 25
4. \$300 by 250
5. \$600 by 50
6. 320 rd. by .125
7. 640 mi. by 12.5
8. 540 ft. by $33\frac{1}{3}$
9. 120 yr. by $66\frac{2}{3}$
10. 2240 lb. by .25
11. 2000 lb. by 2.5
12. 5280 ft. by 37.5 ft.
13. 1728 by 250
14. \$400 by 87.5
15. \$3200 by \$625
16. \$1500 by \$2.50

195. 1. What is the cost of 24 yd. of cloth at 50¢ per yard? at $12\frac{1}{2}$¢ per yard? at $16\frac{2}{3}$¢ per yard? at 75¢ per yard? at $87\frac{1}{2}$¢ per yard? at $37\frac{1}{2}$¢ per yard?

2. How many yards of cloth can be purchased for \$12 at 25¢ a yard? at $12\frac{1}{2}$¢ a yard? at $6\frac{1}{4}$¢ a yard? at $37\frac{1}{2}$¢ a yard? at $33\frac{1}{3}$¢ a yard? at $66\frac{2}{3}$¢ a yard? at \$1.50 a yard? at \$$1.33\frac{1}{3}$ a yard? at \$1.20 a yard?

3. If 40 acres of land cost \$2000, how much will 50 acres cost at the same rate? 60 acres? 100 acres? 45 acres? 55 acres (40 acres + $\frac{1}{4}$ of 40 acres + $\frac{1}{2}$ of $\frac{1}{4}$ of 40 acres)?

4. George White paid Thomas Evans \$12 for the loan of some money for 60 da. At the same rate, how much must he pay for the use of the same sum for 90 da.? for 30 da.? for 75 da. (60 da. + $\frac{1}{4}$ of 60 da.)? for 120 da.? for 70 da.? for 50 da.? for 20 da.? for 80 da.?

REVIEW

196. 1. Draw a square and show the following parts of it: .50, .25, .75, .12$\frac{1}{2}$, .37$\frac{1}{2}$, .87$\frac{1}{2}$, .33$\frac{1}{3}$, .66$\frac{2}{3}$.

2. Draw a square, and divide it into as many equal parts as are necessary to show either $\frac{1}{2}$ or $\frac{1}{3}$ of the square.

3. What is the l. c. m. of 2, 3, 9? of 2, 3, 4, 6?

4. Draw a square, and divide it into as many equal parts as are necessary to show all of the following parts: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$. Show on the square the parts $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{6}{6}$.

5. Describe a cubic foot. Think of a box whose inside measure is 1 ft. by 1 ft. by 1 ft. How many bricks do you think the box will contain? How can you find the exact number it will contain? If this box is watertight, how many gallons will it contain? (1 gal. = 231 cu. in.)

6. A boy made a bookcase. The top of the lower shelf is 6″ from the floor. The space between the lower shelf and the top of the case is 3′ 6″. The case contains four shelves. The space between the two lower shelves is 12″. The other shelves are placed so that the distance between them is exactly equal. How far apart are they if the shelves are each $\frac{7}{8}$″ in thickness? The width of the case is 2′ 6″, and the depth 1′. Draw a diagram, using the scale 2″ = 1′.

197. 1. Divide each by 10: 47; $38.40; $.80; 3.1416.

2. Divide by 100: 2200 lb.; 5280 ft.; 1760 yd.; $457.50.

3. State a short method for multiplying by 10; by 100; by 25; by 33$\frac{1}{3}$; by 66$\frac{2}{3}$; by 75; by 12$\frac{1}{2}$; by .25; by .87$\frac{1}{2}$; by 37$\frac{1}{2}$. Give several illustrations of each.

4. Explain what is meant by the reciprocal of any number. What is the reciprocal of $\frac{2}{3}$? of 8? of $\frac{1}{5}$? of 5$\frac{1}{2}$?

5. Divide $\frac{4}{5}$ ft. by $\frac{8}{15}$ ft.; 6.2 by .02; $\frac{12}{17}$ by 6; $\frac{13}{17}$ by 3.

6. State how to multiply and how to divide a fraction by a fraction; a whole number by a fraction; a fraction by a whole number; a mixed number by a whole number or a mixed number. Illustrate each.

7. State how to determine the place of the decimal point in the quotient in division of decimals. Fix the decimal point in: $.002\overline{)4.8368}$; $2.36\overline{)13.4}$; $34\overline{)4.275}$.

8. The question, How many square inches are there in 1 square foot? is answered by the number 144. Ask a similar question that is answered by each of the following: 3, 12, 9, 4, 2, 60, 24, $5\frac{1}{2}$, 144, 7, 320, 231, 1728, 5280, 365, 640, 27, 52, 160, 128, 2150.42, 8, 32, 100, 16, 2000, 2240.

9. The question, What is $\frac{1}{8}$ of 100? is answered by the number $12\frac{1}{2}$. Ask a question concerning a fractional part of 100 that is answered by each of the following: 20, 75, 25, $12\frac{1}{2}$, $37\frac{1}{2}$, 5, $62\frac{1}{2}$, $66\frac{2}{3}$, $87\frac{1}{2}$, $33\frac{1}{3}$, 40, 10, 80, $16\frac{2}{3}$.

10. The question, What is the ratio of 10¢ to \$1? is answered by the number $\frac{1}{10}$. Ask a question concerning the ratio of some quantity to \$1 that is answered by each: $\frac{1}{2}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{6}$, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{7}{8}$, $\frac{1}{5}$, $\frac{1}{8}$, $\frac{4}{5}$, $\frac{1}{10}$, $\frac{1}{20}$, $\frac{5}{4}$, $\frac{3}{2}$, $\frac{7}{4}$, 2, $\frac{5}{3}$, $\frac{4}{3}$.

198. The first number is the product of two numbers. The second number is one factor. Find the other factor.

1. 36, 6	7. 1.5, .5	13. \$16.40, $\frac{1}{3}$
2. 30, $\frac{1}{2}$	8. \$.15, 3	14. \$20, $.33\frac{1}{3}$
3. 15, .5	9. \$.15, 5¢	15. 96 ft., $.12\frac{1}{2}$
4. $\frac{3}{4}$, 2	10. \$1.80, .06	16. 96 ft., $\frac{1}{8}$
5. $\frac{3}{8}$, $\frac{1}{5}$	11. \$30, .05	17. \$34.40, .08
6. .5, 10	12. \$25.60, .08	18. \$10, .04

199. Oral Exercises.

Fill in the amounts omitted under each heading:

	Cost	Selling Price	Gain	Loss	Part Gained	Part Lost
1.	\$40	\$50	\$10		$\frac{1}{4}$	
2.	\$25	\$30	x		x	
3.	\$60	\$50		x		x
4.	\$80	x		\$20		x
5.	x	\$100	\$25		x	
6.	x	\$150		\$50		x
7.	\$75	x	\$50		x	
8.	x	\$75		x		$\frac{1}{4}$
9.	\$150	x		x		$\frac{1}{6}$
10.	x	\$110	x		$\frac{1}{10}$	

11. Express as hundredths the part gained or lost in each of the above exercises, as $\frac{1}{4}$ gain = .25 gain.

12. Write and solve ten exercises similar to Exs. 1–10 above.

13. Example 5 above may be stated as a problem:

ILLUSTRATION: A man sold a horse for \$100 at a gain of \$25. Find the cost of the horse and what part the gain is of the cost.

14. State Exs. 1–4 and 6–10 as problems.

200. **1.** If .04 of some amount is \$10, what is the amount?

2. By what must \$240 be multiplied to produce \$12?

3. Multiply: \$600 by .06; \$300 by .04; \$80.50 by .07.

4. Find .06 of \$360; of \$4; of \$24; of \$30; of 80 mi.

5. If 4 times some amount is \$16, what is the amount?

201. Oral Exercises.

1. If 12 articles cost $ 36, how may the cost of 6 articles be found without finding the cost of 1 article?

2. What part of the cost of 12 articles must be added to the cost of 12 articles to give the cost of 18 articles? of 15 articles? of 14 articles? of 16 articles? Illustrate.

3. What part of the cost of 6 articles must be added to the cost of 6 articles to give the cost of 9 articles? of 8 articles? of 7 articles? Illustrate each.

4. What part of the cost of 6 articles must be subtracted from the cost of 6 articles to give the cost of 5 articles? of 4 articles? Illustrate each.

5. When the cost of 6 articles is known, how may the cost of 3 articles be found? of 2 articles? of 1 article? of 12 articles? Illustrate each.

6. When the cost of 50 articles is known, how may the cost of $12\frac{1}{2}$ articles be found? of $37\frac{1}{2}$ articles? of $62\frac{1}{2}$ articles? of 75 articles?

202. 1. If .06 times some amount is $12, what is the amount? If .04 of an amount is $ 20, what is the amount? If .03 of an amount is $ 24, what is the amount?

2. $45 is .09 of what amount? $75 is .15 of what amount? $1.60 is .08 of what amount?

3. $15 is 1.25 of what amount? is .20 of what amount?

4. How much is 1.75 of $ 80? of $ 200? of 640 A.?

5. $ 40 is what part of $ 80? $\frac{1}{2} = \frac{x}{100}$.

6. $ 20 is how many hundredths of $40? of $80?

7. .6 of 600 is .12 of what number?

8. .9 of 800 is .3 of what number?

203. Written Exercises.

Keep each result until all the problems have been solved.

1. A farmer rented a field 60 rd. long and 40 rd. wide. Find the number of acres in the field.

2. The annual rent of the field was $8.75 per acre. Find the rent of the field for 1 yr.

3. The farmer planted the field in broom corn, which yielded $\frac{1}{3}$ T. to the acre. Find the total yield of broom corn. Each acre of broom corn yielded 1 T. of seed, valued at $16 per ton. Find the value of the seed.

4. The farmer sold the broom corn at $80 a ton. Find the value of the crop.

5. The farmer paid a commission merchant $4 a ton for selling the broom corn. Find the commission.

6. The commission merchant paid $2.50 per ton freight and $.75 per ton cartage on the broom corn. How much should he remit to the farmer, after deducting these expenses and his commission?

7. The expense of seed and of planting and harvesting the crop amounted to $15 per acre. How much was the farmer's net profit per acre from the crop?

8. If 25 lb. of broom corn are used in making 1 doz. brooms, how many dozen brooms can be made from the yield of 1 A.?

9. If the manufacturer sells the brooms for $2.50 per dozen, how much does he receive for the brooms made from the yield of 1 A.?

10. How much did the broom corn cost per pound at $80 per ton?

11. How much is the cost of the broom corn used in making 1 doz. brooms?

12. If the cost of labor and of the material other than the broom corn is $.80 for each dozen brooms, how much do the brooms cost the manufacturer per dozen?

13. How much is the manufacturer's profit on each dozen brooms? The manufacturer's profit on each dozen brooms is what part of the cost of a dozen?

14. A wholesale merchant bought the brooms from the manufacturer at $2.50 a dozen and sold them to retail dealers at a profit of $\frac{1}{10}$ of the cost. What was the price of the brooms per dozen to the retail dealer?

15. The retail dealer sold the brooms to his customers at a profit of $\frac{1}{5}$ of the cost to him. Find the price paid to the retail dealer for each broom.

16. If a retail dealer's net profit on each dozen brooms is $\frac{1}{3}$ of the gross profit, how much was his net profit on the sale of 1 doz. brooms?

17. Find the difference between the cost of 1 doz. brooms to the manufacturer and the cost to the consumer.

204. Written Exercises.

1. .06 of some amount is $30. Find .03 of the same amount.

SUGGESTION: .03 is one half of .06. Take one half of $30.

2. .06 of an amount is $30. Find .09 of the amount.

SUGGESTION: .09 is one half more than .06.

Follow the above suggestions in the solution of each:

3. Find .06 of $6400. From the answer find .02 of $6400; .03 of $6400; .09 of $6400; .08 ($\frac{1}{3}$ more than .06) of $6400; .04 ($\frac{1}{3}$ less than .06) of $6400; .05 ($\frac{1}{6}$ less than .06) of $6400; .07 of $6400; .12 of $6400.

205. Changing Decimal Fractions to Common Fractions.

1. What is the numerator in .375? What is the denominator?

2. How many decimal places are there in .375? Write .375 as a common fraction. Compare the number of decimal places in .375 with the number of 0's in $\frac{375}{1000}$.

3. Change .875 to a common fraction.

MODEL: $.875 = \frac{875}{1000} = \frac{7}{8}$.

To change a decimal fraction to a common fraction, write the numerator of the fraction over the denominator of the fraction. Reduce to lowest terms.

Change to common fractions:

4. .1, .2, .3, .4, .5, .6, .7, .8, .9.

5. .10, .20, .30, .40, .50, .60, .70, .80, .90.

6. .12, .15, .25, .35, .45, .55, .65, .75, .85, .95.

7. .125, .375, .625, .875, .025, .075, .04, .05, .02.

8. Reduce to mixed numbers: 4.25, 26.5, 8.75, 15.375, 45.125, 7.875, 12.625, 8.20, 35.60, 2.04, 5.40.

9. In the following the cents and mills are expressed decimally as fractions of a dollar. Write with the cents expressed as common fractions; thus, \$6.40 = \$6$\frac{2}{5}$: \$5.25, \$8.20, \$4.10, \$3.50, \$7.80, \$2.75, \$6.60, \$4.125, \$9.625, \$8.375, \$4.875, \$25.30, \$15.05, \$4.01, \$7.90.

10. What decimal is equivalent to $\frac{1}{2}$? $\frac{1}{4}$? $\frac{3}{4}$? $\frac{1}{5}$? $\frac{2}{5}$? $\frac{3}{5}$? $\frac{4}{5}$? $\frac{1}{8}$? $\frac{3}{8}$? $\frac{7}{8}$? $\frac{7}{10}$? $\frac{3}{10}$? $\frac{9}{10}$?

11. Express each as an improper fraction: 1.25, 1.10, 1.20, 1.125, 2.40, 1.375, 1.625, 1.875, 3.6, 1.80.

12. Add as decimals: .25, .125, .4, .875, .2, 4.75, 6.07. Change to common fractions and add.

206. Changing Common Fractions to Decimal Fractions.

1. Change $\frac{3}{4}$ to a decimal fraction.

$3 \div 4$ is the same as $\frac{3}{4}$. A fraction is an indicated division, in which the numerator is the dividend and the denominator is the divisor. The division indicated by $\frac{3}{4}$ may be performed by placing the decimal point after 3 and dividing, thus:

$\frac{3}{4} = 4\overline{)3.00}$ $.75$. The fraction $\frac{3}{4}$ has been reduced to a decimal fraction.

2. Perform the indicated division. Continue the division until there is no longer a remainder. $\frac{2}{5}$, $\frac{3}{8}$, $\frac{4}{5}$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{1}{8}$.

To change a common fraction to a decimal fraction, divide the numerator by the denominator.

3. The prime factors of 10 are 2 and 5. Name all the numbers to 30 which have no other prime factors than 2 or 5. Find by trial whether any fraction whose denominator has any prime factors other than 2 or 5 can be changed to an exact decimal. Which of the fractions in Sec. 207 can be reduced to exact decimals?

207. Written Exercises.

Change the following to decimals. Where the decimal is inexact, continue the division to three places.

1. $\frac{3}{5}$	7. $\frac{2}{5}$	13. $\frac{5}{12}$	19. $2\frac{2}{5}$	25. $\$3\frac{1}{3}$
2. $\frac{5}{8}$	8. $\frac{2}{9}$	14. $\frac{7}{20}$	20. $4\frac{13}{16}$	26. $\$7\frac{4}{5}$
3. $\frac{1}{3}$	9. $\frac{1}{6}$	15. $\frac{5}{7}$	21. $6\frac{3}{16}$	27. $\$8\frac{3}{10}$
4. $\frac{1}{8}$	10. $\frac{1}{7}$	16. $\frac{5}{6}$	22. $3\frac{1}{7}$	28. $\$7\frac{2}{3}$
5. $\frac{2}{3}$	11. $\frac{3}{8}$	17. $\frac{1}{2}$	23. $8\frac{1}{20}$	29. $\$5\frac{1}{8}$
6. $\frac{4}{5}$	12. $\frac{3}{4}$	18. $\frac{1}{50}$	24. $5\frac{4}{25}$	30. $\$6\frac{1}{12}$

208. Changing Fractions to Hundredths.

1. Change $\frac{4}{5}$ to a fraction whose denominator is 100.

MODEL: $\frac{4}{5}=\frac{x}{100}$. 5 is contained in 100 twenty times. Multiply both terms of $\frac{4}{5}$ by 20. $\frac{4}{5}=\frac{80}{100}$.

Since 1 is equivalent to $\frac{100}{100}$, $\frac{1}{5}$ is equivalent to $\frac{20}{100}$, and $\frac{4}{5}$ is equivalent to $\frac{80}{100}$.

2. Change to fractions whose denominators are 100: 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{3}{5}$, $\frac{3}{4}$, $\frac{6}{10}$, $\frac{4}{20}$, $\frac{3}{25}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{1}{12}$.

3. Express as decimal fractions each of the fractions in Ex. 2.

4. Memorize the following:

$1=\frac{100}{100}=1$	$\frac{1}{20}=\frac{5}{100}=.05$	$\frac{1}{3}=\frac{33\frac{1}{3}}{100}=.33\frac{1}{3}$
$\frac{1}{2}=\frac{50}{100}=.50$	$\frac{1}{25}=\frac{4}{100}=.04$	$\frac{2}{3}=\frac{66\frac{2}{3}}{100}=.66\frac{2}{3}$
$\frac{1}{4}=\frac{25}{100}=.25$	$\frac{1}{50}=\frac{2}{100}=.02$	$\frac{1}{8}=\frac{12\frac{1}{2}}{100}=.12\frac{1}{2}$
$\frac{3}{4}=\frac{75}{100}=.75$	$\frac{1}{12}=\frac{8\frac{1}{3}}{100}=.08\frac{1}{3}$	$\frac{3}{8}=\frac{37\frac{1}{2}}{100}=.37\frac{1}{2}$
$\frac{1}{5}=\frac{20}{100}=.20$	$\frac{1}{10}=\frac{10}{100}=.10$	$\frac{5}{8}=\frac{62\frac{1}{2}}{100}=.62\frac{1}{2}$
$\frac{2}{5}=\frac{40}{100}=.40$	$\frac{3}{10}=\frac{30}{100}=.30$	$\frac{7}{8}=\frac{87\frac{1}{2}}{100}=.87\frac{1}{2}$
$\frac{3}{5}=\frac{60}{100}=.60$	$\frac{7}{10}=\frac{70}{100}=.70$	$\frac{1}{6}=\frac{16\frac{2}{3}}{100}=.16\frac{2}{3}$
$\frac{4}{5}=\frac{80}{100}=.80$	$\frac{9}{10}=\frac{90}{100}=.90$	$\frac{1}{7}=\frac{14\frac{2}{7}}{100}=.14\frac{2}{7}$

5. Express as common fractions in lowest terms: .25, .20, .40, .50, .60, .70, .75, .80, .05, .02, .04, .10, .90, .01.

6. Write with the fractional part expressed as a decimal: $7\frac{4}{5}$, $4\frac{3}{4}$, $6\frac{2}{5}$, $8\frac{1}{2}$, $9\frac{1}{5}$, $4\frac{1}{4}$, $3\frac{1}{10}$, $8\frac{3}{10}$, $12\frac{3}{20}$, $9\frac{2}{5}$.

7. Express as dollars and cents and add $\$8\frac{1}{2}$, $\$3\frac{4}{5}$, $\$9\frac{1}{4}$, $\$4\frac{1}{10}$, $\$7\frac{1}{5}$, $\$8\frac{7}{10}$, $\$12\frac{3}{4}$, $\$15\frac{3}{5}$, $\$14\frac{1}{20}$, $\$7\frac{1}{25}$.

209. Written Exercises.

1. There are 2000 lb. in a ton. How many tons of hay are there in 5400 lb.?

2. At $8.50 per ton, how much will 6500 lb. of hay cost?

3. At $10.50 per ton, how much will 950 lb. of coal cost?

4. How many hundredweight (100 lb.) are 575 lb.? At $5.60 per hundredweight, how much will a farmer receive for some hogs weighing 3750 lb.?

5. At $37.50 per ton, how much will a farmer receive for 12,400 lb. of wheat?

6. At $1.25 each, how many hats can be bought for $20?

7. How much will 45.75 A. of land cost at $65.50 per acre?

8. The circumference of a circle is 3.1416 times its diameter. Find the diameter of a tree, the circumference of which is 7.75 ft.

9. Find the circumference of a cylindrical tank, the diameter of which is 4 ft. 9 in. (4.75).

10. At $38.50 each, how much will 14 cows cost?

11. The area of a rectangle is 42.625 sq. in. Its length is 7.75 in. How wide is the rectangle?

12. If a train travels at an average rate of 46.75 mi. per hour, in how many hours will it travel 390.6 mi.?

13. At $5\frac{1}{4}$¢ ($.0525) per pound, how many pounds of sugar can be bought for $4.20?

14. When hay is worth $7.50 per ton, how many tons can be bought for $90?

LUMBER MEASURE

210. 1. The unit used in measuring lumber is the **board foot**, which is the equivalent of a piece of board 1 ft. long, 1 ft. wide, and 1 in. thick.

A board 12 ft. long, 12 in. (1 ft.) wide, and 1 in. or less in thickness contains 12 times 1 board foot, or 12 board feet. In measuring lumber, boards less than 1 in. thick are considered inch boards. The name board foot is generally shortened to "foot." The Roman numeral "M" is used to denote a thousand feet.

2. How many board feet are there in a piece of board 1 ft. long, 12 in. (1 ft.) wide, and 2 in. thick? 1 ft. long, 6 in. ($\frac{1}{2}$ ft.) wide, and 2 in. thick?

3. What part of a board foot is there in a piece of board 1 ft. long, 6 in. ($\frac{1}{2}$ ft.) wide, and 1 in. thick? 1 ft. long, 8 in. ($\frac{2}{3}$ ft.) wide, and 1 in. thick? 1 ft. long, 4 in. ($\frac{1}{3}$ ft.) wide, and 1 in. thick?

4. A piece of board 1 ft. long, 6 in. wide, and 2 in. thick contains 2 times $\frac{1}{2}$ board foot, or 1 board foot. Explain. How many board feet are there in a piece of board 1 ft. long, 6 in. wide, and 3 in. thick?

To find the number of board feet in a piece of lumber, multiply the number of board feet in one foot of the length by the number of feet in the length of the piece.

5. Find the number of board feet in 16 pieces of 3″ by 4″, each 12 ft. long.

Model:

$$16 \times \cancel{12} \times 3 \times \frac{4}{\cancel{12}} \text{ board feet} =$$

192 board feet.

Take either 3″ or 4″ as the width. Taking 4″ as the width, the number of board feet in 1 ft. of the length is $3 \times \frac{4}{12}$ board feet; and in 1 piece 12 ft. long, $12 \times 3 \times \frac{4}{12}$ board feet; and in 16 pieces, $16 \times 12 \times 3 \times \frac{4}{12}$ board feet, or 192 board feet.

211. Oral Exercises.

Find the number of feet in a piece of lumber of the following dimensions:

1. $1'' \times 12''$, 10 ft. long
2. $2'' \times 12''$, 1 ft. long
3. $2'' \times 6''$, 14 ft. long
4. $2'' \times 4''$, 16 ft. long
5. $4'' \times 4''$, 12 ft. long
6. $4'' \times 8''$, 15 ft. long
7. $6'' \times 6''$, 18 ft. long
8. $1'' \times 16''$, 15 ft. long

212. Written Exercises.

1. Find the number of feet in 120 pieces of lumber, each $2''$ by $4''$ by $16'$.

2. Measure various pieces of lumber.

3. Find the cost of lumber for a bridge 10 ft. long, if planks $3'' \times 12'' \times 14'$ are laid over four timbers $8'' \times 8'' \times 14'$. Boards costing \$18 per M; timbers \$20 per M.

4. Find the cost of the lumber for a 5-board fence around an orchard 160 ft. by 240 ft. The boards used are 6 in. by 1 in. by 16 ft. and cost \$14 per M. The posts, 8 ft. in length, are set 8 ft. apart, and are made of pieces 4 in. by 4 in. by 16 ft., costing \$16 per M.

213. Flooring.

1. When *tongued* and *grooved*, a board 3 in. wide is $2\frac{1}{2}$ in. wide when laid. The part of the board thus lost is $\frac{1}{5}$ of the width covered by the board after it has been fitted. Explain. If 168 ft. of flooring 3 in. wide are needed for a certain floor, $\frac{1}{5}$ as much must be added if tongued and grooved flooring 3 in. wide is used. Why?

2. Find the number of feet of flooring needed for a room 24 ft. wide and 30 ft. long, if tongued and grooved flooring 3 in. wide, $\frac{7}{8}$ in. thick, and 12 ft. long is used. What is the cost of the flooring at \$40 per M?

3. What part of $5\frac{1}{2}$ in. is $\frac{1}{2}$ in.? of $2\frac{1}{2}$ in. is $\frac{1}{2}$ in.? Having found the number of feet of lumber needed to floor a certain room with boards 6 in. wide, how may the number of feet needed to floor the same room with boards 6 in. wide that have been tongued and grooved be found?

214. Shingling.

1. The unit of shingling is a **square**, which is an area of 100 square feet.

2. When shingles have been laid, about 4 inches of their length is exposed to the weather. The average width of a shingle is about 4 inches. Consequently the exposed surface of one shingle is about 16 square inches, or about $\frac{1}{9}$ square foot. It will thus take about 900 shingles to cover a square. Allowing for waste, 1000 shingles are estimated for a square. A *bunch* of shingles contains 250 shingles. How many bunches should be allowed to each square?

3. Find the cost of the shingles necessary to cover both sides of a roof, if each side is 24′ by 40′, at \$2.25 per thousand shingles.

MODEL: $24 \times 40 \times 2 \times .01 \times \$2.25 = x.$

The number of square feet in both sides of the roof is $24 \times 40 \times 2$, and the number of squares is .01 times this product. The cost of the shingles is \$2.25 multiplied by the number of squares. Why?

4. Find the cost of the shingles necessary to cover both sides of a roof, if each side is 36′ by 48′, and the shingles cost \$2.50 a thousand.

5. Estimate the cost of shingles to cover the roof of your schoolhouse, at \$2 a thousand.

215. Difference between Dates.

1. Walter Harris was born May 26, 1893. How old was he on January 4, 1907?

MODEL:

yr.	mo.	da.
1907	1	4
1893	5	26
13	7	8

Write the later date as the minuend and the earlier date as the subtrahend. It is evident that some number of days added to 26 da. equal 1 mo. and 4 da. Subtract thus: 26 da. and 4 da. are 1 mo. 4 da. and 4 da. (in the minuend) are 8 da. Carry 1 mo. to 5 mo., as in subtraction of integers. 6 mo. and 6 mo. are 12 mo.; 6 mo. and 1 mo. are 7 mo. Carry 1 yr. to 1893. Complete the subtraction.

2. Find your age by subtraction.

3. Find the time from the landing of Columbus in America to the date when the Declaration of Independence was signed.

4. Frank Thomas borrowed \$750 of Charles Gray on Oct. 8, 1902, and paid it on July 2, 1903. How long did he have the money?

5. When the *exact* number of days between two dates that are less than a year apart is required, it is necessary to take account of the number of days in each month included, as in the following: Find the exact number of days from Jan. 4, 1907 to April 11, 1907. There are 27 full days left in January, 28 days in February, 31 days in March, and 11 days in April (including April 11), or 27 da. + 28 da. + 31 da. + 11 da., or 97 da.

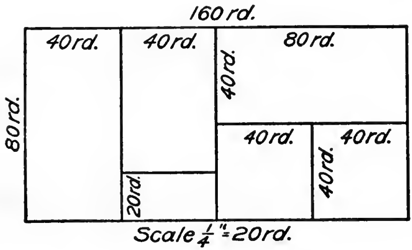
6. Find the exact number of days from the Fourth of July to Christmas; from Christmas to May 1.

7. Mr. Jenkins borrowed a team of Mr. Slate on Aug. 21 and returned it on Nov. 15. At \$1.50 a day, how much did he owe for the use of the team?

216. Review Exercises.

1. Make a drawing to represent a city lot 40 ft. front and 120 ft. deep, using the scale 1 in. = 20 ft. Using the same scale, represent at the back of the lot the space occupied by a barn 20 ft. by 30 ft.

2. A man bought a tract of land 160 rd. long and 80 rd. wide. How many acres did it contain? The tract was divided as shown in the figure. Find the area of each field. Find the cost of fencing the tract as shown in the figure at $1.25 per rod. 12800 ÷ 160 = 80 $100



3. A field containing 20 A. is 40 rd. wide. How long is it? 160 × 20 = 3200 ÷ 40 = 80

4. Mr. James bought Lot 2 (p. 135) for $40 per front foot. After paying for a 6-ft. cement sidewalk costing 12¢ per square foot, he sold the lot at a profit of $320. How much did he receive for it? $1948

5. After selling 60 acres a farmer had $\frac{5}{6}$ of his land left. How many acres had he before making the sale? 110 A

6. If 80 A. of land cost $4000, how much at the same price per acre will 320 A. cost? $1600

7. If hay is worth $12 a ton, how much is 500 lb. of hay worth? How much is 400 lb. worth? $3.00 $2.40

8. If a boat traveled 120 mi. during the first 8 hr. after leaving port, how far at the same rate will it travel in 1 da.? in 2 da.? 360 720

9. At $1 per yard, what is the cost of $\frac{1}{3}$ yd. of silk? of $\frac{2}{3}$ yd.? of $\frac{3}{4}$ yd.? .33⅓ .66⅔ .75

10. If a man's expenses for 3 mo. amount to \$135, at the same rate, how much will his expenses amount to in 1 yr.? \$540

11. If a horse is fed 1 bu. of oats in 5 da., how many bushels will be necessary to feed it for 1 mo. (30 da.)? 6

12. If it costs \$10 to pasture 6 horses for 1 month, how much will it cost to pasture 9 horses for the same length of time?

13. At \$7.25 per ton, how much will 6.75 T. of coal cost?

14. How much more sugar is received for \$1 by buying at $16\frac{1}{2}$ lb. for a dollar rather than at $6\frac{1}{4}$¢ per pound?

15. On the morning of March 7 a ship captain announced that he had on board enough provisions to last 80 da. Give the date on which the provisions would give out.

16. A boat that was due in port on Dec. 25 arrived on Jan. 6. How many days was she overdue? 12

17. Change to decimals: $\frac{3}{5}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{7}{8}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{5}$, $\frac{3}{4}$, $\frac{7}{10}$, $1\frac{1}{4}$, $1\frac{1}{3}$, $1\frac{1}{2}$, $2\frac{3}{4}$, $1\frac{4}{5}$.

18. Change to common fractions: .125, .375, .25, .875, .6, .625, .8, .40.

19. Find .05 of \$200; .06 of \$18.75; 1.04 of \$80.

20. \$282 = .94 of ——; \$375 = .75 of ——; \$60 = $\frac{3}{4}$ of ——.

21. \$60 = $1\frac{1}{4}$ times 48; \$60 = $1\frac{1}{5}$ times ——; \$60 = 1.20 of ——.

22. Change to 100ths: $\frac{4}{5}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{7}{8}$, $\frac{2}{3}$, $\frac{1}{12}$.

23. If $\frac{3}{5}$ of the cost of a city lot is \$1200, how much is the cost of the lot? 2000

24. How many tons of hay at \$12 a ton will equal in value the cost of laying a concrete sidewalk 40 ft. long and 6 ft. wide at $12\frac{1}{2}$¢ per square foot? $2\frac{1}{2}$

PART III

PERCENTAGE

217. Hundredths as Per Cents.

1. Read each: $\frac{25}{100}$, .60, .10, $\frac{45}{100}$, .06, .85, $\frac{125}{100}$, .05.

2. $\frac{5}{100}$, or .05, may be written 5%. It is then read 5 *per cent.* **Per cent** means *hundredths.* 5 per cent means 5 of the 100 equal parts. The sign (%) is called the per cent sign.

3. The unit 1 is equivalent to how many hundredths? to how many per cent?

4. Read the following: 4%, 8%, 25%, 40%, 75%, 100%, 150%, 200%, $6\frac{1}{2}$%.

5. Express as per cents: $\frac{5}{100}$, $\frac{9}{100}$, $\frac{8}{100}$, $\frac{20}{100}$, $\frac{60}{100}$, $\frac{100}{100}$.

6. Express as per cents: .01, .03, .12, .18, .50, .90, .99, 1, 2, .125 ($12\frac{1}{2}$%), .375, .625, .875.

7. Write as common fractions: 7%, 2%, 40%, 85%, 45%, 4%, 100%.

8. Write as decimal fractions: 1%, 5%, 7%, 30%, $\frac{15}{100}$, 75%, 80%, $\frac{90}{100}$, 100%, $37\frac{1}{2}$%, $33\frac{1}{3}$%, $14\frac{2}{7}$%.

218. Finding some per cent of a number.

1. 4% of \$500 is the same as \$500 multiplied by .04. Find 4% of \$500; of \$250; of \$45.50; of \$875.

To find any per cent of a number, multiply the number by the required per cent expressed as a decimal fraction.

2. Find 5% of \$360; of \$60; of \$100; of \$840.25.

3. Find 12% of \$400; of \$350; of \$100; of \$247.25; of \$1300.

4. Find 45% of 650 mi.; 80% of 640 A.; 62% of 400 bu.; 1% of \$400.

5. Find 100% of \$500. Compare 100% of \$500 with \$500.

6. 125% means $\frac{125}{100}$, or 1.25. Find 125% of 300 mi.

7. Name a per cent of \$600 that is the same as \$600; that is less than \$600; that is more than \$600.

8. Is 80% of a number more or less than the number? What per cent of a number is equivalent to one half of the number?

9. A man owes 8% of \$700. How much does he owe?

10. A man borrowed \$800 and agreed to pay 8% of the amount borrowed for the use of it for one year. How much did he pay for the use of \$800 for a year?

11. A man borrowed \$700 and agreed to pay 8% of the amount borrowed for the use of the money each year. How much did he pay for the use of \$700 for 1 year? for 2 years? for 3 years?

12. Money paid for the use of money is called **interest**.

13. A man borrowed \$400 and agreed to pay 6% interest each year. How much interest did he pay in 1 year? in $\frac{1}{2}$ year? in $1\frac{1}{2}$ years? in 2 years? in $2\frac{1}{2}$ years?

14. Find the interest on \$600 for 2 years at 6%.

15. A real estate agent sold a city lot for Mr. Thomas for \$1500. He received for his services 5% of the selling price of the lot. How much did he receive for selling the lot?

16. A real estate agent sold a city lot for Mr. Brown for $2000. He received a commission of 5% of the selling price for his services. How much did he receive for selling the lot? How much did Mr. Brown receive for the lot, after paying the commission?

17. A farmer shipped 25 tons of hay to a commission merchant in a city, who sold it for $8 per ton. The commission merchant received for his services 2% of the amount of the sale. Find the amount of his commission.

18. A commission merchant received a car of broom-corn containing 8 tons, which he sold at $120 per ton. He received a commission of 5% for selling it. Find the amount of his commission.

19. A farmer shipped 40 tons of hay to a commission merchant who sold it for $10 per ton. He received a commission of 6%. Find the amount of his commission. How much did the farmer receive for the hay, after deducting the commission?

20. A farmer had 160 acres of land. He sold 40% of it. How many acres did he sell? What per cent of the land did he have left? If he received $85 per acre for the land sold, how much did he receive for it?

21. A farmer had 320 acres of land. He sold 60% of it for $80 per acre and the remainder for $75 per acre. How much did he receive for the land?

22. Mr. Evans borrowed $250 of Mr. White and paid 7% interest. At the end of the year, how much should Mr. White receive from Mr. Evans, if he received the interest and the money loaned?

23. Find 50% of the number of children in your school-room.

219. Fractions as Per Cents.

1. The unit 1 is equivalent to how many hundredths? to how many per cent?

2. What per cent of a number is equivalent to the number? to $\frac{1}{2}$ of the number? to $\frac{1}{4}$ of the number? to $\frac{1}{10}$ of the number? to 2 times the number? to 5 times the number?

3. State how a common fraction may be reduced to a decimal fraction.

4. Change $\frac{3}{5}$ to per cent.

MODEL: $\begin{array}{r} .60 = 60\,\% \\ 5\overline{)3.00} \end{array}$

Change the fraction to a decimal, extending the reduction to two decimal places. Express hundredths as per cent.

Another Method: Since 1 is 100%; $\frac{3}{5}$ is $\frac{3}{5}$ of 100%, or 60%.

Work: $\overset{20\,\%}{\cancel{100}\,\%} \times \frac{3}{\cancel{5}} = 60\,\%$.

5. Change $\frac{3}{8}$ to a decimal fraction. .375 is the same as $.37\frac{1}{2}$, which is the same as $37\frac{1}{2}\,\%$.

6. State how a common fraction may be changed to hundredths, expressed as a decimal. What is the meaning of per cent?

To change a common fraction to per cent, divide the numerator by the denominator, and carry the reduction to two decimal places in the quotient. Express the quotient as per cent.

7. Change to per cents: $\frac{2}{5}$, $\frac{7}{8}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{2}{3}$, $\frac{9}{10}$, $\frac{7}{9}$.

8. Change to per cents: $\frac{11}{12}$, $\frac{14}{25}$, $\frac{6}{15}$, $\frac{30}{47}$, $\frac{19}{20}$, $\frac{25}{27}$.

9. Change to per cent: $1\frac{4}{5}$.

MODEL: $1\frac{4}{5} = \frac{9}{5}$. $\begin{array}{r} 1.80 = 180\,\% \\ 5\overline{)9.00} \end{array}$

10. Change to per cents: $1\frac{1}{3}$, $1\frac{5}{8}$, $1\frac{1}{2}$, $1\frac{1}{4}$, $1\frac{3}{5}$, $1\frac{3}{8}$.

11. Memorize the following:

$1 = 100\ \%$	$\frac{3}{5} = 60\ \%$	$\frac{3}{10} = 30\ \%$	$\frac{1}{8} = 12\frac{1}{2}\ \%$
$\frac{1}{2} = 50\ \%$	$\frac{4}{5} = 80\ \%$	$\frac{7}{10} = 70\ \%$	$\frac{3}{8} = 37\frac{1}{2}\ \%$
$\frac{1}{4} = 25\ \%$	$\frac{1}{20} = 5\ \%$	$\frac{9}{10} = 90\ \%$	$\frac{5}{8} = 62\frac{1}{2}\ \%$
$\frac{3}{4} = 75\ \%$	$\frac{1}{25} = 4\ \%$	$\frac{1}{12} = 8\frac{1}{3}\ \%$	$\frac{7}{8} = 87\frac{1}{2}\ \%$
$\frac{1}{5} = 20\ \%$	$\frac{1}{50} = 2\ \%$	$\frac{1}{3} = 33\frac{1}{3}\ \%$	$\frac{1}{6} = 16\frac{2}{3}\ \%$
$\frac{2}{5} = 40\ \%$	$\frac{1}{10} = 10\ \%$	$\frac{2}{3} = 66\frac{2}{3}\ \%$	$\frac{1}{7} = 14\frac{2}{7}\ \%$

220. Oral Exercises.

1. Find $16\frac{2}{3}\ \%$ of \$480.

$16\frac{2}{3}\ \%$ of \$480 may be found by multiplying \$480 by $.16\frac{2}{3}$, or it may be found by taking $\frac{1}{6}$ of \$480. Solve by both methods. Which method is the shorter?

2. Certain per cents of quantities may be found more easily by the use of fractional equivalents. One of these is $33\frac{1}{3}\ \%$. Name others.

Solve, using fractional equivalents: *

3. $16\frac{2}{3}\ \%$ of 24 hr.

4. $33\frac{1}{3}\ \%$ of \$15.

5. $14\frac{2}{7}\ \%$ of \$35.

6. $66\frac{2}{3}\ \%$ of 36 mi.

7. $37\frac{1}{2}\ \%$ of 48 yd.

8. 25 % of 320 rd.

9. 50 % of \$1.60.

10. $8\frac{1}{3}\ \%$ of 360 da.

11. $12\frac{1}{2}\ \%$ of 80¢.

12. $62\frac{1}{2}\ \%$ of 640 A.

13. $87\frac{1}{2}\ \%$ of 320 rd.

14. $16\frac{2}{3}\ \%$ of \$7.20.

15. $14\frac{2}{7}\ \%$ of \$84.

16. $66\frac{2}{3}\ \%$ of 60 bu.

17. $33\frac{1}{3}\ \%$ of \$2.10.

18. 75 % of \$400.

19. 80 % of \$25.

20. $16\frac{2}{3}\ \%$ of 30 ft.

* This exercise should be supplemented with oral drills until the pupils are able to find the above per cents readily by the use of their fractional equivalents. The fractional equivalents of the above per cents should be used in subsequent exercises.

221. Oral Exercises.

1. At a sale the following discounts were advertised. (*a*) Find the amount of reduction and (*b*) the selling price:

a. $16\frac{2}{3}\%$ off on carpets marked 90¢ per yard.
b. $33\frac{1}{3}\%$ off on bric-a-brac marked \$6.
c. $14\frac{2}{7}\%$ off on ladies' hats marked \$14.
d. $66\frac{2}{3}\%$ off on damaged cloth marked 30¢ per yard.
e. $12\frac{1}{2}\%$ off on tables marked \$16.
f. $37\frac{1}{2}\%$ off on cloaks marked \$16.

2. At what price should the following be marked:

a. Cloth that cost 80¢ per yard, to make a profit of 25%?

b. Suits that cost \$15, to make a profit of $33\frac{1}{3}\%$?

c. Hats that cost \$2.40, to make a profit of 25%?

d. Shoes that cost \$3 per pair, to make a profit of $33\frac{1}{3}\%$?

e. Silk that cost \$1.50, to make a profit of 50%?

f. Overcoats that cost \$16, to make a profit of $37\frac{1}{2}\%$?

g. Lace at 60¢ per yard, to make a profit of $16\frac{2}{3}\%$?

3. What per cent of a number remains after subtracting 25% of it? 20%? 40%? 75%? 5%? $66\frac{2}{3}\%$? $33\frac{1}{3}\%$? 50%? 60%? 10%? 2%? 100%? 90%?

4. What fractional part of a quantity remains after subtracting 50% of it? 20%? 25%? 30%? 40%? 75%? $33\frac{1}{3}\%$? 80%? $66\frac{2}{3}\%$? 10%? $16\frac{2}{3}\%$? $14\frac{2}{7}\%$? 5%? $12\frac{1}{2}\%$? 100%? 90%? $37\frac{1}{2}\%$? 15%?

5. How much remains of \$24 after deducting 50% of it? 25%? 75%? $16\frac{2}{3}\%$? $33\frac{1}{3}\%$? $66\frac{2}{3}\%$? $12\frac{1}{2}\%$? $37\frac{1}{2}\%$?

6. How much remains of \$36 after deducting 25% of it? 50%? 75%? $33\frac{1}{3}\%$? $66\frac{2}{3}\%$? $16\frac{2}{3}\%$? 100%?

7. Find $\frac{1}{2}$ of 36; 50% of 80; $\frac{1}{3}$ of 90; $33\frac{1}{3}$% of 75; $\frac{1}{5}$ of 200; 75% of 200; 20% of 15 · $\frac{2}{5}$ of 60; 40% of 120; $\frac{25}{100}$ of 40; $12\frac{1}{2}$% of 72; $66\frac{2}{3}$% of 90; $\frac{3}{8}$ of 64.

8. A merchant bought silk at $1.80 per yard and sold it at a profit of $33\frac{1}{3}$%. How much did he make on each yard?

9. A man bought hay at $8 per ton and sold it at a profit of 25%. What was his profit on each ton? What was the selling price per ton?

10. A grocer bought tea at 40¢ per pound and sold it at a profit of 50%. What was the selling price?

11. A suit of clothes marked $20 was sold at a reduction of 20%. Find the amount of the discount and the selling price of the suit.

12. A wagon that cost $72 was sold at a profit of $16\frac{2}{3}$%. What was the selling price of the wagon?

13. A merchant advertised a reduction of 25% on all goods. Find the reduction on suits marked $30; on shoes marked $4; on hats marked $2; on cloth marked 80¢ per yard; on rugs marked $6.

14. A house owned by Mr. West was rented to Mr. James by a real estate firm for one year at $30 per month. If the firm received as commission 10% of the first month's rent, what was the amount of the commission?

15. A hardware merchant invested $5000 in his business. He cleared 15% on the investment in one year. What was the amount cleared during the year?

16. In the catalogue of a carriage manufacturer a certain carriage was listed at $150. It was bought by a retail dealer at a discount of 20% from the list price. How much did the carriage cost the retail dealer?

17. A farmer shipped 50 boxes of apples to a commission merchant, who sold them at 90 ¢ per box. The commission merchant charged a commission of 5 % for his services. Find the amount of his commission. He paid freight charges amounting to $3.50. How much should he remit to the farmer after deducting for commission and freight?

18. Mr. A bought a cow for $ 40 and sold it at a profit of 20 %. What was the selling price of the cow?

19. Mr. A sold a cow for $\frac{6}{5}$ of the cost. He received $ 48 for the cow. Find the cost of the cow.

20. Mr. A sold a cow at a profit of $\frac{1}{5}$ of the cost. His profit was $ 8. Find the cost.

21. A real estate dealer bought a lot for $ 600. After five years he sold it at a profit of 100 % of the cost. What per cent of the cost of the lot did he receive for it?

22. A merchant's stock of goods valued at $ 4500 was damaged by fire. He was obliged to dispose of the goods for $66\frac{2}{3}$ % of their former value. What fractional part of their value did he receive for them? How much did he receive for his stock?

23. A dealer was asked the price of a certain carriage. He replied that he would sell the carriage for $ 200 and allow the purchaser 60 days in which to make the payment, or that he would allow a discount of 2 % for cash payment. Find the cash price of the carriage.

24. Mr. James pays $1\frac{1}{2}$ % taxes on $ 4000. Find the amount of his tax.

25. A man bought a lot for $ 1600. He sold it for $ 1800. How much did he gain on the lot? His gain was what part of the cost?

26. Frank Thomas borrowed $1200 for 1 yr. at 6% interest. How much did he pay for the use of the money?

222. Finding the number of which a given number is a certain per cent.

1. If 4% of a sum of money is $12, what is 1% of it? If 1% of a sum of money is $3, what is 100% of it?

2. When 5% of a selling price is $80, what is 1% of the selling price? What is the selling price?

3. If 8% of a number is 160, what is 1% of the number? What is 2% of the number? What is the number?

4. When 6% of a number is given, how may 1% of it be found? How, then, may the number be found? Any number is equivalent to how many per cent of itself?

5. To find a number when a certain per cent of it is given, find what 1% of it is, then find 100% of it. By this method find the number of which 24 is 8%.

6. When the multiplier and the product are given, how may the multiplicand be found?

7. Some number when multiplied by 4 is 216, what is the number? Some number when multiplied by .08 is 24, what is the number?

8. To say that 7% of a number is 161, is the same as to say that some number when multiplied by .07 gives 161 as a product. The number may be found by dividing 161 by .07.

To find the number of which a given number is a certain per cent, divide the given number by the given per cent expressed as a decimal.

In each of the following, name the multiplier and the product, and state how the multiplicand may be found:

9. 15 % of a number is 60.	**12.** 8 % of some land is 25.6 A.
10. 9 % of a number is 135.	**13.** 6 % of some money is $108.
11. 18 % of a number is 81.	**14.** 45 % of a crop is 135 bu.

223. Written Exercises.

1. $41.49 is 12 % of what amount?

MODEL:

$$\begin{array}{r} \$345.75 \\ .12\overline{)\$41.49} \end{array}$$

Some amount when multiplied by .12 gives $41.49 as a product. The amount is found by dividing the product ($41.49) by the multiplier (.12).

2. 240 mi. is 12 % of how many miles?

3. 128 tons is 8 % of how many tons?

4. A man paid $32 interest for the use of some money for one year at 8 %. What was the sum borrowed?

5. A farmer received 40 % of a crop as rent for his land. His share of the wheat amounted to 400 bu. in one year. What amount of wheat was raised on the farm in that year?

6. A man received $80 interest on some money which he loaned for a year at 10 %. Find the amount of the loan.

7. On a certain day 4 of the pupils in a school were absent. This was 8 % of the number enrolled. How many pupils were enrolled in the school?

8. During one season a baseball team lost 14 games, which was 40 % of the number of games played. How many games did the team play?

9. Two men entered into partnership in a retail hardware store. One agreed to furnish 40 % of the capital and the other 60 % of the capital. The partner who contributed 40 % of the capital invested $2400. Find the whole amount of the capital.

10. A farmer sold 120 acres of land, which was 30 % of his entire farm. How many acres had he before making the sale? What per cent of his farm did he still own?

224. Oral Exercises.

1. A number is how many times 20 % of itself? If 20 % of a number is 8, what is the number?

2. A number is how many times $33\frac{1}{3}$ % of itself? If $33\frac{1}{3}$ % of a number is 25, what is the number?

3. What part of a number is each of the following per cents of the number: 25 %, $37\frac{1}{2}$ %, 50 %, $62\frac{1}{2}$ %, $66\frac{2}{3}$ %, 75 %, $87\frac{1}{2}$ %, $16\frac{2}{3}$ %, $14\frac{2}{7}$ %, $8\frac{1}{3}$ %, $12\frac{1}{2}$ %, 20 %, $33\frac{1}{3}$ %?

4. A number is how many times each of the following per cents of itself: $14\frac{2}{7}$ %, 75 %, 25 %, $16\frac{2}{3}$ %, 50 %, $8\frac{1}{3}$ %, 20 %, $37\frac{1}{2}$ %, $12\frac{1}{2}$ %, $33\frac{1}{3}$ %, $62\frac{1}{2}$ %, $87\frac{1}{2}$ %, $66\frac{2}{3}$ %?

5. If 50 % of the amount of money a boy has is $12, how much money has he? How much money has he if 25 % of his money is $5? if 10 % of his money is $3? if $33\frac{1}{3}$ % of his money is $8? if $16\frac{2}{3}$ % of his money is $10? if $66\frac{2}{3}$ % of his money is $24?

6. $5 is 25 % of —. $8 is $33\frac{1}{3}$ % of —. 4 mi. is 20 % of —. 6 gal. is 50 % of —. 12 yd. is 75% of —. 160 rd. is 10 % of —.

7. Find the number of which 16 is 25 %; 30 is 20 %; 18 is $66\frac{2}{3}$ %; 40 is 200 %; 60 is 300 %; 15 is $37\frac{1}{2}$ %; 25 is 50 %; 50 is $62\frac{1}{2}$ %; 70 is $33\frac{1}{3}$ %; 75 is 100 %.

8. When $14\frac{2}{7}\%$ of a number is given, how may the number be found? $14\frac{2}{7}\%$ of a farm is 25 acres. How many acres are there in the farm?

9. A man sold 45 head of cattle, which was 25% of the number he had. How many head of cattle had he?

10. A merchant sold goods at a discount of $16\frac{2}{3}\%$ from the cost price and lost \$60. What was the cost?

11. In $37\frac{1}{2}\%$ of a farm there are 90 acres. How many acres are there in the farm?

12. A merchant made $12\frac{1}{2}\%$ on the cost of some goods by selling them at a profit of \$6. Find the cost of the goods. Find the selling price of the goods.

13. A number is how many times $\frac{2}{3}$ of itself? $\frac{5}{8}$ of itself? $\frac{3}{4}$ of itself? $\frac{3}{8}$ of itself?

14. If $66\frac{2}{3}\%$ of a number is 120, what is the number?

15. If $87\frac{1}{2}\%$ of a number is 70, what is the number?

16. In a spelling test a boy spelled correctly 30 words, which was 75% of the number of words in the test. Find the number of words in the test.

17. Mr. A sold a cow at a profit of 25%. His profit amounted to \$10. Find the cost.

18. Mr. A sold a cow at a loss of 25% of the cost. For what part of the cost did he sell the cow? He received \$30 for the cow. Find the cost.

19. A fruit grower planted 120 apple trees. 20 of them died. What per cent of the trees died?

20. If 12 trees are 25% of the number planted by a fruit grower, how many trees did he plant?

21. Eight pupils were absent from school on a certain day, which was 20% of the pupils enrolled. How many pupils were enrolled in the school?

225. Written Exercises.

1. A farm was sold for \$6000, which was 25 % more than it cost. Find the cost of the farm.

The fractional equivalents of per cents should be used whenever the work can be made easier or shorter by their use.

MODEL A: $\frac{5}{4}$ of the cost of the farm = \$6000.

$$\frac{4}{4} \text{ of the cost of the farm} = \frac{4}{\cancel{5}} \text{ of } \overset{\$1200}{\$\cancel{6000}}, \text{ or } \$4800.$$

Since the farm was sold for $\frac{5}{4}$ (125%) of its cost, the cost of the farm is $\frac{4}{5}$ of the selling price.

MODEL B: $1.25\overline{)\$6000.00}$ = \$48 00.

Since \$6000 is 125 % of the cost of the farm, the cost of the farm may be found by dividing \$6000 by 1.25. (See Sec. 223.)

2. A city lot was sold for \$1200, which was 20 % more than it cost. Find the cost of the lot.

3. After increasing his stock $33\frac{1}{3}$ %, a merchant found that he had \$12,000 invested. Find the amount of his investment before the increase.

4. A sum of money was borrowed for a year at 8 % interest. At the end of the year the money borrowed and the interest amounted to \$432. What per cent was this of the sum borrowed? Find the sum borrowed.

5. If the population of a certain city in 1905 was 81,250, and this was an increase of 25 % over the population in 1895, what was the population in 1895?

6. A dealer sold a carriage for \$96, at a loss of 20 %. What per cent of the cost of the carriage did he receive for it? How much did the carriage cost him?

7. A firm sold a carriage to a retail dealer for \$119, which was at a discount of 15 % from the list price of the carriage. Find the list price of the carriage.

226. Finding what per cent one number is of another.

1. Each of the following fractions is equivalent to what per cent: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{1}{12}$, $\frac{1}{10}$, $\frac{3}{10}$, $\frac{7}{10}$, $\frac{1}{20}$, $\frac{1}{25}$, $\frac{1}{50}$?

2. Each of the following is equivalent to what per cent: $1\frac{1}{2}$, $1\frac{1}{3}$, $1\frac{1}{4}$, $1\frac{1}{5}$, $1\frac{3}{4}$, $1\frac{2}{3}$, $1\frac{1}{8}$, $1\frac{1}{6}$, $1\frac{1}{7}$, $1\frac{1}{12}$, $1\frac{3}{8}$, $1\frac{5}{8}$, $1\frac{7}{8}$, $1\frac{2}{5}$, $1\frac{4}{5}$?

3. 8 is what part of 16? What is the ratio of 8 to 16? $\frac{1}{2}$ of a number is what per cent of the number? 8 is what per cent of 16?

4. 24 is what part of 36? 24 is what per cent of 36?

5. 12 is what per cent of 36? of 24? of 48? of 60?

6. Express as a common fraction the ratio of 6 to 8; of 20 to 25; of 25 to 20; of 30 to 35; of 40 to 60.

7. Express as hundredths in decimal form the ratio of 3 to 5; of 5 to 8; of 4 to 5; of 24 to 30.

8. 16 is what per cent of 20?

MODEL A: $5\overline{)4.00}$ $.80 = 80\%$

$\frac{16}{20} = \frac{4}{5}$. Reduce $\frac{4}{5}$ to a decimal and carry the reduction to two decimal places in the quotient. .80 is the same as 80%.

MODEL B: $20\overline{)16.00}$ $.80 = 80\%$

Some per cent of 20 is 16. 20 is the multiplicand and 16 is the product. The multiplier may be found by dividing the product (16) by the multiplicand (20). The multiplier is .80, which is the same as 80%.

To find what per cent one number is of another, express as a common fraction the ratio of the one to the other, and reduce the fraction to a decimal, carrying the reduction to two decimal places in the quotient. Express the result as per cent.

227. Written Exercises.

1. What per cent of 120 mi. is 90 mi.?

2. \$45 is what per cent of \$50?

Find what per cent of:

3. 50 is 20.
4. 25 is 50.
5. 18 is 15.
6. 48 is 60.
7. 54 is 27.
8. 240 mi. is 180 mi.
9. 360 bu. is 600 bu.
10. 320 rd. is 80 rd.
11. 640 A. is 120 A.
12. 360 da. is 30 da.
13. 5280 ft. is 1760 ft.
14. 5000 ft. is 1000 ft.
15. 2000 mi. is 6000 mi.
16. 2000 lb. is 750 lb.

17. A man owned 320 A. of land. He sold 80 A. What per cent of his land did he sell? What per cent of it did he have left?

18. A coal dealer bought 240 tons of coal. He sold 160 tons. What per cent of it did he sell? What per cent of it did he have left?

228. Oral Exercises.

1. $1\frac{1}{3}$ times a number is what per cent of the number? If $1\frac{1}{3}$ times a number is 36, what is the number? If $133\frac{1}{3}\%$ of a number is 48, what is the number?

2. What per cent of a number is $1\frac{1}{2}$ times the number? If 150 % of a number is 12, what is the number?

3. If $1\frac{2}{3}$ times a number is 20, what is the number? If $166\frac{2}{3}\%$ of a number is 60, what is the number?

4. If 6 % of a certain amount is \$30, what is 1 % of the amount?

229. Per Cent of Gain or Loss.

1. Mr. A bought a cow for $40 and sold it at a gain of $8. $8, the gain, is what per cent of $40, the cost?

2. Mr. Clark bought a cow for $40 and sold it for $48. Find the gain. The gain is what per cent of the cost?

3. Mr. Brown bought a horse for $120 and sold it for $100. Find the amount of his loss. His loss is what per cent of the cost of the horse?

4. When the cost price and the selling price are given, how is the amount of the gain or loss found?

5. The per cent which the amount of gain or loss is of the cost is called the gain or loss per cent. The gain or loss per cent is always some per cent of the cost.

To find the gain or loss per cent, find what per cent the amount of gain or loss is of the cost.

6. A furniture dealer bought some rocking-chairs for $4 each and sold them for $6 each. How much did he make on each chair? What was his gain per cent?

7. A bicycle that cost $40 was sold for $30. What was the loss per cent?

8. A fruit dealer bought berries at 6¢ per box and sold them at 10¢ per box. What was his gain per cent?

9. A man bought a cow for $30 and sold it for $40. What was his gain per cent?

10. A newsboy bought papers for 3¢ each and sold them for 5¢ each. What was his gain per cent?

11. A newsboy bought papers for 1¢ each and sold them for 2¢ each. What was his gain per cent?

230. Oral Exercises.

Find the gain or loss per cent:

	Cost	Selling Price		Cost	Gain	Loss
1.	$10	$15	6.	$16	$4	
2.	$15	$10	7.	$12		$4
3.	$25	$30	8.	$15		$3
4.	$30	$25	9.	$20	$2	
5.	$40	$45	10.	$25		$5

231. Written Exercises.

Find the value of x in each:

	Cost	Selling Price	Gain	Loss	Gain %	Loss %
1.	$80	$100	x		x	
2.	$75	x	$25		x	
3.	x	$120		$30		x
4.	$50	x		$5		x
5.	x	$60	$20		x	
6.	x	$4.80		$1.20		x
7.	$20	x	x		8 %	
8.	$36	x		x		5 %
9.	x	$80		x		20%
10.	x	$24	x		20 %	

11. Ex. 1 above may be stated in the form of a problem, thus: A man bought a horse for $80 and sold it for $100. Find the gain or loss per cent. State problems for Exs. 1–10 above.

12. A certain baseball team won 6 games out of 10. What per cent of the games did the team win?

232. Written Exercises.

1. A real estate agent bought a city lot for $1200 and sold it for $1500. What was the gain per cent?

2. A merchant disposed of a stock of goods valued at $8000 for $6000. What was the loss per cent?

3. An agent received $40 for selling hay at a commission of 5%. Find the selling price of the hay.

4. The interest on a sum of money for one year at 6% was $72. On what amount was interest paid?

5. A farmer lost 45% of his wheat crop by fire. His loss amounted to 600 bushels. What was the amount of his entire crop?

6. After suffering a loss of 35% of the value of his stock of goods, a merchant found that the remainder of his stock was worth $13,000. What was the value of his stock before the loss?

7. A stock of goods valued at $4500 was partly destroyed by fire. After the fire the stock was estimated to be worth $3000. What was the per cent of loss?

8. Mr. Thomas bought a farm for $5250. He rented the farm for $420 a year. His rent amounted to what per cent of his investment?

9. Mr. Bunker bought a lot for $1500 and built a house on it costing $3000. He rented his property for $300 a year. His rent amounted to what per cent of his investment?

10. A business block in a city was advertised for sale for $75,000. This block rented for $500 per month. The income from the rent amounted to what per cent of the price asked for the property?

REVIEW

233. Oral Exercises.

1. By selling land at \$25 per acre more than it cost him, a farmer gained 20 % of the cost of the land. Find the cost of the land.

The gain, or \$25 per acre, amounts to 20 % of the cost, or $\frac{1}{5}$ of the cost. Since \$25 per acre is $\frac{1}{5}$ of the cost, the cost is 5 times \$25 per acre, or \$125 per acre.

2. By selling a carriage for \$15 more than it cost him, a dealer gained $12\frac{1}{2}$ % of the cost of the carriage. Find the cost of the carriage.

3. A city lot increased \$200 in value, which amounted to an increase of $33\frac{1}{3}$ % of its cost. Find the cost of the lot.

4. A gain of $66\frac{2}{3}$ % of the cost amounted to a gain of \$120. Find the cost.

5. A horse was sold for \$150, which was 120 % ($\frac{6}{5}$) of the cost. Find the cost of the horse.

6. By selling an overcoat for \$35, a merchant made a profit of $16\frac{2}{3}$ % of the cost. What fraction expresses the ratio of the selling price to the cost? Find the cost.

7. A boy sold a pony for \$6 more than it cost him. His profit amounted to $16\frac{2}{3}$ % of the cost of the pony. Find the cost and the selling price.

8. After selling 80 % of his land, a farmer had what per cent of it left? After selling 80 % of his land, a farmer had left 40 acres. How many acres had he before making the sale?

9. By selling a cow for \$32, a farmer lost 20 % of the cost price. What fraction expresses the ratio of the selling price to the cost? Find the cost of the cow.

10. A liveryman made 40 % on the cost of a horse by selling the horse for \$140. What fraction expresses the ratio of the selling price to the cost? Find the cost.

11. By selling a lot for \$640, a dealer lost 20 % of the cost price. The selling price was what fraction of the cost of the lot? Find the cost of the lot.

12. A field of wheat was damaged by floods to the extent of 25 % of the expected yield. The yield amounted to 30 bushels of oats to the acre. This was what fractional part of the expected yield? What was the expected yield?

13. A watch that cost \$25 was sold for 200 % of the cost. Find the selling price of the watch.

14. A painting that cost \$60 was sold for $33\frac{1}{3}$ % less than it cost. It was sold for what fractional part of its cost? Find the selling price.

15. A merchant made a profit of 25 % of the cost of silk by selling it for \$.80 per yard. Find the cost of the silk per yard.

16. A sum of money loaned at 7 % yields \$42 interest each year. Find the sum loaned.

17. \$20 is what part of \$100? A carriage that cost \$100 was sold for \$120. It was sold for what per cent of the cost price?

18. A stove that costs \$40 is sold for \$36. The loss is what part of the cost of the stove? The loss is what per cent of the cost of the stove? The selling price is what per cent of the cost price?

19. A farm that costs \$60 per acre is sold for \$70 per acre. The gain on each acre is what part of the cost per acre? The gain is what per cent of the cost?

234. Written Exercises.

1. Hay that cost \$40 for 5 tons was sold at \$9 a ton. What was the profit on each ton? the gain per cent?

2. A clothing merchant advertised a reduction of 20% on all goods. Find the amount of reduction and the sale price of suits marked \$35, hats marked \$2, suspenders marked 50¢, shoes marked \$3.50, neckties marked 25¢, overcoats marked \$20, cuffs marked 20¢ per pair, collars marked 2 for 25¢.

3. A jeweler sold a watch for \$26, which was at a profit of $33\frac{1}{3}$%. Find the cost of the watch.

4. Goods damaged by fire were sold for \$2400, which was at a loss of 40%. What was their original value?

5. What per cent of his earnings does a man save who earns \$80 a month and saves \$300 each year?

6. A farmer paid \$4000 for a farm and sold it for \$4200. Find the gain per cent.

7. A man's yearly income from a farm valued at \$6000 is \$1500. The income is what per cent of the value of the farm?

8. By selling a carriage for $12\frac{1}{2}$% more than it cost him, a dealer made a profit of \$15. How much did the carriage cost him?

9. Two men entered into partnership to purchase a boat that cost \$300. Each contributed one half of the capital. One of the men sold his share of the boat for \$120. Did he gain or lose, and what per cent?

10. A house that was valued at \$2400 was rented so that the yearly rent amounted to 12% of the value of the property. What was the monthly rent of the house?

11. By selling a cow for $15 more than it cost him, a farmer gained $33\frac{1}{3}$ % of the cost of the cow. Find the cost of the cow. Find the selling price.

12. Tea that was sold at 60¢ per pound was sold at a profit of $33\frac{1}{3}$ %. Find the cost of the tea.

13. A piano dealer sold two pianos for $240 each. On one he made a profit of 20 % and on the other he lost 20 %. How much did each of the pianos cost him? Did he gain or lose on the two pianos?

14. How should goods that cost $1.20 per yard be marked to sell at a profit of 20 %? 25 %? $33\frac{1}{3}$ %? 50 %?

15. Three men bought some land for $3600. One furnished $1500, another $1200, and the third $900. They sold the land for $4200. What per cent of the capital did each furnish? What per cent of the profit should each receive? What was each man's share of the profit?

16. A man had $800 in a bank. He drew out first $200 and then $300. What per cent of his money did he draw out? What per cent was left in the bank?

17. The salary of a clerk was increased from $60 per month to $75 per month. What per cent of increase was made in his salary? The increase would amount to how many dollars in two years?

18. The population of Los Angeles was 50,300 in 1890 and 102,479 in 1900. The population in 1900 was what per cent of the population in 1890? What was the increase in population from 1890 to 1900? What was the per cent of increase in the ten years?

19. A newsboy sold 25 papers at 5¢ each, which had cost him 3¢ each. What was the amount of his profit? What was his gain per cent?

235. Oral Exercises.

1. Find 25 %, 50 %, and 75 % of each of the following: \$100; \$ 80; \$120; 40 A.; 36 in.; 2000 lb.; 16 oz.; 12 mo.; 24 hr.; 360 da.; 144 sq. in.

2. What is $33\frac{1}{3}$ % and $66\frac{2}{3}$ % of each of the following: \$120? \$1200? 360 da.? 36 in.? 180 mi.? 27 ft.? \$1500? 12 mo.? 60 min.? 24 yd.?

3. Find $12\frac{1}{2}$ %, $37\frac{1}{2}$ %, $62\frac{1}{2}$ %, and $87\frac{1}{2}$ % of each of the following: 24 hr.; \$240; \$1; 20 mi.; 2000 lb.; 144 sq. in.; 360 da.; \$1200; 640 A.; 72 yd.; 16 oz.; 216 cu. in.; 320 rd.; \$4000; \$.48.

4. Express each in per cent: $\frac{1}{5}$, $\frac{1}{4}$, $\frac{2}{5}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{1}{6}$, $\frac{3}{8}$, $\frac{7}{10}$, $\frac{2}{3}$, $\frac{7}{8}$, $\frac{1}{3}$, $\frac{3}{5}$, $\frac{5}{4}$, $\frac{4}{3}$, $\frac{6}{5}$, $\frac{9}{8}$, $\frac{3}{2}$, $\frac{1}{7}$, $\frac{7}{5}$, $2\frac{1}{2}$, $1\frac{1}{4}$, $1\frac{1}{3}$, $1\frac{2}{3}$, $1\frac{4}{5}$, $2\frac{1}{4}$, $1\frac{1}{8}$, $1\frac{2}{5}$, 2, 8, 10.

5. Express each as a common fraction in lowest terms: 80 %, 50 %, $33\frac{1}{3}$ %, 25 %, 125 %, 20 %, 120 %, 70 %, 40 %, 150 %, $66\frac{2}{3}$ %, $14\frac{2}{7}$ %, 75 %, $133\frac{1}{3}$ %, 175 %, 180 %, $12\frac{1}{2}$ %, 140 %, 160 %, $112\frac{1}{2}$ %, $16\frac{2}{3}$ %, 90 %, $37\frac{1}{2}$ %, $87\frac{1}{2}$ %, $62\frac{1}{2}$ %, $137\frac{1}{2}$ %, $187\frac{1}{2}$ %, 110 %, 130 %.

6. Find 125 %, 150 %, 175 %, $112\frac{1}{2}$ %, $137\frac{1}{2}$ %, $162\frac{1}{2}$ %, and $187\frac{1}{2}$ % of each of the following: 24 hr.; 320 rd.; 640 A.; 360 da.; 16 oz.; 2000 lb.; \$1200; \$4000; \$80.

7. Find $133\frac{1}{3}$ %, 120 %, $166\frac{2}{3}$ %, 140 %, and 160 % of each of the following: \$150; 30 da.; 360 da.; \$6000; 120 rd.; \$250; 60¢; \$1.80.

8. Find the number of which 30 is $33\frac{1}{3}$ %; 60 is 25 %; 20 is 40 %; 36 is $66\frac{2}{3}$ %; 35 is 125 %; 120 is 120 %; 48 is $37\frac{1}{2}$ %; 90 is 150 %; 180 is $12\frac{1}{2}$ %; 180 is $112\frac{1}{2}$ %; 42 is 175 %; 50 is 200 %; 24 is 160 %; 200 is 40 %; 80 is $66\frac{2}{3}$ %; 15 is $166\frac{2}{3}$ %; 24 is 4 %; 30 is 5 %; 18 is 6 %; 45 is 9 %. 33 is 110 %; 12 is 2 %; 130 is 200 %.

236. Written Exercises.

1. A sum of money borrowed, together with the interest on it for one year at 7 %, amounted to \$909.50. This was what per cent of the money borrowed? Find the sum borrowed.

2. A boy spelled correctly 45 words in a test of 50 words. What per cent should he receive as his standing in the test?

3. A girl missed 4 problems in an arithmetic test containing 10 problems. What per cent of the problems did she miss? What per cent did she have correct?

4. 5 % of a certain amount is \$20. Find the amount.

5. Find the amount when 8 % of the amount is \$240; \$80.

6. A farmer had 24 cows and sold 16 of them. What per cent of the cows did he sell? What per cent did he have left?

7. A house and lot was advertised for sale for \$8000. This property was rented for \$32.50 per month. The rent amounted to what per cent of the price asked for the property?

8. If a man spent 60 % of his savings in building a barn and had \$400 left, how much had he saved?

9. A liveryman made 40 % on the cost of a horse by selling it at a profit of \$36. Find the cost of the horse.

10. An article that cost a retail merchant \$14 was sold to a customer at a profit of $14\frac{2}{7}$ %. How much did the customer pay for the article?

11. The total enrollment in a certain school was 180 pupils. On a certain day 150 pupils were present. The number present was what per cent of the enrollment?

237. Oral Exercises.

1. By selling a horse for 20 % more than it cost him a liveryman gained \$30. How much did the horse cost him? For how much did he sell it?

2. What is $2\frac{1}{2}$ % of \$400? $3\frac{1}{2}$ % of \$60? 5 % of \$1400? 6 % of \$250? 10 % of 2000 lb.? $5\frac{1}{2}$ % of \$200? 7% of \$150? 8 % of \$2500?

3. What is the difference between 1 % and .1 %? between $\frac{1}{4}$ of a number and $\frac{1}{4}$ % of a number?

4. What is $\frac{1}{2}$ % of \$200? .2 % of \$400? $\frac{1}{4}$ % of \$8000?

5. 8 is what part of 24? 8 is what per cent of 24? 20 is what per cent of 25? \$20 is what per cent of \$30? \$40 is what per cent of \$30?

6. A boy missed 1 word in a spelling lesson of 20 words. At the same rate, how many would he have missed in a lesson of 100 words?

7. After having his salary raised \$10 a month, a clerk's yearly salary amounted to \$1620. What was his monthly salary before receiving the increase?

8. A carriage that cost \$120 was sold for \$80. The sale price was what per cent of the cost?

9. A man's monthly salary was raised from \$60 to \$75. What per cent was his salary increased?

10. 25¢ is what per cent of 30¢? The cost of 3 bars of soap when bought at 3 bars for 25¢ is what per cent of the cost when bought at 10¢ a bar?

11. What per cent of profit is made when articles are bought at 40¢ a dozen and sold at 5¢ apiece?

12. What per cent of profit is made when articles are bought at 10¢ a dozen and sold at 2 for 5¢?

238. Oral Exercises.

1. If $\frac{5}{8}$ of the value of a piece of property is \$1500, what is the value of the property?

2. If $\frac{2}{3}$ of a man's yearly salary is \$1200, what is his yearly salary?

3. A clerk saved \$40 a month, which was $\frac{2}{5}$ of his monthly salary. What was his monthly salary?

4. What part of his income does a man save who saves \$60 a month from an income of \$1200 a year?

5. Frank has a certain sum of money and James has $\frac{2}{3}$ as much. They both together have 60¢. How much money has each?

The money of both together is how many thirds of Frank's money?

6. Two boys took a piece of work to do for \$6. One boy worked twice as many hours as the other boy. How much should each receive?

7. A man gave Henry \$3 as many times as he gave Walter \$4. He gave \$14 to the two boys. How much did each receive?

8. Separate \$45 into two amounts in the ratio of 5 to 4; \$36 into three parts in the ratio of 2, 3, and 4.

9. In a school of 120 pupils there were 5 girls to every 3 boys. Find the number of boys and girls.

10. Rob, Fred, and Ada together received \$2.40 from their father. For every 15¢ that Rob received Fred received 10¢, and Ada 5¢. How much did each receive?

11. A newsboy wished to make an estimate of his yearly earnings, so he kept account of his earnings for 3 weeks and found that he earned \$6 the first week, \$4 the second week, and \$5 the third week. At the same rate, how much would he earn in a year?

239. Commission.*

A person who transacts business for another frequently receives as his pay a certain rate per cent of the amount involved in the transaction. This is known as his **commission.** One who buys or sells for another on commission is called a **commission merchant**, a **broker**, or an **agent.**

240. Written Exercises.

1. Find 2% of $2400.

2. A commission merchant sold $2400 worth of hay for a farmer and charged 2% for his services. Find the amount of his commission. How much should he remit to the farmer, after deducting his commission and $300 for freight charges and $150 for cartage?

3. An agent received $16 as his commission for selling a bill of goods at a commission of 5%. Find the amount of his sales.

4. A farmer shipped 40 sacks of potatoes to a commission merchant, who sold them at 95¢ a sack. After deducting his commission of 5%, how much should he remit to the farmer?

5. A merchant's profits averaged 15%. His total sales for January, 1906, amounted to $13,800. Find the cost of the goods sold. Find the profits for the month.

6. A farmer shipped 18 tons of hay to a commission merchant, who sold it at $9.50 per ton. How much did the merchant remit to the farmer, after deducting his commission of 5% and freight and cartage charges amounting to $1.75 per ton?

* For a more extended treatment of Commission, see Appendix, pp. 262–264.

7. Find the net proceeds of the sale of 860 lb. of butter at 18¢ per pound, commission 6%.

8. A real estate agent received a commission of 5% for selling a city lot. Find the sale price of the lot, if the agent's commission amounted to $62.50.

9. If the salary of a traveling salesman is $20 a week and a commission of $1\frac{1}{2}$% on the amount of his sales, how much does he earn in a week in which his sales amount to $2254.75?

10. A carriage dealer offered to sell a certain carriage for $250 on two months' time, or to allow a discount of 2% for cash. Find the cash price of the carriage.

11. If a collector retains 10% of the amount of a certain bill for collecting it, what per cent of the amount of the bill does the creditor receive? A collector remitted to a creditor $126 as the net proceeds of a collection, after retaining his commission of 10%. Find the amount of the bill collected.

12. After deducting his commission of 4%, an agent remitted $79.20 to a shipper. Find the amount of the sales.

13. The amount received by a shipper, after a commission of 5% has been deducted, is what per cent of the amount of the sales? A shipper received $60.80 as the net returns of a sale of some potatoes, after paying a commission of 5%. Find the amount of the sale.

14. A dairyman shipped 1250 lb. of butter to a commission merchant, who sold it at 22¢ per pound. If the cost of shipping was $2.40 and the cartage amounted to $1.75, how much did the shipment net the dairyman, after paying a commission of 4%?

241. Oral Exercises.

1. If a boy sells 1 newspaper for what 2 papers cost him, what per cent of profit does he make?

2. If a baker sells 2 pies for what 3 pies cost him, what per cent of profit does he make?

3. A merchant sold 5 yd. of cloth for what 6 yd. cost him. What per cent of profit did he make?

4. What per cent of profit does a grocer make who buys canned tomatoes at the rate of 3 cans for 25¢ and sells them at the rate of 2 cans for 25¢?

5. A dealer marked his goods so that he would make 30% profit on them. In order to dispose of his goods, he was obliged to sell them at a discount of 10% from the marked price. What per cent of profit did he make?

6. A collector was allowed a commission of 20% on a bill of $80. What amount did the creditor receive?

7. A dealer marked his goods at 20% above cost. In order to close out his stock, he was obliged to sell the goods at a discount of 25%. Did he gain or lose, and what per cent?

8. There are 4 boys and 8 girls in a class in arithmetic. What per cent of the pupils in the class are girls?

9. The enrollment of pupils in a class was 25 in 1905 and 30 in 1906. What was the per cent of increase?

10. On a certain day a boy missed 3 words out of 15 in spelling. What was the per cent of words correctly spelled?

11. An agent received a commission of 5% for selling a lot for $1500. Find the amount of his commission.

12. An agent's commission of 5% for selling a city lot amounted to $60. For what amount did he sell the lot?

242. Insurance.* **1.** Owners of buildings, merchandise, etc., generally protect themselves against loss by fire by having such property *insured.* Insurance of property against loss by fire is called **fire insurance**, against loss by sea **marine insurance.** What is *life insurance? accident insurance?* Name other forms of insurance.

2. The written agreement between the insurance company and the person protected is called a **policy.** Examine a fire insurance policy. The amount paid for insurance is called the **premium.** The rates of insurance are expressed as a rate per cent on the face of the policy, or as a specified sum for each \$100, or for each \$1000, of the face of the policy.

243. Written Exercises.

1. Mr. Wilson insured his store for \$6000. The insurance cost him $1\frac{1}{2}\%$. Find the premium.

2. Mrs. Hardy insured her house, valued at \$8000, for $\frac{3}{4}$ of its value. Find the amount of the face of the policy. The insurance cost her \$1.40 on each \$100 and extended for three years. How much did the insurance cost her?

3. If 90% of a sum is \$28.80, what is the sum?

4. For what price was a city lot sold if the agent's commission of 5% amounted to \$87.50? How much did the owner receive?

Find the premium on the following amounts of insurance at the rates given:

5. \$4000 at $1\frac{1}{4}\%$.	**8.** \$5600 at \$1.20 per \$100.
6. \$2400 at $1\frac{3}{4}\%$.	**9.** \$1400 at \$1.35 per \$100.
7. \$12,000 at $1\frac{1}{2}\%$.	**10.** \$4250 at \$1.80 per \$100.

* For a more extended treatment of Insurance, see Appendix, pp. 278–283.

11. Mr. Rogers built a house that cost him \$4500. It cost him \$1800 additional to furnish it. To protect himself against the complete loss of his property by fire, he insured his house for \$3000 and his household goods for \$1200. The insurance for three years cost him $1\frac{1}{4}\%$ of the face of the policy.

a. Find the cost of the insurance.

b. If the house and contents were destroyed by fire, how much insurance would he receive?

c. What would be the amount of his loss, including the amount paid for insurance?

d. If the house were damaged to the extent of \$400, how much would he receive?

12. Two men own a store in partnership. One has \$16,000 invested in it, and the other has \$10,000. What part of the store does each own? If the store were sold for \$39,000, what part of this amount would each receive? How much would each receive? If the store were damaged by fire to the extent of \$13,000, how much would each lose?

13. A hotel valued at \$80,000 was insured for \$50,000 in one company and for \$25,000 in a second company. How much would each company be liable for (*a*) if the hotel were totally destroyed; (*b*) if it were damaged to the extent of \$12,000? of \$30,000?

14. What was the amount of commission received by an architect who charged a commission of 5% for drawing the plans and supervising the construction of a house that cost \$4500, exclusive of the architect's fees?

15. Write five insurance problems based on conditions in your community.

244. Oral Exercises.

Express the part and the per cent the first quantity is of the second:

1. \$30, \$40.	6. \$2.50, \$3.	11. \$1200, \$1500.
2. \$40, \$50.	7. 80 A., 160 A.	12. 45 T., 60 T.
3. 20 mi., 25 mi.	8. 10 yd., 16 yd.	13. 80 A., 320 A.
4. 40 ft., 60 ft.	9. \$4, \$24.	14. 2000 ft., 2200 ft.
5. \$100, \$120.	10. 60 lb., 100 lb.	15. \$12, \$200.

16. Express the ratio of the second quantity to the first in each of the above in the form of a fraction in lowest terms and in per cent.

245. Oral Exercises.

1. A collector's commission of 5% amounted to \$30. Find the amount of the bill collected.

2. After deducting his commission of 20%, a collector remitted \$24 to the creditor. Find the amount of the bill collected.

3. Mr. Wright has \$4500 out on interest at 6%. His annual taxes on the money are \$20. What is his net annual income from the \$4500?

4. The yield from a certain field was 30 bu. of oats to the acre in 1904 and 40 bu. to the acre in 1906. What was the per cent of increase in the yield in 1906 over the yield in 1904?

5. The enrollment in a certain school in 1906 was 36 pupils, which was an increase of 20% over 1905. What was the number of pupils enrolled in 1905?

6. 40% of the pupils in a certain school are boys. There are 24 girls in the school. How many pupils are there in the school?

246. Taxes.* **1.** What are some of the expenses of a city government? of a state government? of the national government? The money necessary for the maintenance of state and local governments is derived mainly from taxes levied upon persons, property, and business.

All movable property, such as household goods, money, cattle, ships, etc., is called **personal property.** Immovable property, such as lands, buildings, mines, etc., is called **real estate,** or **real property.** Both forms of property are subject to taxation.

2. For the purpose of taxation, the value of all taxable property is estimated by a public officer called an **assessor.** Property is not generally assessed at its full value.

3. The rate of taxation is expressed as a per cent on the assessed valuation, or as a specified sum on each \$1, or on each \$100, of assessed valuation. Thus, a tax rate of $1\frac{1}{2}\%$ may be stated as a tax of $1\frac{1}{2}$¢ (on each \$1), or of \$1.50 (on each \$100).

4. The national government is supported mainly by revenues derived from taxes levied upon goods imported from other countries, called **duties,** or **customs,** and from **internal revenues,** which consist chiefly of taxes levied upon the manufacture of liquors and tobacco products.

Some imports are admitted without duty. These are said to be on the *free list.* Nearly all imports are subject either to an *ad valorem* or a *specific* duty, or both.

5. An **ad valorem** duty is a tax of a certain rate per cent on the cost of the goods.

6. A **specific** duty is a tax of a specified amount per pound, yard, etc., without reference to the cost of the goods.

7. **Customhouses** have been established at all ports where vessels are authorized to land cargoes. The revenues are collected by federal officers stationed at ports of entry.

* For a more extended discussion of Taxes, see Appendix, pp. 269–273.

247. Written Exercises.

1. A man had \$6000. He invested \$1500 in a city lot. What per cent of his money did he invest?

2. A certain city had an assessed valuation of \$8,000,000. The amount needed to defray the expenses of the city for a year was estimated at \$100,000. The amount needed for expenses was what per cent of the assessed valuation?

3. The assessed valuation of a certain city is \$12,000,000 and the amount to be raised by taxation is \$180,000. What rate of taxation is necessary in order to raise this amount?

4. What is the amount of an agent's commission for selling real estate for \$150,000 at a commission of $1\frac{1}{2}\%$?

5. What is the amount of a man's taxes on property assessed at \$6000 if the tax rate is \$1.20 on each \$100?

6. A real estate agent received \$84 for selling a piece of property at a commission of 2%. Find the selling price of the property.

7. The assessed valuation of a certain farm is \$3600. This is 40% less than the actual value of the farm. Find the value of the farm.

8. What per cent on his investment did a boy make who bought a pony for \$40 and sold him for \$50?

9. The assessed valuation of the property in a county is \$42,000,000, and \$672,000 is to be raised by taxation. Express the rate of taxation in three ways.

10. Find the rate of taxation on:

   a. Valuation, \$450,000; taxes, \$6000.
   b. Valuation, \$275,000; taxes, \$2475.
   c. Valuation, \$360,000; taxes, \$6300.

11. What rate of commission was charged by a collector who charged \$15 for collecting a debt of \$225?

12. The premium on an insurance of \$4500 is \$60. What is the rate of premium?

13. The premium received for insuring a store at $1\frac{1}{2}\%$ was \$105. What was the amount of insurance?

14. At the rate of $1\frac{1}{4}\%$, how much is the tax on property assessed at \$4500?

15. When the valuation and the rate of taxation are given, how may the tax be found? Find the tax on:

a. \$12,000 at $1\frac{3}{4}\%$; at $.8\%$; at 1.4%; at $1\frac{3}{5}\%$.
b. \$10,000 at \$1.20 per \$100; at \$.80 per \$100.
c. \$6000 at 8 mills on a dollar; at 7.6 mills on a dollar.
d. \$3600 at \$.007 on a dollar; at \$.014 on a dollar.

16. If a broker received a commission of $1\frac{1}{2}\%$ for his services, find the amount of his brokerage for buying 2450 cwt. of wheat at \$1.34 per cwt. If this wheat was bought for a milling company, what was the total cost of the wheat to the company? the cost per cwt.?

17. If a traveling salesman sells on an average \$400 worth of goods every week, which of the following offers should he accept from the wholesale firm: (*a*) a salary of \$25 per week and expenses; (*b*) a salary of \$15 a week and expenses, and a commission of 5% on all sales over \$300 per week; (*c*) or his expenses and a commission of 8% on all sales?

18. When the tax and the rate of taxation are given, how may the valuation be found? Find the valuation:

a. Tax, \$60; rate of taxation, $1\frac{1}{2}\%$.
b. Tax, \$120; rate of taxation, 8 mills on a dollar.
c. Tax, \$96; rate of taxation, \$1.20 per \$100.

19. What was the amount of insurance if the premium received for insuring a house at \$1.40 per \$100 was \$49?

20. Furniture valued at \$600 was insured for \$400. For what part of its value was the furniture insured? The premium paid for 3 years was \$8. What was the rate of premium paid?

21. The pupils of the advanced arithmetic class in a certain school were told that the school building was insured for $\frac{3}{4}$ of its estimated value, and that the annual premium at 1% was \$75. They were asked to find the estimated value of the building. One pupil found the value to be \$5625. Was his answer correct?

22. A certain school district voted \$12,000 to erect a new schoolhouse. The assessed valuation of the property in the district was \$600,000. Find the rate of taxation.

23. If a tax collector in a certain city receives a commission of 2% for collecting taxes, what per cent of the amount collected does the city receive? Find the amount of taxes that must be levied in order that a city may receive \$19,600, after allowing a collector a commission of 2% for collecting.

\$19,600 = 98% of the sum levied.

24. Property worth \$9000 was assessed at \$6000. The rate was \$1.50 for each \$100 of assessed valuation. Had this property been assessed at its full value, what rate of taxation would have yielded the same amount of taxes?

25. Examine a tax receipt. Is a separate entry made for taxes on personal property and on real property? Is there an entry for school taxes?

26. Make and solve five problems in taxes, using when possible the actual rates in your county or city.

248. Customs and Duties. The following rates of customs are from the schedule adopted by Congress in 1897, commonly known as the Dingley Tariff:

Newspapers, periodicals, free.
Coffee, free.
Musical instruments, 45% ad. val.
Potatoes, 25¢ per bu.
Tea, free.
Hay, $4 per ton.
Carpets (velvet), 60¢ per sq. yd. and 40% ad. val.
Table knives, 16¢ each and 15% ad. val.
Paintings, 20% ad. val.

249. Written Exercises.

1. Find the duty on 60 sq. yd. of velvet carpet worth $1.50 per square yard.

2. What is the duty on 45 tons of hay?

3. What is the duty on a violin worth $80?

4. A painting valued at $2500 was purchased in Italy and brought to the United States. Find the amount of customs on it.

5. Find the amount of the duty on 6 doz. table knives worth $1.80 per dozen.

6. Why are tea and coffee on the free list, while a duty of 25¢ per bushel is placed upon potatoes?

7. What were the net proceeds of an auction sale, if the sales amounted to $1215.40, and the auctioneer received a commission of 10%?

8. After deducting his commission of 5% and $12.50 for freight and cartage, a commission merchant remitted $633.50 to the shipper. Find the amount of the sales.

9. A city lot that cost $1600 was sold for $1800. Find the gain per cent.

250. Oral Exercises.

1. What is the price of coal a ton when it is selling at \$.25 a hundredweight?

2. When hay is selling at \$12 a ton, what is its price per hundredweight?

3. If $\frac{3}{5}$ of the length of a certain bridge is 240 ft., how long is the bridge?

4. If the interest for one year at 5% is \$80, what is the sum on which the interest is paid?

5. A boy shot 10 times at a target and hit it 8 times. Express as per cent the ratio of the number of accurate shots to the number of shots taken.

6. On a certain day a girl missed 3 out of 12 words in a spelling lesson. What per cent of the words did she spell correctly?

7. A baseball team played 8 games and lost 3 of them. What per cent of the games played did the team win?

8. A girl was absent from school 4 days and present 16 days during a school month. What per cent of the time was she present?

9. A man paid a tax of $1\frac{1}{2}$% on property valued at \$4000. Find the amount of his tax.

10. A commission merchant received \$20 for selling \$1000 worth of produce. What was his rate of commission?

11. If a spelling lesson consists of 25 words, what per cent of the lesson is each word? What per cent of the words does a boy spell correctly who misspells 4 words?

12. A boy caught a ball 6 times and missed it 2 times. The number of times he caught the ball is what per cent of the number of chances he had to catch it?

251. Trade Discount.* 1. Manufacturers and wholesale dealers issue catalogues describing articles sold by them and giving their *list prices.* A discount from the list price is made to retail dealers and sometimes to other customers, particularly when goods are purchased in large quantities. Such a discount is generally known as **trade discount,** or **commercial discount.**

2. Several successive discounts are sometimes allowed. Thus, an article may be sold subject to discounts of 25 %, 10 %, and 5 %; that is, a discount of 25 % is made from the list price, and a second discount of 10 % is made from the price after making the discount of 25 %, and a third discount of 5 % is made on the price after the two discounts have been made. A separate **cash discount** is usually allowed when payment is made within a specified time after the purchase of the goods.

252. Written Exercises.

1. Find the net cash price to a retail hardware merchant of a stove listed at $45, trade discounts of 20 % and 10 %, and a cash discount of 5 %.

MODEL:		
	$45,	list price.
	9,	first discount.
	$36,	second price.
	3.60,	second discount.
	$32.40,	third price.
	1.62,	cash discount.
	$30.78,	net price.

The first discount is 20 % of $45, or $9. The price after making this discount is $45 – $9, or $36. The second discount is 10 % of $36, or $3.60. The price after making the second discount is $36 – $3.60, or $32.40. From this a cash discount of 5 % is deducted, leaving the net price $30.78.

Instead of deducting each discount separately, the sum of the several discounts may be stated as a single discount. This may be

* For a more extended discussion of Trade Discount, see Appendix, p. 265.

found thus: A discount of 20% from the list price reduces the cost to 80% of the list price; and the second discount reduces it to 90% of this, or to 90% of 80% of the list price, which is 72% of the list price, and the cash discount reduces it to 95% of 72% of the list price, or to 68.4% ($68\frac{2}{5}$%) of the list price.* Compare $68\frac{2}{5}$% of $45 with the answer found.

In figuring discounts, use the shortest method in every part of the problem.

2. Which is the greater discount, a single discount of 25% or a discount of 20% and a second discount of 5%?

3. If the hardware merchant (Prob. 1) sold the stove for $45, how much was his profit if he paid out $2.50 for freight and cartage? What per cent profit did he make?

4. What was the net cash price to a jeweler of a watch listed at $35, discounts 30%, 15%, 5%, and a cash discount of 2%? If the jeweler sold the watch for $35, what per cent profit did he make on it?

5. A piano firm bought a piano listed at $350 and received discounts of 40%, 20%, and 10%. How much did the firm make on the piano by selling it at $350? What per cent profit was made by the firm?

6. How much does a dealer make on a carriage listed at $120, if he buys it at a discount of 20%, 5%, and takes advantage of a cash discount of 2%, and sells it at the list price?

7. How much less does a dealer pay for a wagon listed at $150, if he is allowed a single discount of 35% than if he is allowed successive discounts of 15%, 15%, and 5%?

8. What per cent of $48 is $60? of $50 is $30? of $120 is $100? of $100 is $120? of $150 is $200?

* The single equivalent discount may be found by adding together 20%, 10% of 80%, or 8%, and 5% of 72%, or 3.6%. 20% + 8% + 3.6% = 31.6%.

253. Oral Exercises.

1. A man bought a house for \$ 4000 and sold it at a gain of 20 %. For how much did he sell it?

2. A farmer sold 200 sacks of potatoes, which was 80 % of his entire crop. How much was his entire crop?

3. If the price of steak was raised from 12 ¢ a pound to 15 ¢ a pound, what was the per cent of increase in price?

4. The total enrollment in a certain school was 40 pupils, and the average number in daily attendance was 35. The average daily attendance was what per cent of the enrollment?

5. An agent received \$ 6 for collecting a bill of \$ 30. What was his per cent of commission?

6. A man bought a farm for \$ 4000 and sold it for \$ 5400, what was his per cent of gain?

7. At what price must a dealer sell carriages that cost \$ 120 to make a profit of $33\frac{1}{3}$ %? 20 %? 25 %? $12\frac{1}{2}$ %? 10 %?

8. After increasing his capital by \$ 1200, a merchant had \$ 4200 invested in his business. What amount had he invested before increasing his capital? By what per cent of itself was the original capital increased?

9. By selling a city lot for \$ 1500, a man gained 25 %. Find the cost of the lot.

10. Property worth \$ 6000 was assessed for purposes of taxation at \$ 4000. For what per cent of its value was the property assessed?

11. What monthly rental must a man get from property valued at \$ 3000 to yield a net income of 6 %, if it costs him \$ 60 a year to maintain his property?

INTEREST

254. 1. On July 1, 1907, Charles H. Thomas borrowed of Joseph R. White $ 300, which he promised to return in one year, with interest at 6 per cent. As an acknowledgment of his indebtedness and as a promise to pay, Mr. Thomas gave Mr. White his **note**, of which the following is a copy:

> *$300. Berkeley, Cal., July 1, 1907.*
>
> *-----One year-----after date, for value received,---I---*
>
> *promise to pay to Joseph R. White, or order,----------*
>
> *Three hundred and $\frac{00}{100}$ ------------------------ Dollars,*
>
> *with interest thereon at 6% per annum from date until paid.*
>
> *Charles H. Thomas.*

2. The sum specified in a note is called its **face**, or the **principal**.

3. What is the principal and the rate of interest specified in the above note? When and where was the note executed? On what date did the above note fall due?

4. The person to whom, or to whose order, the amount named in a note is to be paid is called the **payee**, and the person by whom the note is to be paid is called the **payer**. In the above note who is the payer and who the payee? What is the meaning of the words *or order? per annum?*

5. Has any provision been made in the above note for the payment of interest beyond the period of one year?

6. Where in the note did Mr. Thomas acknowledge that he had received of Mr. White something of the value of $ 300?

7. As the note was made payable to Mr. White, *or order*, it is said to be **negotiable**; that is, it may be passed from one person to another, and it becomes payable to the person to whom it is ordered paid. Six months after the note was executed, Mr. White bought a city lot of J. C. Anderson, and as part payment for the lot, he transferred the note to Mr. Anderson. In making the transfer, Mr. White wrote on the back of the note, over his own signature, "Pay to J. C. Anderson, or order." By this indorsement, the note was made payable to Mr. Anderson.

8. Find the interest on the note to July 1, 1908.

9. The sum of the principal and interest is called the **amount**.

10. What was the amount of the note on July 1, 1908?

255. Method of Aliquot Parts.

1. The unit of time for which interest is computed is usually one year. The interest on a given note for three years is how many times the interest for one year? The interest for six months is what part of the interest for one year?

2. What part of the interest for one year is the interest for 3 months? for 2 months? for 4 months? for 1 month?

3. The interest for 1 month is what part of the interest for 6 months? for 4 months? for 3 months? for 2 months?

4. The interest for 5 months is the interest for 4 months plus the interest for what part of 4 months?

5. Find the interest on $400 for 1 yr. and 5 mo. at 7 %.

MODEL:

	$400	
	.07	
	$28.00	= interest for 1 yr.
4 mo. = $\frac{1}{3}$ yr.	9.33	= interest for 4 mo.
1 mo. = $\frac{1}{4}$ of 4 mo.	2.33	= interest for 1 mo.
	$39.66	= interest for 1 yr. and 5 mo.

EXPLANATION. First find the interest for 1 yr. by multiplying the principal by .07. Next find the interest for 4 mo. by dividing the interest for 1 yr. by 3; then find the interest for 1 mo. by dividing the interest for 4 mo. by 4. The sum of these three amounts is the interest for 1 yr. and 5 mo. In dividing drop all fractions of cents.

6. From the interest for one year, the interest for any number of months may be found by taking the following parts:

1 mo. = $\frac{1}{12}$ yr. 3 mo. = $\frac{1}{4}$ yr.
2 mo. = $\frac{1}{6}$ yr. 4 mo. = $\frac{1}{3}$ yr.
6 mo. = $\frac{1}{2}$ yr.
5 mo. (4 mo. and 1 mo.) = $\frac{1}{3}$ yr. plus $\frac{1}{4}$ of $\frac{1}{3}$ yr.
7 mo. (6 mo. and 1 mo.) = $\frac{1}{2}$ yr. plus $\frac{1}{6}$ of $\frac{1}{2}$ yr.
8 mo. (4 mo. and 4 mo.) = $\frac{1}{3}$ yr. plus $\frac{1}{3}$ yr.
9 mo. (6 mo. and 3 mo.) = $\frac{1}{2}$ yr. plus $\frac{1}{2}$ of $\frac{1}{2}$ yr.
10 mo. (6 mo. and 4 mo.) = $\frac{1}{2}$ yr. plus $\frac{1}{3}$ yr.
11 mo. (6 mo. and 5 mo.) = $\frac{1}{2}$ yr. plus $\frac{1}{3}$ yr. plus $\frac{1}{4}$ of $\frac{1}{3}$ yr.

Into what other suitable parts for finding interest may each be divided: 9 mo.? 8 mo.? 10 mo.?

256. Written Exercises.

Find the interest and the amount of:

1. $250, 1 yr. 6 mo., 8 %.
2. $560, 2 yr. 4 mo., 6 %.
3. $875, 3 yr. 5 mo., 7 %.
4. $100, 1 yr. 9 mo., 7 %.
5. $620, 7 mo., 4½ %.
6. $260, 1 yr. 10 mo., 8 %.
7. $720, 1 yr. 3 mo., 5 %.
8. $1200, 10 mo., 6 %.
9. $2400, 2 yr. 3 mo., 6 %.
10. $500, 1 yr. 8 mo., 4 %.

257. Interest for Years, Months, and Days.

1. It is sometimes necessary to find the interest for years, months, and days, in which case thirty days are usually regarded as one month.

2. When the interest for one month is known, how may the interest be found for 15 da.? for 10 da.? for 6 da.? for 5 da.? for 3 da.? for 1 da.? When the interest for 6 da. is known, how may the interest be found for 1 da.?

3. Find the interest on \$150 for 1 yr. 7 mo. 14 da. at 8%.

MODEL:	\$150	
	.08	
	\$12.00	= interest for 1 yr.
6 mo. = $\frac{1}{2}$ yr.	6.00	= interest for 6 mo.
1 mo. = $\frac{1}{6}$ of $\frac{1}{2}$ yr.	1.00	= interest for 1 mo.
10 da. = $\frac{1}{3}$ mo.	.33	= interest for 10 da.
3 da. = $\frac{1}{10}$ mo.	.10	= interest for 3 da.
1 da. = $\frac{1}{3}$ of 3 da.	.03	= interest for 1 da.
	\$19.46	= interest for 1 yr. 7 mo. 14 da.

4. From the interest for one month, the interest for any number of days may be found as in the following:

22 da. (10 da. and 10 da. and 2 da.) = $\frac{1}{3}$ mo. plus $\frac{1}{3}$ mo. plus $\frac{1}{5}$ of $\frac{1}{3}$ mo.

18 da. (6 da. and 6 da. and 6 da.) = $\frac{1}{5}$ mo. taken 3 times. Is this easier than to separate 18 da. into the parts 15 da. and 3 da., or into the parts 10 da. and 6 da. and 2 da.? Explain and illustrate.

5. Determine for each number of days, from 1 to 29, how the interest can be found most readily, when the interest for one month is known. Compare your results with those determined by other pupils, to see who has the best method. Test each method by taking some amount as the interest for one month.

6. Write a note, naming some pupil as payee and yourself as the maker, and find the amount of the note for 1 yr. 5 mo. 12 da.

258. Written Exercises.

Find the interest on:

1. $250 for 1 yr. 9 mo. 15 da. at 7%.
2. $700 for 2 yr. 8 mo. 21 da. at 5%.
3. $684.50 for 7 mo. 25 da. at 6%.
4. $1200 for 3 yr. 4 mo. 14 da. at $5\frac{1}{2}$%.
5. $300 for 1 yr. 10 da. at 8%.
6. $45.75 for 2 yr. 8 mo. 12 da. at 4%.
7. $2500 for 9 mo. 13 da. at 8%.
8. $560 for 2 yr. 3 mo. 23 da. at 7%.
9. $645.40 for 2 yr. 7 mo. 26 da. at 9%.
10. $820.15 for 1 yr. 5 mo. 20 da. at 4%.
11. $125 for 11 mo. 17 da. at 8%.
12. $214.45 for 4 yr. 2 mo. 19 da. at 6%.
13. $750 for 1 yr. 6 mo. 28 da. at 7%.

Find the interest on each of the following at 6%; at 5%; at 7%; at $4\frac{1}{2}$%:

14. $800 from Oct. 1, 1904 to May 10, 1906.
15. $475 from June 11, 1903 to Nov. 18, 1904.
16. $240.60 from April 8, 1904 to Feb. 21, 1905.
17. $350 from Jan. 1, 1904 to Nov. 20, 1904.
18. $1340 from June 8, 1903 to Dec. 29, 1904.
19. $26.48 from Sept. 12, 1905 to Aug. 10, 1906.
20. $1700 from March 24, 1905 to Aug. 15, 1906.
21. $48.62 from Nov. 18, 1902 to July 20, 1904.
22. $5000 from Sept. 7, 1903 to Dec. 23, 1903.
23. $467.89 from April 4, 1904 to July 26, 1905.

259. Sixty Days Method.

1. Money loaned for less than one year is usually loaned for 90 da., 60 da., or less. The best unit of time to use in finding the interest is 60 da., and the best rate is 6 %, as the interest at 6 % for 60 da. is 1 % (.01) of the principal, found by moving the decimal point.

2. Find the interest on $2700 for 60 da. at 7 %.

MODEL:

$27	= interest at 6 % for 60 da.
4.50	= interest at 1 % for 60 da.
$31.50	= interest at 7 % for 60 da.

3. What part of 60 da. is 30 da.? 10 da.? 20 da.? 5 da.? 15 da.? 12 da.? 6 da.? 3 da.? 2 da.?

4. From the interest for 60 da., how may the interest be found for 30 da.? for 15 da.? for 6 da.? for 20 da.? for 10 da.? for 5 da.? for 90 da.? for 120 da.?

5. Find the interest on $4000 at 6 % for 60 da.; for 90 da.; for 30 da.; for 20 da.; for 120 da.

6. From the interest for 30 da., how may the interest be found for 15 da.? for 45 da.? for 10 da.? for 5 da.?

7. From the interest at 6 %, how may the interest be found at 7 %? at 8 %? at 9 %? at 5 %? at $5\frac{1}{2}$ %?

8. Find the interest on $500 for 90 da. at 7 %.

MODEL:

$5	= interest at 6 % for 60 da.
2.50	= interest at 6 % for 30 da.
$7.50	= interest at 6 % for 90 da.
1.25	= interest at 1 % for 90 da.
$8.75	= interest at 7 % for 90 da.

Find the interest on:

9. $600 at 6 % for 30 da.

10. $1000 at 7 % for 90 da.

11. $1200 at 5 % for 60 da.

12. $10,000 at 7 % for 45 da.

260. Cancellation Method. (For problems, see Sec. 258.)

1. What part of 360 da. are 90 da.? The interest for 90 da. is what part of the interest for 1 yr. (360 da.)?

2. Find the interest on \$600 for 90 da. at 7%.

MODEL: $\overset{\$150}{\$\cancel{600}} \times .07 \times \dfrac{\overset{1}{\cancel{90}}}{\underset{4}{\cancel{360}}} = \$10.50.$

The interest for 1 yr. is \$600 × .07, and the interest for 90 da. is $\frac{90}{360}$, or $\frac{1}{4}$ of \$600 × .07.

3. What part of 1 year's interest is the interest for 6 mo.? for 2 mo.? for 8 mo.? for 9 mo.? for 10 mo.? for 15 da.? for 45 da.? for 1 mo. 15 da.? for 3 mo. 20 da.? for 1 yr. 3 mo.? for 1 yr. 6 mo.? Find the interest on \$600 at 5% for each of these periods.

261. Six Per Cent Method. (For problems, see Sec. 258.)

1. Interest is sometimes calculated by a method commonly known as the Six Per Cent Method. By this method, the interest at 6% for the given time is found, and from this the interest at the required per cent.

2. As 1 mo. is $\frac{1}{12}$ of 1 yr., the rate of interest for 1 mo. is $\frac{1}{12}$ of 6%, or $\frac{1}{2}$% (.005); and as 1 da. is $\frac{1}{30}$ of 1 mo., the rate of interest for 1 da. is $\frac{1}{30}$ of .005, or $.000\frac{1}{6}$.

The interest at 6% for 1 yr. = .06 of the principal.
The interest at 6% for 1 mo. = .005 of the principal.
The interest at 6% for 1 da. = $.000\frac{1}{6}$ of the principal.

3. Find interest on \$350 for 2 yr. 7 mo. 21 da. at 7%.

MODEL:	Rate for 2 yr. at 6%	.12
	Rate for 7 mo. at 6%	.035
	Rate for 21 da. at 6%	.0035
	Rate for 2 yr. 7 mo. 21 da.	.1585

\$350 × .1585 = \$55.475 = int. on \$350 for 2 yr. 7 mo. 21 da. at 6%.
9.245+ = int. on \$350 for 2 yr. 7 mo. 21 da. at 1%.
\$64.72 = int. on \$350 for 2 yr. 7 mo. 21 da. at 7%.

262. Promissory Notes.

1. A written promise to pay a definite sum of money at a specified time is called a **promissory note.**

A promissory note is usually called a *note.*

2. Compare the promissory note on p. 207 with the following. The note on p. 207 is a *time note,* as the time of payment is specified in it.

A DEMAND NOTE

$300 Oakland, Cal., Jan. 4, 1907.

On demand, for value received I promise to pay to Joseph R. White, or order, three hundred dollars, with interest thereon at 6% per annum from date until paid.

Charles H. Thomas.

A JOINT NOTE

$600 Seattle, Wash., July 16, 1906.

Sixty days after date, we, or either of us, promise to pay to Frank F. Kenyon, or order, six hundred dollars, with interest thereon at 5% per annum from date until paid.

Value received.

Walter J. Bunker,
John R. Davies.

3. Each maker of a joint note is liable for its payment in full.

4. The following should appear in a note:

a. The time and place where the note was executed.

This is usually written at the top and toward the right.

b. The sum to be paid, including the rate of interest, if any is paid.

The face of the note is usually written in figures at the top and toward the left and in words in the body of the note.

c. The signature of the maker or makers.

d. The time of payment.

When no time of payment is specified, the note is payable on demand.

e. Notes usually contain the words *value received.*

Answer each concerning the two notes in Sec. 262.

5. When was the note executed? Where?
6. When is the note payable?
7. What is its face?
8. Who is the maker? the payee?
9. Is the note negotiable?

10. When a note becomes due it is said to **mature.** In some states a note matures three days after the time specified in the note. The three additional days are called **days of grace.** Days of grace have been abolished in most states, and are not computed in the answers given in this book.

11. Each state has fixed its own *legal* rate of interest, which is the rate allowed on claims drawing interest when no rate of interest has been otherwise arranged. What is the legal rate in the state in which you live?

12. Many states have fixed a *maximum* rate of interest that can be collected by agreement. A higher rate than that authorized by law is called **usury.** Is there a law against usury in the state in which you live?

13. Write a demand note; a time note; a joint note.

263. Partial Payments. — Mercantile Rule.* **1.** On Jan. 1, 1906, James Smith of Los Angeles, Cal., borrowed of Frank Adams $1000 for one year at 6%, giving his note for this amount. Write the note.

When the loan was made, it was agreed that if James Smith made any payments on the note before its maturity, he would be credited with each *partial* payment and would be credited with the same rate of interest as he was paying, 6%, from the date of each payment until the time of final settlement.

2. On July 1, 1906, James Smith paid Frank Adams $400. Indorse this payment by writing "July 1, 1906, $400" on the back of the note. Final settlement was made Jan. 1, 1907.

Under these conditions James Smith had the use of $1000 borrowed of Frank Adams for 1 yr., and Frank Adams had the use of $400 paid by James Smith for 6 mo. (from July 1, 1906, to Jan. 1, 1907).

At the time of final settlement James Smith owed Frank Adams $1000, with interest for 1 yr., at 6%, or $1060; and Frank Adams owed James Smith $400, with interest for 6 mo. at 6%, or $412. In settling the note, James Smith paid Frank Adams $1060 – $412, or $648.

Notes and accounts which do not run for more than one year, on which partial payments are made, are often settled by business men as above.

MERCANTILE RULE. 1. *Find the amount of the face of the note at the time of settlement.*

2. *Find the amount of each payment from the date of payment to the date of settlement.*

3. *Subtract the sum of the amounts of the payments from the amount of the face of the note.*

3. Write a note, naming some pupil as payee and yourself as payer. Make three partial payments and have them indorsed to your credit. Settle the note.

* For the United States Rule of Partial Payments, see Appendix, p. 266.

264. Compound Interest.*

1. When the unpaid interest is added to the principal, as it becomes due, to form a new principal on which interest is computed, the interest is called **compound interest.**

Interest may be added to the principal annually, semiannually, quarterly, etc., according to agreement.

The payment of compound interest cannot usually be enforced by law, but if the debtor is willing to pay compound interest, it may be collected without violating the law against usury.

2. Savings banks generally pay interest semiannually. When it is not collected by the depositor, it is added to his deposit and he is paid compound interest.

3. If interest is collected when due and reinvested at once at the same rate of interest, the result is the same as when compound interest is received.

4. Find the amount of \$600 for 2 yr. 6 mo. at 8%, interest compounded annually. Find the difference between the compound interest and the simple interest.

MODEL:

\$600	= principal for first year.
48	= interest for first year.
\$648	= amount, or principal, for second year.
51.84	= interest for second year.
\$699.84	= amount, or principal, for third year.
27.99	= interest for 6 mo.
\$727.83	= amount for 2 yr. 6 mo. at 8%.

Compound interest	= \$727.83 − \$600 =	\$127.83.
Simple interest	=	120.
Difference	=	\$ 7.83.

5. If a man invests \$1000 at compound interest at 6% when he is 30 years of age and keeps it earning at the same rate until he is 50 years of age, what will be the amount of the \$1000 at that time? (Use table, p. 320.)

* For table of compound interest, see Appendix, p. 320.

265. Bank Discount and Proceeds.

1. Banks usually collect interest in advance on sums loaned. Thus, if George White borrows \$100 at a bank for 60 da. at 6%, his note will be made out for \$100, and the bank will deduct from this amount the interest on \$100 for 60 da. at 6%, or \$1. Mr. White will receive \$99. At the end of 60 da. he will pay the bank the face of the note, or \$100.

2. If interest is collected in advance, how much money will a person receive at a bank on a note for \$2000 for 60 da., if the bank charges 8% interest?

3. On April 8 J. J. Dow bought \$600 worth of goods of D. C. Brown, on 90 da. time, giving his note for the amount without interest. On the same day D. C. Brown sold the note to a bank, the bank deducting 6% interest for the term of the note (90 da.). Find the amount received for the note by D. C. Brown.

4. Interest paid in advance upon the amount due on a note at its maturity is called **bank discount.** Bank discount is computed from the date of the purchase of the note by the bank to the legal date of maturity.

Some banks include both the day of purchase and the day of maturity in the discount period. When days of grace are allowed, these are included in the discount period.

5. The sum paid for a note when sold is called the **proceeds** of the note. The proceeds on a note is the amount due at maturity, less the bank discount.

6. C. W. Smith held a note against R. E. Orr for \$4000 for 60 da. without interest. After 20 da., he sold it to a bank at a discount of 6%; that is, the bank deducted 6% int. on the note for the 40 da. between its purchase and expiration. Find the bank discount and the proceeds.

7. On April 24, 1906, James J. Hall sold a horse to G. M. Bruce for \$150, taking in payment his note for 1 year with interest at 6%. Find the amount of the note at maturity.

8. Mr. Hall (Ex. 7) needed money, so he sold the note to a bank on the same day, the bank discounting it at 6%. How much did Mr. Hall receive for the note?

MODEL:	Face of note	= \$150
	Interest for 1 yr. at 6%	= 9
	Amount at maturity	= \$159.
	Discount 1 yr. at 6%	= 9.54 (computed on \$159)
	Proceeds	= \$149.46

9. If the note (Probs. 7 and 8) had been discounted at 8% instead of 6%, what amount would Mr. Hall have received?

10. If the note (Probs. 7 and 8) had been discounted three months after date of issue, or on July 24, 1906, the bank would have deducted interest on the amount due at maturity (\$159) for the exact number of days from July 24, 1906 to April 24, 1907 (7 da. + 31 da. + 30 da. + 31 da. + 30 da. + 31 da. + 31 da. + 28 da. + 31 da. + 24 da.), or for 274 da. Find the amount which Mr. Hall would have received.

11. A 90-da. note for \$500, without grace, dated Aug. 5, 1905, with interest at 5%, was discounted at a bank on Aug. 25 at 6%. Find the day of maturity, the amount at maturity, the bank discount, and the proceeds.

12. A man borrowed \$1000 of a bank for 1 yr. at 6%, paying interest in advance. 6% interest in advance on \$1000 is equivalent to what rate paid at the end of the year?

266. Review.

1. During a certain school month a boy worked 209 problems, of which 194 were correct. Find the per cent of correct work.

2. A baseball team won 43 games and lost 15 games one season. Find the per cent of games won.

3. The rent of a house was raised from $ 30 a month to $ 35; this was an increase of what per cent?

4. A person bought a house for $ 6000. The taxes, insurance, repairs, and other expenses connected with the property amounted to $ 120 a year. For how much a month must the property be rented to net 6 % on the investment?

5. A man built two flats costing him $ 5000 on a lot which cost $ 2000. He rented one of the flats for $ 40 a month and the other for $ 35. The expenses connected with the property amounted to $ 200 a year. The net income amounted to what per cent on the investment?

6. An electric light meter registered 80,000 watt hours on Oct. 9, and 106,000 watt hours on Nov. 9. Find the amount of the bill for the month at 9¢ for each 1000 watt hours.

7. A gas meter registered 29,800 cu. ft. on April 24, and 31,800 cu. ft. on May 24. Find the amount of the bill for the month at $ .90 per 1000 cu. ft.

8. The population of a certain city was 47,235 in 1900, and 60,624 in 1910. Find the increase per cent.

9. At 65¢ a sack (100 lb.), what is the price of coal per ton?

10. Find the interest on $ 2800 from June 8 to Jan. 16 at 6 % per annum.

PART IV

FORMS AND MEASUREMENTS *

267. 1. Lines are **vertical** |, **horizontal** —, and **oblique** \ /.

2. These are **right angles.** 

3. A **rectangle** has four right angles. 

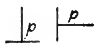
4. These are **right triangles.** 

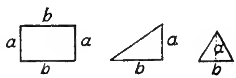
5. These are **acute angles.** 

6. These are **acute-angled triangles.** 

7. These are **obtuse angles.** 

8. These are **obtuse-angled triangles.** 

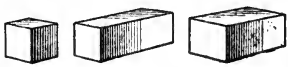
9. **Perpendicular** (p) means at right angles to. 

10. These figures have a **base** (b) and an **altitude** (a). 

11. These lines are **parallel.**

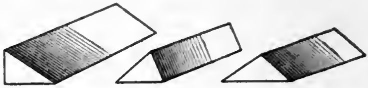
12. These are **quadrilaterals.** 

13. These quadrilaterals are **parallelograms.** 

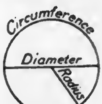
14. These are **rectangular prisms.** 

* With complete reviews.

15. These are **triangular prisms**.



16. Circumference, diameter, and **radius** belong to the **circle**.



Circle.

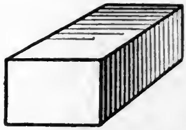
17. These are **cylinders**.

268. Relation of Forms.

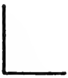
Study the relation of these forms:

RIGHT ANGLE.

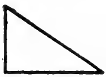
RECTANGLE.



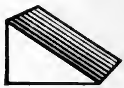
RECTANGULAR PRISM.



RIGHT ANGLE.



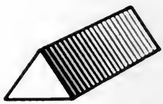
RIGHT TRIANGLE.



RIGHT TRIANGULAR PRISM.

ACUTE ANGLE.

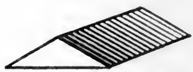
ACUTE TRIANGLE.



ACUTE TRIANGULAR PRISM.

OBTUSE ANGLE.

OBTUSE TRIANGLE.



OBTUSE TRIANGULAR PRISM.

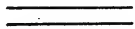
CIRCLE.

SPHERE.

CYLINDER.

269. Lines.

1. Lines that extend in the same direction and are the same distance apart are called **parallel lines.**



PARALLEL LINES.

2. Suspend a weight by a string. When the weight is at rest, the line represented by the string is called a **vertical line.**

3. The surface of the water in a tank or a pond is said to be level, or **horizontal.** A slanting line is called an **oblique** line.

A vertical line is represented on a page by a line parallel to the sides, and a horizontal line by a line parallel to the top and bottom.

4. Hold your pencil in a vertical position; in a horizontal position; in an oblique position.

5. Point to surfaces in the schoolroom that are horizontal, vertical, oblique, parallel.

6. Draw two vertical parallel lines on the blackboard; two horizontal parallel lines; two oblique parallel lines.

270. Right Angles.

1. Two lines that meet form an **angle,** ∠. When two lines form a square corner, the angle between them is called a **right angle.**

2. Draw four right angles.

RIGHT ANGLES.

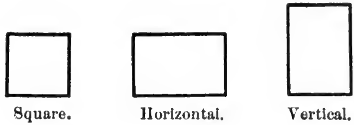
3. Point to surfaces in your schoolroom that meet at right angles.

4. Two lines that form a right angle are said to be **perpendicular** to each other. Draw perpendicular lines.

PERPENDICULAR LINES.

5. Point to lines or surfaces in the schoolroom that are perpendicular to each other.

271. Rectangles.



RECTANGLES.

1. A figure whose angles are all right angles is called a **rectangle.** *Rect* means right. Rectangle means *having right angles.*

2. A rectangle whose sides are all the same length is called a **square.**

A rectangle having two opposite sides longer than the other two opposite sides is sometimes called an **oblong.**

3. Draw a square; a vertical rectangle; a horizontal rectangle. Point to surfaces in your schoolroom that are rectangles. Are any of these squares?

4. How many sides has a rectangle? How many angles has a rectangle? Are the sides of a rectangle parallel? Name surfaces not in your schoolroom that are rectangles.

272. 1. Draw a square whose side is 1 foot. This is called a **square foot.** Draw and name a square whose side is 1 inch.

2. Draw a square yard. Divide it into square feet. How many square feet are there in a square yard?

3. Divide a square foot into square inches. How many square inches are there in a square foot?

4. A square $16\frac{1}{2}$ feet each way is called a **square rod.** Mark out a square rod on the school grounds.

5. Draw a square whose side is 2 inches. Divide it into square inches. How many are there?

6. Draw 3 inch squares and a 3-inch square. Compare the size of a 3-inch square and 3 square inches.

7. The number of square units in a surface is called its **area.**

273. Areas of Rectangles.

1. Repeat the Table of Linear Measure (§ 99). Review p. 81. Repeat the Table of Square Measure (§ 101).

2. Using the scale $\frac{1}{4}$ in. = 1 rd., make a drawing to represent a rectangle 16 rd. long and 10 rd. wide. Express the area of the rectangle in acres.

3. Find the area of a flower bed that is 12 ft. 9 in. long and 8 ft. 4 in. wide.

4. How many acres are there in a tract of land 80 rd. by 80 rd.?

5. A farm that contains 80 A. is $\frac{1}{4}$ mi. wide. How long is the farm?

6. Find the number of square yards of surface in the walls and ceiling of your schoolroom, deducting for the doors and windows.

7. Find the value of a field 40 rd. long and 20 rd. wide at $85 an acre.

274. Written Exercises.

1. Reduce 2 yd. 2 ft. 7 in. to inches.

2. Find the sum of 8 ft. 6 in., 7 ft. 4 in., 9 ft. 11 in., and 6 ft. 5 in.

3. Find the perimeter of a rectangle whose length is 24 ft. 8 in. and whose width is 15 ft. 10 in.

4. How many rods of fence are required to inclose a rectangular 20-acre field whose length is 80 rd.?

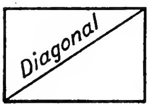
5. How many bundles of shingles are necessary to shingle a surface 50 ft. by 16 ft., if the shingles are laid 4 in. to the weather?

6. How many yards of carpet are necessary to cover a floor 16 ft. by 12 ft., if the carpet is 27 in. in width?

275. Right Triangles.

1. Draw a right angle. The point at which the lines meet is called the **vertex** of the angle.

2. Draw a rectangle. Draw a straight line joining the vertices of the opposite angles of the rectangle. This line is called the **diagonal** of the rectangle. The diagonal divides the rectangle into two equal *triangles*.

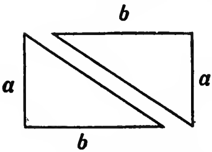


3. A figure having three angles is called a **triangle.** *Tri* means three. Triangle means *having three angles.*

4. A triangle having one right angle is called a **right triangle.**

276. Area of Right Triangles.

1. The **base** of a figure is the side on which it is assumed to rest, and the **altitude** is the perpendicular distance between the top and the base, or the base produced.



RIGHT TRIANGLES.

Consider the length of a rectangle as its base and the width as its altitude.

2. The area of a right triangle is what part of the area of a rectangle having the same base and altitude?

The area of a right triangle is equal to one half the product of its base and altitude.

The work is sometimes shorter if the altitude is multiplied by one half the base, or if the base is multiplied by one half the altitude.

Dimensions must be expressed in like units. By the product of the *lines* is meant the product of the *numbers* denoting them.

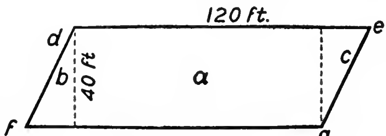
3. Find the area of right triangles of the following dimensions: base, 12 in., altitude, 8 in.; base, $6\frac{1}{2}$ ft., altitude, $9\frac{1}{4}$ ft.; base, 40 rd., altitude, 80 rd.

277. Parallelograms.

1. A plane (flat) figure bounded by four straight lines is called a **quadrilateral.** *Quadri* means four and *lateral* means sides. Quadrilateral means *having four sides.*

2. A quadrilateral whose opposite sides are parallel is called a **parallelogram.** Is a rectangle a parallelogram? Draw a quadrilateral that is not a parallelogram.

3. This figure represents a city lot. Is the form of the lot a parallelogram? The form of the lot may be regarded as composed of a rectangle, *a*, and two right triangles, *b* and *c*. Since the figure is a parallelogram, triangle *b* is the same size as triangle *c*. If triangle *b* were cut off and placed alongside triangle *c*, with the side *fd* on the side *ge*, what change, if any, would this make in the size of the lot? What change, if any, would it make in the form of the lot? What would be the dimensions of the resulting lot? How should you find its area?



PARALLELOGRAM.

4. The sum of the areas of triangles *b* and *c* is equivalent to the area of a rectangle having the same base and altitude as the triangles. Therefore the number of square feet in the lot may be found by multiplying 120 by 40.

The area of a parallelogram is equal to the product of its base and altitude.

5. Cut several parallelograms out of paper, and show by the method of Prob. 3 that each parallelogram has the same area as a rectangle having the same base and altitude.

6. Draw several parallelograms. Assign the dimensions, and find the area of each.

278. Trapezoids.

1. Fig. 1 represents a field whose area is to be found. It may be regarded as composed of a rectangle, *a*, and a right triangle, *b*. Its area is the sum of the areas of these two parts. The number of square rods in the rectangle is 10×20, and in the triangle is 10×3. The number in the entire field is 10×23. Explain.

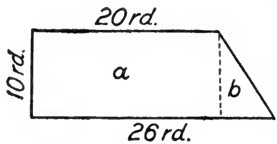


FIG. 1.—TRAPEZOID.

2. What is the average length of the two sides of the field (Fig. 1)? The field has the same area as a field of the same width whose length is one half of the sum of 20 rd. and 26 rd., or 23 rd. Explain.

3. Draw five figures similar to Fig. 1. Assign the dimensions, and find the area represented by each.

4. Figure 2 represents a cross section of a foundation wall. It may be regarded as composed of a rectangle, *a*, and two right triangles, *b* and *c*. What is the combined length of the bases of the two triangles? If triangle *b* were cut off and placed in an inverted position alongside triangle *c*, what change would it make in the form of the figure? What would be the dimensions of the resulting figure? How would you find its area?

5. The area of triangle *b* (Fig. 2) is equivalent to the area of a rectangle of the same altitude whose base is one half of the base of the triangle. Is the same true of the area of triangle *c*? The area of the surface represented by the entire figure is equivalent to the area of a rectangle of the same altitude but whose base is one half the sum of the two bases *de* and *fg*.

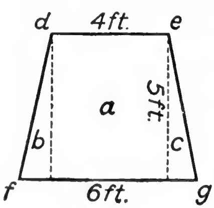


FIG. 2.

6. A quadrilateral that has only two sides parallel is called a **trapezoid.** See Figs. 1 and 2, p. 228.

The area of a trapezoid is equal to the product of its altitude and one half the sum of its bases.

7. Draw five figures similar to Fig. 2, p. 228. Assign the dimensions, and find the area of each.

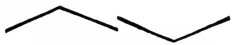
The area of any quadrilateral may be found by resolving it into triangles or into rectangles and triangles.

279. Area of Triangles.

1. An angle that is less than a right angle is called an **acute angle.** *Acute* means sharp or pointed.

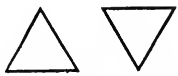
ACUTE ANGLES.

2. An angle that is greater than a right angle is called an **obtuse angle.** *Obtuse* means dull or blunt.



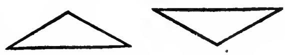
OBTUSE ANGLES.

3. A triangle whose angles are all acute is called an **acute-angled triangle.**



ACUTE-ANGLED TRIANGLES.

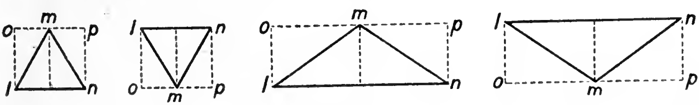
4. A triangle one of whose angles is obtuse is called an **obtuse-angled triangle.**



OBTUSE-ANGLED TRIANGLES.

5. Can you draw a triangle having more than one obtuse angle? Draw an acute-angled triangle; an obtuse-angled triangle.

6. Any triangle may be divided into two right triangles.



7. Triangle *lmn* in each of the above figures is one half of the rectangle *opln.* Explain.

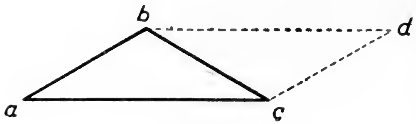


Fig. 1.

8. Any triangle may be considered as one half of a parallelogram of the same base and altitude. Triangle *abc* in Fig. 1 is one half of the parallelogram *abdc*.

9. Draw five triangles. Show by the methods given in Probs. 7, 8 that the triangles are each one half of a parallelogram having the same base and altitude.

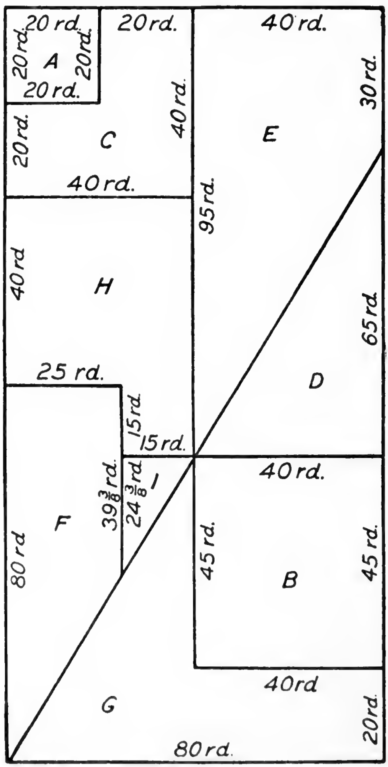


Fig. 2.

The area of a triangle is equal to one half the product of its base and altitude.

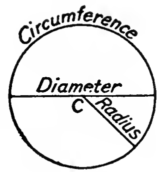
10. Draw five triangles. Assign their dimensions and find the area of each.

The area of any figure may be found by resolving it into triangles.

11. Figure 2 represents a farm 80 rd. by 160 rd. Find the area of each field.

12. The two sides of a field, one 132 rods, the other 152 rods in length, are parallel. The perpendicular distance between the two sides is 80 rods. Diagram and find area.

280. Circles.



1. Draw a circle. Mark the center (C) of the circle. The line bounding the circle is called the **circumference.** Draw a line from the center of the circle to the circumference. This is called a **radius** of the circle. Draw a straight line through the center of the circle from circumference to circumference. This is called the **diameter** of the circle. Compare the length of the diameter with the length of the radius.

2. Measure the diameter of a circle. Measure the circumference of the same circle. Divide the circumference by the diameter. The answer should be nearly 3.1416 ($3\frac{1}{7}$). This is the ratio of the circumference to the diameter. This ratio is commonly denoted by the symbol π, which is a Greek letter named pī.

3. The circumference of a circle is 3.1416 times the diameter. How can you find the circumference when the radius is given? When the circumference is given, how can you find the diameter? the radius?

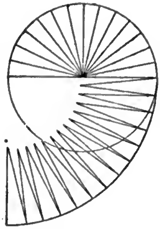
4. If a wagon wheel is $3\frac{1}{2}$ ft. in diameter, what is its circumference? How many times will it turn in going 1 mile?

5. Find the circumference of a circle whose diameter is 24 in.; $7\frac{1}{2}$ in.; 2 ft. 6 in.; 40 mi.; 80 rd.

6. Find the circumference of a circle whose radius is 2 in.; 2 ft. 4 in.; 6 ft.; 40 rd.; $3\frac{1}{2}$ yd.

7. Find the diameter and radius of a circle whose circumference is 24 ft.; $12\frac{1}{2}$ in.; 1 mi.; 36 rd.

8. Find the equatorial diameter of the earth if its equatorial circumference is 24,900 mi.



281. Area of a Circle.

1. A circle may be regarded as composed of an infinite number of triangles, the sum of whose bases is the circumference of the circle and whose altitude is the radius of the circle. Therefore the area of a circle is the area of the triangles composing it.

The area of a circle is equal to one half the product of its circumference and radius.

2. The circumference of a circle is 3.1416 times the diameter, or π times 2 times the radius $= 2\,\pi r$.

Area of circle (Prob. 1) $= \dfrac{\text{Cir.} \times r}{2}$.

Substituting $2\,\pi r$ for Cir.,

Area of circle $= \dfrac{2\,\pi r \times r}{2} = \pi \times r \times r$, or πr^2, read *pi r square*.

The area of a circle is equal to πr^2 $(3.1416 \times r \times r)$.

3. Find the area of a circle whose radius is 2 in.; 12 in.; 24 ft.; 4.5 rd.; 64 yd.; 18 ft.; 3 ft. 7 in.; $8\frac{3}{4}$ in.

4. Find the area of a circle whose circumference is 24 in.; 2 ft. 4 in.; 40 rd.; 28 yd.; $\frac{1}{2}$ mi.; 1 mi.

5. The diameter of a circular flower bed is 6 ft. What is its area?

6. The atmospheric pressure is about 15 lb. to the square inch. Find the pressure on a surface of a watch crystal $1\frac{1}{2}$ in. in diameter.

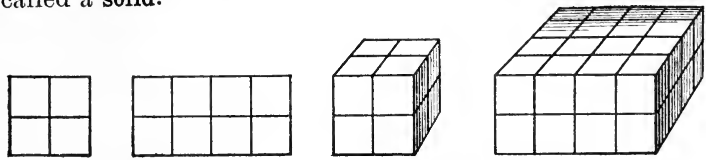
7. Over how many square feet of surface can a horse graze when tied with a rope 20 ft. long?

8. Which has the greater surface, a rectangular table top that is 3′ 6″ by 3′ 6″ or a circular table top that is 3′ 9″ in diameter?

282. Volume of Prisms and Cylinders.

1. Draw a rectangle. A rectangle has two dimensions; namely, *length* and *width*. If it is given a third dimension, *thickness*, it becomes a **rectangular solid** or a **rectangular prism**.

2. Anything that has length, breadth, and thickness is called a **solid**.



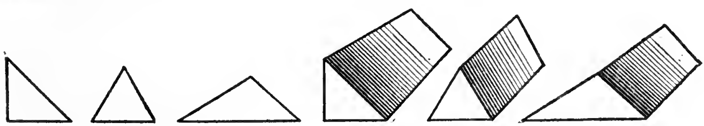
RECTANGLES. RECTANGULAR PRISMS.

3. How many rectangular faces has a rectangular prism? Name some rectangular prisms that you have seen.

4. A rectangular prism whose faces are all equal squares is called a **cube**.

5. Construct rectangular prisms out of cardboard or paper.

283. 1. Draw a triangle. How many dimensions has a triangle? If it is given a third dimension, it becomes a **triangular prism**. The ends of a triangular prism are triangles and the sides are rectangles.

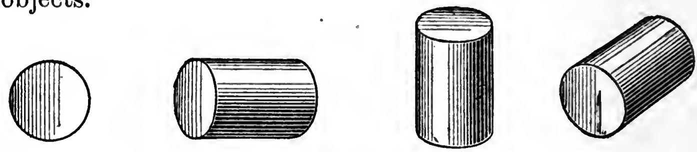


TRIANGLES. TRIANGULAR PRISMS.

2. How many faces has a triangular prism? How many of the faces are triangles? How many are rectangles?

3. Construct a triangular prism out of cardboard or paper.

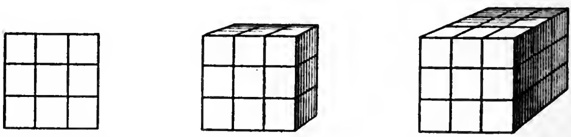
284. 1. Draw a circle. If a circle is given three dimensions, it becomes a **cylinder**. Mention some cylindrical objects.



CYLINDERS.

2. Construct a cylinder out of paper.

3. The number of cubic units that a solid contains is called its **volume**, or **capacity**.



9 square units. FIG. 1. — 9 cubic units. FIG. 2. — 3 times 9 cubic units. FIG. 3.

285. 1. The area of the end, or base, of a prism or a cylinder tells how many square units that surface contains (Fig. 1). There are as many cubic units in one unit of length as there are square units in the surface of the end, or base (Fig. 2). Explain. There are as many cubic units in the prism or cylinder as the product of the number of units in the area of the end, or base, and the number of units in the length or altitude of the prism or cylinder (Fig. 3). Explain.

2. The **volume** of a prism or a cylinder is equal to the product of the area of the end, or base, and the length, or altitude.

3. Find the capacity of a cylindrical tank whose diameter is 18 in. and whose height is 4 ft. 6 in.

Number of square feet in area of base $= 3.1416 \times \frac{3}{4} \times \frac{3}{4}$ $(\frac{3}{4} \times \frac{3}{4} = r^2)$.
Number of cubic feet in capacity $= 3.1416 \times \frac{3}{4} \times \frac{3}{4} \times 4\frac{1}{2}$.

4. There are 231 cubic inches in a gallon. How many gallons will the tank (Prob. 3) hold?

5. Find the number of cubic feet in a bin 8 feet long, 4 feet wide, and 6 feet deep.

6. There are 2150.42 cubic inches in a measured bushel. How many bushels will the bin (Prob. 5) hold?

7. Find the number of cubic feet of air in a room 16 feet long, 10 feet wide, and 9 feet high.

8. Find the number of cubic yards of earth that must be removed in excavating a basement 8 feet deep, 36 feet long, and 24 feet wide.

286. Surfaces of Prisms and Cylinders.

1. How many surfaces has a cube? a rectangular prism? a triangular prism? Construct each out of cardboard. State how the area of the combined surfaces of a prism may be found.

2. Find the area of the surfaces (excluding the ends) of a timber 12″ by 12″ and 16′ in length. What name is given to such a solid?

3. Bring together the ends of a sheet of paper so that the sides form circles. What name is given to the figure formed by the sheet? The length of the sheet becomes the circumference of the base of the cylinder and the width of the sheet becomes its altitude. The area of the cylinder (excepting the bases) is therefore the area of the rectangle forming its convex surface.

4. Find the convex surface of a cylindrical tank whose diameter is 6 feet and whose altitude is 8 feet.

287. Written Problems.

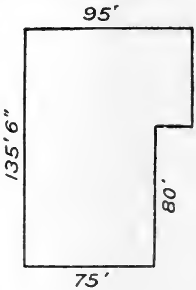
1. If it takes .98 cu. yd. of crushed stone, .47 cu. yd. of sand, and 1.56 bbl. of cement to make 1 cu. yd. of concrete, how much of each will make 100 cu. yd. of concrete?

.08 cu. yd.

2. How much crushed stone (Prob. 1), sand, and cement will be required to build a concrete wall 6 ft. high, 18 in. thick, and 60 ft. long, if $3\frac{3}{4}$ cu. ft. of cement is a barrel? (Express each answer as a whole number, since a fractional part of these units cannot be purchased.)

3. If the sand costs 80¢ per cubic yard, the cement \$2 per barrel of $3\frac{3}{4}$ cu. ft. each, the stone \$1.50 per cubic yard, and the labor for building the wall 80¢ per cubic yard, find the cost of the wall (Prob. 2).

4. Mr. Adams owns a 50-ft. lot on which he has recently built a house. He wishes to have a 6-ft. cement sidewalk laid along the front of the lot 2 ft. in from the curb, and a 4-ft. cement sidewalk laid from the curb to the front steps of the house, 20 ft. in from the curb. Find the cost of the walks at \$.12$\frac{1}{2}$ per square foot. First make a diagram of the walks.



5. The figure represents a lot owned by Mr. Morse. Find the area of the lot. Find the cost of excavating a basement 8 ft. deep on the property at \$1.25 per cubic yard.

6. How much will it cost at \$ .32 a cubic yard to remove 1 ft. of dirt from a lot 100 ft. by 150 ft.?

7. At \$5.50 a cord, how much will a pile of wood 24 ft. long, 4 ft. wide, and 8 ft. high cost?

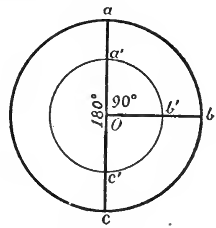
180 ÷ 2 1

MEASUREMENT OF CIRCLES AND CIRCUMFERENCES

288. 1. For the purpose of measurement circumferences of circles are considered to be divided into 360 equal parts, called **degrees** (°).

2. A portion of a circumference is called an **arc**.

3. What portion of the circumference of the larger circle is arc *ab*? What portion of the circumference of the smaller circle is arc *a′b′*?



4. How many degrees are there in arc *ab*? in arc *ac*? in arc *a′b′*? in arc *a′c′*?

5. If the circumference of the larger circle is 24,900 mi., how long is each degree on the circumference? If the circumference of the smaller circle is 6000 mi., how long is each degree on the circumference?

6. As the angle at *O*, the common center of the two circles, increases or diminishes as fast as the arc suspended by its sides increases or diminishes, the angle is also measured in degrees. Thus, when the arc between two radii is 90°, the angle formed at the center of the circle by the radii is an angle of 90°, or a **right angle**.

7. How many arcs of 90° are there in a circumference? How many right angles can be formed at the center of a circle?

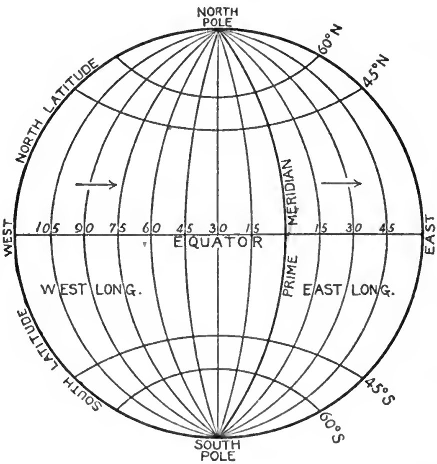
289. Arc and Angle Measure.

60 seconds (″) = 1 minute (′)
60′ = 1 degree (°)
360° = a circumference
360° = 4 right angles (rt. ∠s)

290. Latitude and Longitude.

1. Using a map, point to the equator; to a meridian. The equator is midway between what two points? Do the meridians extend around the earth or only from pole to pole? What is a meridian? How must two or more places be located to have the same meridian? How must two or more places be located so as not to have the same meridians? Are the meridians of all places shown on the map?

2. Places on the earth's surface may be located by two measures taken from two lines intersecting at right angles.



3. The lines taken for locating places on a map are the **equator** and some selected meridian, called the **prime meridian.** The meridian of the Royal Observatory at Greenwich (near London), England, is taken by most nations as the prime meridian.

4. The distance in degrees north and south from the equator is called **latitude**; and the distance in degrees east and west from the prime meridian is called **longitude.** Places north of the equator are in north latitude. What places are in south latitude? in east longitude? in west longitude?

5. In the figure, point to a place located 60° west longitude and 45° north latitude; 90° west longitude and

45° north latitude; 30° east longitude and 45° south latitude; 45° west longitude and 45° south latitude. Give the latitude and longitude of each point at which lines intersect in the figure.

6. What is the difference in degrees between two places, one 60° west longitude and the other 15° east longitude? one 105° west longitude and the other 30° west longitude? one 45° north latitude and the other 60° south latitude?

7. How many degrees is it from the equator to the North Pole? from the equator to the South Pole? from the North Pole to the South Pole? How many degrees is it from any point on the equator halfway around the earth? one fourth way around the earth?

8. The equatorial circumference of the earth is 24900 mi. What is the length in miles of a degree on the equator?

9. What is the latitude of your home? If the polar circumference of the earth is 24800 mi., how far do you live from the equator? from the North Pole?

10. Which is longer, a degree on the Arctic Circle, on the Tropic of Capricorn, or on the equator? Explain.

11. What is the greatest latitude that a place can have?

12. What is the greatest longitude that a place can have either east or west from the prime meridian? Why?

13. Where must a place be located to have a latitude of 0°? Where must a place be located to have a latitude of 0° and a longitude of 0°?

14. If the prime meridian in the figure on p. 238 is the meridian of Greenwich, locate on the figure your own home; the city of New York; Chicago; San Francisco; Rio Janeiro; Berlin.

291. Longitude and Time — Local Time.

1. What part of the earth's surface receives the light of the sun at any one time? Why does the sun appear to move from east to west?

2. How many hours does it take the earth to rotate once on its axis? Through how many degrees does any meridian pass during each rotation?

3. How many degrees of longitude pass under the sun's rays during 24 hours? during 1 hour?

4. Which passes under the sun's rays first, the meridian of your home or that of a place 15° east of you? west of you?

5. When the vertical rays of the sun fall on any part of the meridian of your home, it is noon by sun time at all places on the meridian. Is it then before noon or after noon by sun time at places east of your home? west?

6. Since the earth rotates through 360° in every 24 hours, it must rotate through 15° each hour. Therefore the difference in sun time between places 15° apart, in an east and west line, is 1 hour.

7. When it is noon by sun time on the prime meridian, what is the time at a place 15° E.? 15° W.? 30° E.? 30° W.? 45° W.? 60° E.? 75° W.? 90° W.? 105° W.?
8 5 6 7

8. When it is noon by sun time on the principal meridian, what is the longitude of a place at which it is 11 A.M.? 1 P.M.? 10 A.M.? 2 P.M.? 9 A.M.? 4 P.M.?
15 E 15 W 30 E 30 W 45 E 60 W

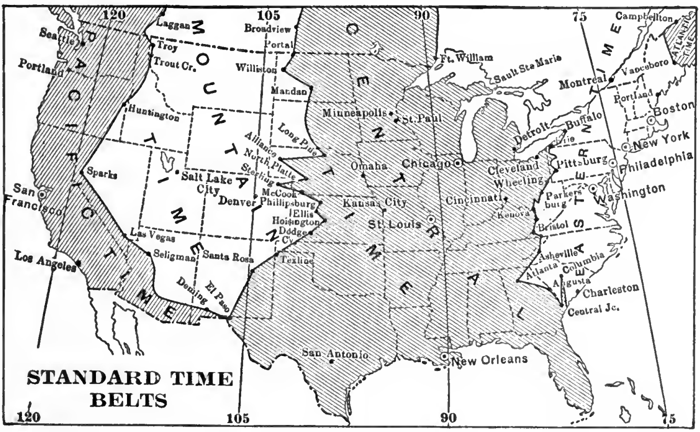
9. What is the difference in sun time between places 30° W. and 30° E.? 45° W. and 60° E.? 30° W. and 60° W.? 60° W. and 105° W.?

10. When it is noon on the prime meridian, where is it midnight? 9 A.M.? 9 P.M.? 6 A.M.? 6 P.M.?

292. Standard Time.

To avoid the confusion that would arise if every place used its own local time, in 1883 the railroads of the United States and Canada agreed upon a system known as **standard time.** Under this system the United States is divided into four time belts, each approximately 15° in width, and each having the local time of its central meridian, which is some multiple of 15°. These divisions are named after the sections of the country embraced by them as follows: **Eastern,** having the time of the meridian of 75° W.; **Central,** having the time of the meridian of 90° W.; **Mountain,** having the time of the meridian 105° W.; and **Pacific,** having the time of the meridian 120° W.

5 A.M. 6 A.M. 7 A.M. 8 A.M.



While the time belts are theoretically 15° in width, they are actually wider or narrower than 15°. The irregularities of the divisions are due to the fact that the railways find it convenient to make the changes in time at the division termini that are nearest to $7\frac{1}{2}°$ east or west of the central meridians.

293. Map Questions.

1. When it is noon in Philadelphia, what time is it in Chicago? in Denver? in San Francisco? in New York?

2. When it is 9 A.M. in Chicago, what time is it in San Francisco? in Washington? in New Orleans? in Denver? in Seattle? in New York?

3. A telegram was sent from Washington at 2 P.M. and was received in San Francisco at 11 A.M. of the same day. Explain.

4. At 10 P.M. the people of Los Angeles, Cal., were reading the election returns of New York, which had been compiled at 11 P.M. of the same day. Explain.

5. The passengers on a west-bound train arrived in Sparks, Nev., at 6.05 A.M. and after a stop of 10 min. started on their journey at 5.15 A.M. Explain.

6. On leaving North Platte, Neb., the passengers on an east-bound train found that their watches were all 1 hr. behind time. Explain.

7. How many times must a person reset his watch in traveling from Boston to San Francisco, if he wishes to have correct time on the journey?

8. If the telegraph office in Chicago, Ill., closes at 6 P.M., what is the latest time a message can be sent from San Francisco in time to reach this office before it closes, allowing 30 min. for delays in transmission?

9. When it is noon by standard time in western Iowa, is it earlier or later than noon by local time? Name some place where it is 6 P.M. by standard time before it is 6 P.M. by local time.

RATIO

294. **1.** What is the ratio of 4 ft. to 6 ft.? Compare the ratio of 4 ft. to 6 ft. with the ratio of 2 ft. to 3 ft., and with the ratio of 8 ft. to 12 ft.

2. What effect upon the ratio of two quantities has (*a*) multiplying both terms by the same number? (*b*) dividing both terms by the same number?

3. Name two quantities whose ratio is $\frac{4}{5}$. Name two other quantities having the same ratio.

4. Name numbers whose ratio is expressed by the fraction $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{7}{8}$, $\frac{5}{3}$, $\frac{3}{2}$, $\frac{1}{3}$.

5. Name whole numbers whose ratio is as $\frac{1}{2}$ to 2; as 3 to $\frac{1}{3}$; as $\frac{2}{3}$ to 6; as 6 to $\frac{3}{4}$.

6. Write ratios equivalent to the following, but with one or both terms a fraction: 1 to 2; 1 to 4; 2 to 3; 2 to 1; 3 to 7.

7. Supply the number in place of x: $\frac{4}{5}=\frac{x}{10}$. The fractions are stated as equivalent fractions. Compare their terms to find the number in the place of x.

8. The ratio between two numbers may be stated in the form of a fraction. In $\frac{3}{4}=\frac{6}{8}$ we have an equality of ratios. The equality between ratios is called a **proportion**.

9. Solve: $\frac{2}{5}=\frac{4}{x}$; $\frac{6}{10}=\frac{x}{30}$; $\frac{5}{7}=\frac{10}{x}$; $\frac{x}{8}=\frac{12}{16}$; $\frac{x}{4}=\frac{10}{12}$.

10. If 20 bbl. of flour cost \$80, how many barrels of flour can be bought for \$120? (\$120 is one half again as much as \$80.)

11. In a certain city the ratio of the number of school-census children to the total population is 1 to $4\frac{1}{2}$. If the school census is 20,000, what is the population of the city?

1 : 4.5 = 20[illegible] : x

PART V

POWERS AND ROOTS

295. $3 \times 3 = 9$. 3 is used *twice* as a factor to give 9. 9 is called the **second power** of 3. What number is the second power of 2? of 4? of 5? of 10? of 1? of 12?

2. The second power of a number is called its **square**, as the number of units in the area of a square surface is found by taking the second power of the number denoting the length of a side of the square.

3. The square of 3 may be indicated thus: 3^2. Indicate the square of 4; of 5; of 1; of 10; of 12. Give the value of each: 7^2, 8^2, 2^2, 6^2.

The small figure written at the right and above indicates how many times the number is to be taken as a factor and is called the **exponent** of the number.

4. $3 \times 3 \times 3 = 27$. 27 is the **third power**, or **cube**, of 3. What number is the cube of 1? of 2? of 4? of 5? of 6? of 10? of 12?

5. The cube of 3 may be indicated by an exponent, thus: 3^3. Indicate the cube of 7; of 8; of 9. Give the value of each: 1^3, 2^3, 10^3, 12^3.

5^4 is read *the fourth power of* 5, or 5 *to the fourth power*; it means $5 \times 5 \times 5 \times 5$. Read and tell meaning of: 6^4, 3^5, 2^6.

6. Find the volume of a cube whose edge is 5 in. Find the cube of 5.

7. Give the square of each of the numbers from 1 to 12.

8. Square $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, .5, 1.5, .04, $\frac{1}{10}$, $2\frac{1}{4}$.

9. Find and memorize the cubes of 1, 2, 3, 4, 5, 6, 10, 12.

The process of finding a power of a number is sometimes called **involution**.

10. A number that is the square of some integer or fraction is called a **perfect square**. Thus, 25 (5×5) and $\frac{25}{36}$ $(\frac{5}{6} \times \frac{5}{6})$ are perfect squares. Is 24 a perfect square?

11. Square each: 20, 30, 40, 50, 60, 70, 100.

12. Is the square of 2 plus the square of 3 the same as the square of 5?

296. 1. Which is the more and how much, $20^2 + 5^2$ or 25^2?

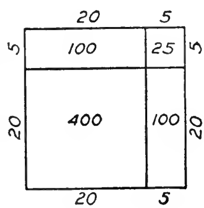
2. The square of any number composed of tens and units may be found thus:

$$\begin{array}{rl}
20 + 5 & \\
20 + 5 & \\
\hline
100 + 25 & (20 + 5) \times 5 \\
400 + 100 \quad\quad & (20 + 5) \times 20 \\
\hline
400 + 2(100) + 25 & = 20^2 + 2(20 \times 5) + 5^2
\end{array}$$

The square of 25 is seen to be the square of the tens, plus twice the product of the tens and the units, plus the square of the units.

3. Square as above: 23, 47, 105 $(100 + 5)$.

4. The figure represents a square whose side is 25 units. The square whose side is 20 units contains 400 square units. The two rectangles 20 by 5 contain 100 square units each. The square is completed by the addition of the small square 5 by 5, containing 25 square units. The area of the square is $(400 + 2(20 \times 5) + 25)$, or 625 square units.



5. Construct a square whose side is $10 + 5$ units.

297. Roots.

1. Since 9 is the square of 3, 3 is the **square root** of 9; that is, it is one of the two equal factors of 9. What number is the square root of 4? of 25? of 64? of 36? of 49? of 16? of 144? of 100? of 81? of 121? of 1?

2. Since 27 is the cube of 3, 3 is the **cube root** of 27. What is meant by the cube root of a number? What number is the cube root of 1? of 125? of 8? of 1000? of 1728?

3. The sign ($\sqrt{}$) is called the **radical**, or **root sign**, and is placed over a number to show that its root is to be taken. The root to be taken is indicated by a small figure, called an **index**, written in the radical thus, $\sqrt[3]{27}$, which is read the cube root of 27. The index 2 for square root is usually omitted.

4. Read and give the roots: $\sqrt{64}$, $\sqrt[3]{64}$, $\sqrt{49}$, $\sqrt{100}$, $\sqrt[3]{125}$, $\sqrt{81}$, $\sqrt{36}$, $\sqrt{144}$, $\sqrt[3]{81}$.

The process of finding the root of a number is sometimes called **evolution**.

298. Finding Roots by Factoring.

Roots of perfect squares may be found by **factoring.**

1. Find the square root of 324.

By factoring, $324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$.

Arranging the factors into two like groups,

$$324 = (2 \times 3 \times 3) \times (2 \times 3 \times 3).$$
$$\sqrt{324} = 2 \times 3 \times 3, \text{ or } 18.$$

2. Find the cube root of 2744.

Factor 2774. Group the factors into three like groups. The product of one of these groups is the cube root.

3. The square root of a fraction is the square root of its numerator over the square root of its denominator, thus: $\sqrt{\frac{4}{9}} = \frac{2}{3}$.

299. Find the roots indicated:

1. $\sqrt{\frac{25}{36}}$	4. $\sqrt[3]{3375}$	7. $\sqrt[3]{8000}$	10. $\sqrt{1296}$
2. $\sqrt{900}$	5. $\sqrt{129,600}$	8. $\sqrt{\frac{64}{81}}$	11. $\sqrt[3]{15,625}$
3. $\sqrt{11,664}$	6. $\sqrt[3]{512}$	9. $\sqrt{\frac{1}{16}}$	12. $\sqrt{6\frac{1}{4}}$

300. 1. Compare $\sqrt{1} = 1$, $\sqrt{1|00} = 10$, and $\sqrt{1|00|00} = 100$. Notice that there is one figure in the square root for each period of two figures each into which the square can be separated, beginning at units. The period at the left may contain only one figure. By separating any number into such periods, the number of figures in the square root may be told.

2. How many figures are there in the square root of each: 11,664? 129,600? 11,025?

3. $1.2^2 = 1.44$; $9.9^2 = 98.01$; $1.22^2 = 1.4884$. Notice that there are two decimal places in the square for each decimal place in the root.

4. How many decimal places are there in the square root of each: 4.1616? 1190.25? 2550.25?

301. Square Root.

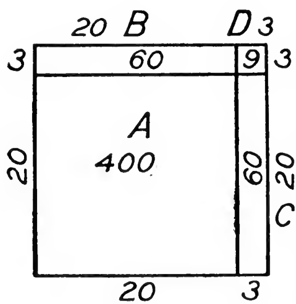
a. Find the square root of 529. *b.* Find the side of a square whose area is 529 square units.

As the square root of some numbers cannot be found by factoring, another method of finding the square root of numbers is necessary. From Sec. 296 we see that the square of a number is the *square of the tens, plus twice the product of the tens and units, plus the square of the units;* and from Sec. 300 we see that the number of figures in the square root of any number is the same as the

number of periods of two places each, beginning with units into which the number can be separated.

MODEL:	$5'29\lvert\underline{23}$
$20^2 =$	$\underline{4}$
$2 \times 20 = 40$	129
$(40 + 3) \times 3 =$	$\underline{129}$

a. As 529 can be separated into two periods, its square root consists of tens and units. Since the square of tens is hundreds, 5 hundreds must include the square of the tens of the root. The largest perfect square in 5 hundreds is 4 hundreds. The square root of 4 hundreds is 2 tens. Write this in the answer at the right. The square of 2 tens is 4 hundreds. Subtract 4 hundreds from 529. The remainder is 129. This remainder must be twice the product of the tens and the units, plus the square of the units. Twice 2 tens is 40. The units' figure of the root is found by taking 40 as a partial divisor. 40 is contained in 120 (omitting the 9, as it is evidently the square of the figure in units' place, or a part of its square) three times. Write 3 as the units' figure of the root. Use 43 as the complete divisor. $3 \times 43 = 129$, which exhausts the remainder.



b. As the largest perfect square in 5 hundred square units contains 4 hundred square units, its side is 20 units (*A*). 129 square units remain to be added in such form as to keep the figure a square. It is evident that these units must be added along two adjacent sides, as *B* and *C*, and at the corner, as *D*. The combined length of the two rectangles, *B* and *C*, is 40 units. Their width may be determined from the fact that their combined areas, plus the area of the small square *D*, is 129 square units. Omitting the 9, as it evidently is the number (or a part of the number) of square units in the small square, 120 square units $\div$ 40 square units $= 3$, the number of units in the width of the rectangles, and also in the side of the small square.

302. To extract the square root of a number:

1. Separate the number into periods of two figures each, beginning at the decimal point.

2. Find the greatest square in the left-hand period, and write its root for the left-hand figure of the required root.

3. Subtract the square from the left-hand period, and bring down the next period to form the complete dividend.

4. Double the part of the root already found, and place it at the left of the dividend for a partial divisor. Disregarding the right-hand figure of the dividend, divide by the partial divisor. The quotient (or quotient diminished) will be the next figure of the root.

5. Annex the root figure last found to the partial divisor for a complete divisor. Multiply the complete divisor by the root figure last found. Subtract the product from the dividend, bring down the next period to form the complete dividend, and continue as before.

303. Written Exercises.

Square roots of numbers that are not perfect squares may be approximated by annexing periods of two decimal ciphers and continuing the process to several decimal places in the roots.

Extract the square root of each:

1. 841 2 9	4. 56.25	7. 1600
2. 104,976	5. .6724	8. 10.24
3. 3844	6. 160	9. .007225

10. Find the side of a square whose area is 256 sq. rd.

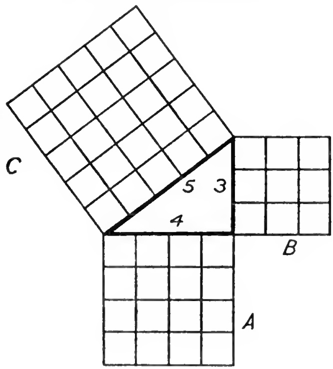
11. Find the side of a square field whose area is 10 A.

12. Find the perimeter of a square 40-acre field.

304. 1. Draw a right angle. Draw a rectangle. Draw a diagonal through the rectangle. Into how many equal triangles does the diagonal divide the rectangle? What kind of triangles are they?

2. The longest side of a right triangle is called its **hypotenuse**, and the other two sides are called its **legs.**

3. Draw a right triangle whose legs are 6 in. and 8 in. Measure the length of the hypotenuse. Construct squares upon each of the three sides, and divide them into square inches. Compare the number of square inches in the square on the hypotenuse with the number in the other two squares together.



4. The figure represents a right triangle whose legs are 3 units and 4 units and whose hypotenuse is found to be 5 units. Compare the number of units in the square upon the hypotenuse with the number of units in the sum of the squares upon the other two sides.

The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.

Answer the following from the figure:

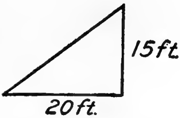
5. If the number of squares in A and B are given, how may the number in C be found?

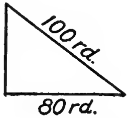
6. If the number of squares in B and C are given, how may the number in A be found?

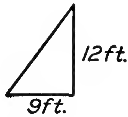
7. If the number of squares in A and C are given, how may the number in B be found?

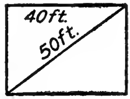
305. Written Exercises.

1. Find the length of the third side of each:









2. Find the length of the diagonal of the floor of a rectangular room 14 ft. by 16 ft., to the nearest thousandth of a foot.

3. A boy stood on the ground 45 ft. from the foot of a tree 60 ft. in height. How far was it in a straight line from the boy's feet to the top of the tree?

4. How much less is the distance along a diagonal path across a rectangular field 40 rd. by 80 rd. than the distance around two sides of the field?

5. How long must a rope be to reach from the top of a 60-ft. pole to a point on the ground 30 ft. from the foot of the pole?

6. Find the diagonal of a square field whose side is 40 rd.

7. Find the side of a square whose diagonal is 60 ft.

8. Find the diagonal of a rectangular room 20 ft. by 26 ft. If the ceiling of the room is 10 ft. from the floor, what is the distance from one of the lower corners of the room at one end of the diagonal on the floor to the upper corner at the other end?

9. If A is 85 mi. south of B, and C is 75 mi. west of B, how far is it from A to C?

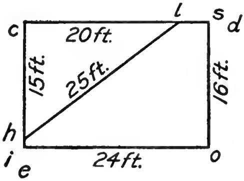
10. One side of a rectangular field is 40 rd. The diagonal is 50 rd. Find the other side.

11. Find the diagonal of a 10-acre square field.

306. Laying off a Rectangle.

1. When the two sides of a rectangle are in the ratio of 3 units to 4 units, the diagonal is one fourth more than the longer side, thus: If the sides of a rectangle are 18 ft. and 24 ft., the diagonal is 24 ft. plus 6 ft., or 30 ft. Prove that this is correct and that it holds with various rectangles when the ratio of the sides is as 3 to 4.

2. A farmer asked two schoolboys to lay off a rectangle 16 ft. by 24 ft. to mark the foundation of a carriage house. The boys used two pieces of cord and a measure. They tied the cords to two stakes so that they crossed at *c*, a corner of the rectangle, and extended one cord in the direction of *cd* and the other in the direction of *ce*. To make the angle at *c* a right angle and thus to get the positions of the two sides of the rectangle, they measured off from *c* 15 ft. on one cord and 20 ft. on the other cord. With these as the legs of a right triangle, they adjusted the position of the cords so as to make the distance *hl* 25 ft. Having determined the direction of the sides *cs* and *ce*, they measured 24 ft. on *cs* and 16 ft. on *ce* and marked the corners *s* and *i*. From *s* they extended a line in the direction *so*, and from *i* a line in the direction *io*. They measured 16 ft. on the line *so* and 24 ft. on the line *io*. They marked the point *o* where the two measured lines met. They tested their work, finding that the diagonals *co* and *si* were equal.



3. Using cords and a measure, lay out on the school grounds rectangles 12 ft. by 16 ft.; 20 ft. by 30 ft.

4. Lay off a baseball "diamond" whose side is 60 ft.

SIMILAR SURFACES AND SOLIDS

307. 1. Draw squares whose sides are 1 inch; 2 inches; 3 inches. Find their areas. The areas of the three squares are to each other as 1^2, 2^2, and 3^2.

2. Express the ratio of the areas of a 4-inch square and a 6-inch square.

3. Draw circles whose diameters are 4 inches; 6 inches. As the area of a circle is πr^2, the ratio of the areas of these two circles is as 2^2 is to 3^2. Explain.

4. Is the ratio of the squares of the diameters of two circles the same as the ratio of the squares of their radii?

5. Is the ratio of the squares of the diagonals of two squares the same as the ratio of the squares of their sides?

6. Figures that are of exactly the same shape are called **similar figures.** Draw two similar figures. Are similar figures necessarily the same in size?

The areas of similar plane figures are proportional to the squares of their corresponding lines.

7. Find the volume of a 2-inch cube; of a 3-inch cube. Their volumes are in the ratio of 2^3 to 3^3.

The volumes of similar solids are proportional to the cubes of their corresponding lines.

8. Compare the volumes of two spheres, one 5 inches in diameter and the other 10 inches in diameter.

As the diameter of the larger sphere is twice the diameter of the smaller, the volume of the larger is 2^3 times the volume of the smaller. Explain.

9. Compare the weights of two solid iron spheres of the same density, if one is 2 inches in diameter and the other is 4 inches in diameter.

MISCELLANEOUS EXERCISES

308. **1.** A merchant gained \$28 by selling some goods at a profit of 20%. Find the cost of the goods.

2. An agent who canvassed for a book received 40% of the amount of the sales for selling and delivering the books. Find the amount of his commission for selling and delivering 60 copies at \$1.25 per copy.

3. If the amount received for goods sold averages 20% more than the cost of the goods, find the net profit for a month when the sales amounted to \$24,000 and the expenses for the month amounted to \$3000.

4. 20% of the pupils enrolled in a certain school were absent one stormy day. Twenty-four pupils were present. Find the number enrolled.

5. A commission merchant sold \$6000 worth of produce at a commission of $1\frac{1}{2}$%. Find the amount of his commission.

6. If a collector received 20% commission for collecting a bill of \$17.75, what was the amount of his commission? What per cent of the amount of the bill did the creditor receive?

7. A man paid \$42 taxes when the tax rate was 2%. What was the assessed valuation of his property?

8. If a person pays \$50 tax on property when the rate is $1\frac{1}{4}$%, what is the assessed valuation of his property?

9. A miller bought a ton of wheat through a broker, who charged a commission of 2%. What was the amount paid for the wheat if the cost of the wheat and the brokerage amounted to \$25.50?

The cost of the wheat plus 2% of the cost of the wheat, or 102% of the cost of the wheat, was \$25.50. Prove your answer.

10. A fruit grower shipped 25 boxes of apples to a commission merchant, who sold them at 85¢ per box, charging 4% commission. He was directed to invest the proceeds in groceries, after deducting a commission of 2% for making the purchase. Find the amount expended for the groceries.

The net proceeds of the sale of the apples was $20.40, which was 102% of the amount invested in groceries. Prove your answer.

11. After selling 25% of his interest in a flour mill, a man considered his remaining interest worth $9000. At this rate, what was the value of his interest before making the sale?

12. A real estate broker bought two 60-ft. lots, adjoining, for $1500 apiece, and divided them into three lots of equal frontage which he sold for $1200 apiece. What per cent of profit did he make?

13. At $9 a ton, find the cost of two sacks of coal, each weighing 100 lb.

14. A man bought a lot for $1200 and sold it for $1400. He bought it back for $1500 and resold it for $1600. How much did he make on the lot?

15. A man bought four 50-ft. lots and divided the land into 40-ft. lots, which he sold at the same price per lot as he had paid. Find his gain per cent.

16. At 22¢ a square foot, find the cost per front foot of paving a street 60 ft. wide. Find the cost per front foot to a property owner who pays for half the width of the street. Find the cost to the property owner of paving the street in front of a 45-ft. lot.

17. If 27 tons of coal cost $243, how many tons can be bought for $189?

PART VI

APPENDIX

CORPORATIONS, STOCKS, AND BONDS

309. 1. Corporations. A large business enterprise frequently requires more capital than one person may care to invest in it. Provision is made in the laws of the various states whereby a number of persons may organize a company, called a **corporation**, to engage in business as one body. Sometimes all the necessary capital is subscribed by the persons who organize the corporation, but often the organizers of a company subscribe only a part of the capital.

The laws regulating the incorporation of companies differ considerably in the several states. Frequently a corporation intending to transact business in one state will incorporate in another state, because of certain advantages to be derived thereby.

2. Railway companies, mining companies, express companies, oil companies, and banking institutions are among the largest business corporations.

310. Shares of Stock. 1. Each corporation is capitalized for a special amount, as \$25,000, \$50,000, \$1,000,000, etc. The capital is divided into **shares**, usually of \$100 each or of \$1 each. Thus, a corporation that is capitalized for \$100,000 may issue 1000 shares of the face value of \$100 each, or 100,000 shares of the face value of \$1 each, etc. These shares are bought by persons who invest in the enterprise. Each person who owns one or more shares of stock is called a **stockholder**. The several stockholders constitute the corporation.

2. Every stockholder receives a **certificate of stock**, showing the number of shares he owns and the face value, or **par value**, of each. These certificates are negotiable, and a record of their transfer is usually made on the books of the corporation.

The affairs of a corporation are managed through a board of **directors**, elected by the stockholders, each stockholder having as many votes as the number of shares of stock he owns.

311. Value of Stock. **1.** The price at which stocks are bought and sold in the stock market is called their **market value.** When the market value of stock is the same as its face value, the stock is said to be **at par.** Stock is said to be at a *premium*, or **above par**, when its market value is more than its face value, and **at a** *discount*, or **below par**, when its market value is less than its face value.

2. Examine the stock quotations in a newspaper. Can you tell from the quotations what the face value of the stock is? Tell which stock is at par, above par, below par.

312. Dividends. The net earnings of a corporation, after a surplus sufficient to cover the probable needs has been reserved, are divided among the stockholders according to the number of shares owned by each. These divided profits are called **dividends.** Dividends are computed on the par value of the stock, and are declared annually, semiannually, quarterly, etc.

ILLUSTRATION. If the amount of capital stock is \$500,000 and the amount to be divided among the stockholders is \$25,000, the **rate of dividend** is \$25,000 ÷ \$500,000, or 5%. A stockholder who owns stock of the face value of \$10,000 will receive 5% of \$10,000, or \$500.

Stock that regularly pays a large dividend is usually regarded as a good investment, and is likely to be above par. When the dividends are not equivalent to a fair rate of interest on the investment, the stock is likely to be below par.

313. Corporations sometimes issue two kinds of stock, called **preferred** and **common.** When both kinds of stock are issued, the holders of common stock are not entitled to participate in the profits until a fixed rate of dividend has been paid to holders of preferred stock.

314. Stock Brokers. **1.** A person who is engaged in buying and selling stocks for others is called a **stock broker.** Stocks are usually bought and sold through brokers, generally at a regular meeting place for transacting such business, called a **stock exchange.**

2. The commission of a broker is called **brokerage.** The rate of brokerage varies in different parts of the country from $\frac{1}{8}\%$ of the **par value** for buying and also for selling, to $\frac{1}{2}\%$ or more. A minimum amount is fixed for making small sales and purchases.

The standard amount of stock bought and sold is 100 shares, although a smaller amount may be negotiated. As a rule, fractions of a share cannot be bought.

In stock quotations fractions are always expressed in halves, fourths, and eighths. See quotations, p. 259.

315. Assessments. When the funds of a corporation are not sufficient to carry on its business, an **assessment** is sometimes levied upon the stock. The assessment is usually some number of cents per share, and the failure to pay it is generally punishable by the forfeiture of the stock.

316. Examine a newspaper for stock quotations. What is meant by par value? premium? dividend? market value? brokerage? Name some of the corporations engaged in business in the state in which you live. How many shares of stock of the par value of \$100 each are issued by a corporation that is capitalized for \$500,000? for \$2,000,000? What is a stockholder?

Stock Quotations

317.

New York Stock Exchange	Mar. 9, 1907	Jan. 22, 1907	San Francisco Stock and Exchange Board — Mining	Mar. 9, 1907	Jan. 22, 1907
Adams Express	295	300	Caledonia	.43	.67
American Express	223	242	Confidence	1.05	1.30
C. & N. W.	$154\frac{1}{2}$	$190\frac{1}{4}$	Combin. Frac.	4.45	5.50
Denver & Rio G.	$31\frac{5}{8}$	$39\frac{1}{4}$	Jumbo Exten.	1.85	1.90
Denver & Rio G., pfd.	72	81	Mustang	.27	.21
Illinois Central	$147\frac{1}{2}$	166	Ophir	2.65	3.10
Northern Pacific	$137\frac{5}{8}$	$155\frac{1}{2}$	Red Top Exten.	.50	.37
Southern Pacific	$84\frac{1}{2}$	$95\frac{3}{8}$	Silver Pick	1.35	1.45
Pullman Car Co.	165	$173\frac{1}{2}$	St. Ives	1.82	.93
Union Pacific	$156\frac{1}{2}$	$176\frac{3}{4}$	Utah	.06	.08
Do. preferred	88	$92\frac{1}{2}$	Vernal	.20	.25
Wells Fargo Ex.	280	300	West End	1.40	1.90
Western Union	$80\frac{5}{8}$	$83\frac{3}{4}$	Yellow Jacket	1.05	1.15

The par value of the stock quoted in the left-hand column is $100 per share, and in the other column is $1 per share. On Jan. 22 the market was *high*, and on March 9 *low*. Which of the stocks quoted are above par? below par?

At the above prices, find the cost without brokerage, on March 9, of 100 shares of Wells Fargo Express stock; of 100 shares of Chicago and Northwestern Railway stock; of 100 shares of Western Union stock; of 100 shares of stock in the Ophir mine; of 100 shares of stock in the Silver Pick mine. Find the same for the prices given for Jan. 22.

318. Written Exercises.

1. Find the cost of 100 shares of Western Union stock at $30\frac{5}{8}$, including $\frac{1}{8}\%$ brokerage.

Market price of each share,	$\$80\frac{5}{8}$.
Brokerage on each share,	$\$\ \frac{1}{8}$.
Cost of each share (including brokerage),	$\$80\frac{6}{8}$, or $80.75.
Cost of 100 shares (including brokerage),	$8075.

2. Find the cost of 100 shares of Northern Pacific stock as quoted for March 9, including $\frac{1}{8}\%$ brokerage.

3. Find the net proceeds of the sale of 100 shares of Southern Pacific stock as quoted for Jan. 22, allowing $\frac{1}{8}\%$ brokerage.

Market price of each share,	$\$95\frac{3}{8}$.	
Brokerage on each share,	$\$\ \frac{1}{8}$.	
Net proceeds on each share,	$\$95\frac{2}{8}$,	or $95.25.
Net proceeds on 100 shares,	$9525.	

4. Find the net proceeds from the sales of 400 shares of Illinois Central stock as quoted for March 9, allowing $\frac{1}{8}\%$ brokerage.

5. How much would a man clear by buying 100 shares of stock at $142\frac{1}{4}$ and selling them at $150\frac{3}{4}$, allowing $\frac{1}{8}\%$ brokerage both for buying and for selling?

6. If the Pullman Car Company declares a dividend of 10 %, how much will a person receive who owns 100 shares of the stock?

Par value of 100 shares, $10,000. His dividend will amount to 10% of $10,000.

7. If the Union Pacific Railway declared a dividend of 8 %, how much did a person receive who owned 500 shares of the stock?

8. Find the cost of 100 shares of stock at $1.20 a share, including $\frac{1}{2}\%$ brokerage. If a 10 % dividend is declared on this stock, how much does a person receive who owns 100 shares of the stock?

9. What per cent does a person receive on the amount invested who buys 100 shares of stock at $1.20, paying $\frac{1}{2}\%$ brokerage, and receives a dividend of 10 %?

The dividend amounts to 10% of $100. This amount is what per cent of $120.50, the cost of the stock?

10. A man bought 100 shares of Red Top Extension stock at 37 cents, paying $1 brokerage. Find the cost of the stock. If he received a dividend of 5 %, what per cent did he receive on his investment?

$5, the dividend, is what per cent of $38, the amount invested?

11. What per cent did a man make on his investment who bought 100 shares of stock at 45 ¢ and sold them at 55 ¢, paying $1 brokerage both for buying and for selling?

Cost of the shares, $46; received for the shares, $54; net profit, $9. $9 is what per cent of $46?

319. Corporation Bonds. The promissory note of a corporation, issued under seal, is called a bond. The bond of a corporation is secured by a mortgage.

320. A city or an incorporated village is called a **municipal corporation.**

321. 1. Governments, states, cities, counties, etc., are sometimes obliged to issue bonds to meet urgent demands for money and to provide needed improvements. Such bonds are not secured by mortgages. The integrity of the government issuing a bond is accepted as sufficient security for its payment. The bond of a government is a certificate of indebtedness, with a promise to pay a certain sum to the holder of the bond, with a fixed rate of interest at a specified time, as at the expiration of five years, twenty years, fifty years, etc.

2. Under what conditions is it sometimes necessary for the United States government to borrow money? For what purposes are cities frequently bonded? What is a bond election? Which is the better security, the note of an individual or the bond of the national government?

322. Bonds that are registered by number and in the name of the holder are called **registered bonds.** Bonds having interest certificates attached in the form of coupons are called **coupon bonds.**

Bond Quotations

323. Examine a newspaper for bond quotations. The following quotations are from a newspaper report of prices on the New York Stock Exchange.

Japan 6's	$99\frac{5}{8}$	Southern Pacific 4's	84
Mexican Central 4's	83	U. S. New 4's reg.	$129\frac{1}{2}$
Northern Pacific 4's	$100\frac{7}{8}$	Do. coupon	$129\frac{1}{2}$

The par value of the bonds quoted is $100 each. Bonds are bought and sold in the market in the same manner as stocks. Government bonds are exempt from taxation.

Find the cost of Mexican Central bonds of the face value of $1000, including $\frac{1}{8}\%$ brokerage.

COMMISSION AND BROKERAGE

324. Producer and Consumer. A person who grows agricultural products or manufactures useful articles out of crude materials is called a **producer.** A person who uses up products is called a **consumer.** Name some producers of foods, of clothing, of fuel, of building materials. Name some products that are consumed by nearly every one. Name some classes of persons who are consumers but are not producers.

In early times there was comparatively little buying and selling, and the exchange of products was very limited. Each family produced nearly everything that it consumed. At that time, most of the trade was directly between the producer and the consumer. With the invention of machinery and with improved means of transportation, cities increased rapidly in number as centers of manufacture and trade. People began to devote themselves more particularly to special lines of work. As trade conditions grew more complex, it became more difficult for the producer to trade directly with the consumer. Wholesale and retail establishments developed as agencies for marketing products.

Middleman. A person who deals between the producer and the consumer is known in trade as a *middleman.* Products are often handled by several middlemen before they reach the consumers. The middlemen are generally persons who make buying and selling products their special occupation. Is the retail dealer a middleman? Which of the following

are middlemen: farmers? stock buyers? hardware merchants? carpenters? shoemakers? hay and grain dealers? What are some of the conditions that make it inconvenient for persons living in large cities to buy agricultural products directly from farmers? What are some of the conditions that make it inconvenient for farmers to buy their clothing and tools directly from the manufacturers? Which contributes to the wealth of a country: the producer, the consumer, or the middleman?

325. Commission. A person who transacts business for another frequently receives as his pay a certain rate per cent on the amount involved in the transaction. This is known as his **commission.** One who buys or sells produce for another, receiving as his pay a certain rate per cent on the cost of the products bought or on the selling price of the products sold, is called a **commission merchant.** A **commission house** is an establishment conducted by a commission merchant, where products are received and sold to retail dealers or consumers. Why are the commission houses located in the cities? If your home is in a city in which there are commission houses, tell in what section of the city they are located. If your home is in the country, tell what products are shipped from your community to commission merchants.

Farmers frequently dispose of their products by shipping them to commission merchants in the cities, who sell the products at the market prices and retain as their pay a certain per cent of the selling price. Commission merchants do not usually buy the products shipped to them, but they act merely as the agents of the shippers in receiving and selling the goods. The freight charges for shipping and the cartage charges for hauling the goods are generally paid by the commission merchants from the proceeds of the sale of the products. After deducting their commission and the charges for freight and cartage, the commission merchants remit to the shippers the balance of the sum received for the products sold.

The entire amount received from the sale of goods, before any deduction is made for expenses, etc., is called the *gross* receipts of the sale. The amount remaining from the sale of goods after all expenses have been deducted is called the *net* receipts of the sale.

Where do city retail dealers buy their supplies of fruits, vegetables, etc.? Farmers frequently sell or exchange small quantities of agricultural products at the general merchandise stores in small cities and villages. How do these merchants dispose of the products?

326. Market Reports. Newspapers publish *market reports,* giving the prices at which grain, live stock, dairy products, fruits, etc., were sold on the previous day. Of what use are such reports? Read a recent market report. The price of produce is affected by the supply offered for sale and the demand for it. What effect upon prices has an increase in supply and a decrease in demand? How is the price affected by an increase in demand and a decrease in supply?

327. Brokerage. 1. One who acts as an agent for others to contract for the purchase or sale of goods, receiving as his pay a certain rate of commission, is called a **broker**. The commission of a broker is called **brokerage**. Commission merchants usually take possession of the goods bought and sold by them, while brokers merely contract for the sale or purchase of goods in the name of the person buying or selling, without taking possession of the goods. Brokers deal in stocks, bonds, grain, etc.

2. What is real estate? One who buys and sells lands, exchanges and leases property, etc., is called a **real estate agent**, or a **real estate broker**. He usually receives as his commission a certain rate per cent on the selling price of property, or on a month's rent when property is leased by the month. The rates of commission charged for selling and renting property vary in different communities. Find what rate of commission is charged by agents for selling and renting property in your community. What are some of the conditions that make it inconvenient for each person to sell or rent his own property? When a person wishes to rent a house in a city, how does he find out what houses are for rent?

3. Traveling salesmen, store clerks, auctioneers, insurance agents, etc., frequently receive as pay for their services a commission on the amounts of their sales. Can you name any other business in which those employed receive a commission for their services?

TRADE DISCOUNT

328. **1.** Manufacturers and wholesale dealers issue catalogues and price lists in which the articles manufactured by them are described and their prices given. Most manufacturers sell their goods to wholesale dealers, who supply, in turn, the retail dealers. The prices quoted in catalogues and price lists are commonly known as the **list prices** of goods. A discount from the list prices is generally made to retail dealers, and sometimes to others when goods are bought in large quantities. This discount is reckoned as a certain rate per cent of the list price. Such a discount is generally known as **trade discount.**

2. Often several discounts are allowed. Thus, an article may be sold subject to discounts of 25 %, 10 %, and 5 % from the list price. In such a case, the first discount is from the list price, and the second discount is from the price after deducting the first discount, and the third discount is from the price after deducting the second discount. Usually the amount of discount allowed is varied as the market prices change.

3. A special discount is usually made when cash is paid for goods. The amount of cash discount varies considerably in different lines of trade, ranging generally from $\frac{1}{2}$ % to 6 % of the amount of the bill after deducting the trade discount, and averaging about 2 %. Generally the retail dealer is given until about the tenth day of the following month in which to make cash remittances. Instead of allowing a special discount for cash payments, sometimes an arrangement is made by which the retail purchaser is given 30 days, 60 days, 90 days, 6 months, or even a longer time in which to make his payment. The bill then becomes *due* at the end of the specified time, and in case it is not paid when the time has expired, the purchaser of the goods agrees to pay a specified rate of interest upon the amount of the bill from the time the bill is due until it is paid.

PARTIAL PAYMENTS

329. Instead of paying the whole amount of a note at one time, the maker sometimes pays it in two or more parts. Such payments are called **partial payments.** A record of each payment is indorsed on the back of the note.

330. United States Rule. The method of computing indebtedness when partial payments have been made, illustrated in the following problem, was adopted by the Supreme Court of the United States, and is commonly known as the United States Rule. This method is the legal one in most states.

In states where other methods are legal, teachers should follow them.

1. A note for \$2000 dated May 1, 1906, at 6%, was indorsed as follows: July 25, 1906, \$150; Dec. 16, 1906, \$40; Feb. 12, 1907, \$100. Find the amount due May 1, 1907.

Principal, May 1, 1906	\$2000	
Interest on \$2000 to July 25 (2 mo. 24 da.)	28	
Amount, July 25, 1906	\$2028	
First payment (July 25, 1906)	150	
New Principal, July 25, 1906	\$1878	
Interest on \$1878 to Dec. 16, 1906 (4 mo. 21 da.)		44.13
Second payment, which is less than the interest due, \$40		
Interest on \$1878 from Dec. 16, 1906, to Feb. 12, 1907 (1 mo. 26 da.)		17.53
Amount, Feb. 12, 1907		\$1939.66
Third payment, \$100, which is to be added to the second, \$40		140
New Principal, Feb. 12, 1907		\$1799.66
Interest on \$1799.66 to May 1, 1907 (2 mo. 19 da.)		23.69
Amount due May 1, 1907		\$1823.35

If the second payment, \$40, which *was less than the interest due*, had been deducted from the amount due at the time the payment was made, and if the remainder had been regarded as a new principal, the effect would have been to increase the amount on which interest was paid. Hence, the interest must be reckoned to the time of the third payment.

RULE. *Find the amount of the principal to a time when a payment, or the sum of two or more payments, equals or exceeds the interest due.*

Subtract the payment or payments from the amount.

Treat the remainder as a new principal and proceed as before.

2. Write a note for $800, naming the teacher as payee and yourself as maker. Make three partial payments and have them indorsed on the note. Find the amount due at the time of settlement.

3. Write a note for $1000, naming some pupil as payee and yourself as maker. Make two payments, such that the first is less than the interest to date, but the sum of both exceeds the interest to time of the second payment. Find the amount due at time of settlement.

INTEREST

331. BANKERS' TABLE OF DAYS BETWEEN DATES

	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
Mar.	306	337	365	31	61	92	122	153	184	214	245	275
Apr.	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

The exact number of days from any day of one month to the same day of another month within a year is found thus: Find the number of days from April 12 to Aug. 12. Starting from April, in the left-hand column, pass the pencil across to the column headed August. The number 122 in the column headed August denotes the number of days from any day in April to the corresponding day in August. Hence it is 122 days from April 12 to Aug. 12.

1. How many days must be added to 122 days if you are required to find the number of days from April 12 to Aug. 14? to Aug. 20? How many days must be subtracted from 122 days if you are required to find the number of days from April 12 to Aug. 8? to Aug. 3?

2. What change would be made in the number of days given in the table if Feb. 29 of a leap year should intervene between dates?

332. Written Exercises.

Using the table in Sec. 331, find by the 60-day method (p. 212) the interest on:

1. \$2000 from April 1 to July 4 at 6%.
2. \$4500 from Dec. 20 to Feb. 15 at 7%.
3. \$20,000 from May 12 to Aug. 5 at 5%.
4. \$80,000 from April 16 to June 4 at 6%.
5. \$25,000 from July 4 to Sept. 16 at 8%.
6. \$4370 from Aug. 4 to Dec. 7 at 6%.

EXACT INTEREST

333. 1. In the methods of computing interest in Secs. 255–261, 360 days were taken as one year in reckoning the interest for periods less than a year. Interest computed on the basis of 365 days to the year is called **exact interest.**

2. Exact interest is found by taking such a fractional part of a year's interest as the exact number of days is of 365 days.

3. Find the exact interest on \$400 from March 2 to June 15 at 8%.

Find the exact number of days from March 2 to June 15. Do not count March 2, but count June 15. From March 2 to June 15 there are 29 da. + 30 da. + 31 da. + 15 da. or 105 da.

MODEL:

$$\$400 \times .08 \times \frac{\overset{21}{\cancel{105}}}{\underset{73}{\cancel{365}}} = \$9.20+.$$

4. Compare the interest on \$400 for 105 da. at 8 % computed in the ordinary way with the exact interest.

5. Exact interest for whole years is the same as interest computed by the ordinary method. For periods of less than a year, the exact interest is $\frac{360}{365}$ of the interest computed by the ordinary method, or $\frac{5}{365}$ ($\frac{1}{73}$) less than the ordinary interest. Compared with interest computed by the ordinary method, is exact interest favorable to the person loaning money or to the person borrowing it?

6. Exact interest is computed by the United States government and sometimes by others.

7. How much interest does the United States government save by paying exact interest rather than ordinary interest on \$5,000,000 from Jan. 1 to April 1, 1907, at 3 % ?

8. Find exact interest on each of the amounts in Sec. 332 for the time specified.

TAXES

334. 1. Every person in the United States derives some benefits from one or more of the following divisions of government: *national* (also called *federal*), *state*, *county*, *township*, *school district*, *city*, and *village*. Explain the purpose of each of these divisions of government and name some of the benefits you derive from each. The maintenance of these several forms of government necessarily involves the expenditure of money. This money is derived from **taxes** levied upon the persons, property, incomes, or business of individuals. Mention some purpose for which money is expended by each division of government named above.

2. **Direct taxes** are sums of money levied upon persons, property, incomes, or business of individuals for the support of state or local governments. They are called direct taxes because they are levied directly upon the persons, lands, buildings, etc., of individuals and are payable directly to public officials authorized to collect taxes.

3. **Indirect taxes** are sums levied upon goods imported from other countries, upon certain products manufactured in our own country, and upon the privilege of engaging in certain pursuits, as selling liquors, etc. They are called indirect taxes because they are paid indirectly by the consumers.

4. Ascertain how the money is raised for building schoolhouses, for repairing roads or streets, etc., in the community in which you live.

STATE AND LOCAL TAXES

335. Poll Tax. Each male inhabitant of voting age (generally with certain exceptions) is usually taxed a fixed sum annually, regardless of the ownership of taxable property. This is called a **poll tax.** *Poll* means *head.* Is a poll tax a direct or an indirect tax? It is the only form of personal tax levied in the United States. What are the provisions relating to poll tax in your state?

336. 1. Property Tax. Property is classified either as *personal property* or as *real property.* All movable property, as household goods, money, cattle, ships, etc., is called **personal property.** Immovable property, as lands, buildings, mines, etc., is called **real property.** Both forms of property are subject to taxation.

2. For the purpose of taxation, the value of all taxable property is estimated by a public officer called an **assessor.** This officer prepares a list of all taxable property in his district, showing the names of the owners, the location of the property, and its assessed valuation. Such a list is commonly called an **assessment roll.** The assessed valuation of property is generally less than its actual value.

3. Taxes are usually collected by a public officer called a **tax collector,** who deposits the amounts collected with another public officer called a **treasurer.**

4. What is *public property?* What properties, other than public property, are exempt from taxation in your state?

5. The total assessed valuation of the property in any state is the sum of the assessed valuation of the property in the several counties of the state. How is the assessed valuation of the county in which you live determined? The state and local taxes are usually levied and collected together. Ascertain how state, county, and local taxes are levied and collected in your state.

6. Examine a receipt for the payment of taxes on real property. How is the location of the property described?

337. Written Exercises.

1. The assessed valuation of a certain county is $8,000,000. Find the rate of taxation that will yield revenue sufficient to pay for a new courthouse costing $100,000. How much must Mr. Thomas pay toward the building of the courthouse if his taxable property is assessed at $4000?

2. Find the value of Mr. White's property if his taxes amount to $67.50 when the tax rate is $.01$\frac{1}{2}$ and his property is assessed at $\frac{3}{4}$ of its value.

3. The rate of taxation in a certain county is $.01$\frac{3}{4}$ when the assessed valuation is $\frac{2}{3}$ of the actual value of the property. What would the rate of taxation have been to yield the same amount had the property been assessed at its full value?

4. Make and solve five problems in taxes, using if possible the actual tax rates of the community in which you live.

CUSTOMS AND INTERNAL REVENUE

338. 1. Funds for the maintenance of the national government are derived chiefly from *customs* and *internal revenue.*

2. **Customs**, or **duties**, are taxes levied by the national government upon goods imported from other countries. **Internal revenue** is a tax levied by the national government upon the manufacture or sale of certain articles in the United States.

339. The following tables show the receipts and expenditures of the national government during the fiscal year ended June 30, 1906:

Revenues, Fiscal Year 1906	
Customs	$300,251,877.77
Internal Revenue	249,150,212.91
Lands	4,879,833.65
Miscellaneous	40,172,197.34
Total, exclusive of Postal	$594,454,121.67

Expenditures, Fiscal Year 1906	
Civil and Miscellaneous	$159,823,904.50
War Department	119,704,113.09
Navy Department	111,166,784.35
Indians	12,746,859.08
Pensions	141,034,561.77
Interest	24,308,576.27
Total, exclusive of Postal	$568,784,799.06

The following table shows the receipts from the principal objects of internal taxation during the same fiscal year:

Internal Revenue Receipts, Fiscal Year 1906	
Distilled Spirits	$143,394,055.00
Tobacco	48,422,997.88
Fermented Liquors	55,641,858.56
Oleomargarine	570,037.93
Mixed Flour	2,567.23
Adulterated Butter	9,258.43
Renovated Butter	138,078.09
Playing Cards	489,347.26
Penalties	283,991.62
Collections	150,494.88

340. 1. **Customs.** A list or schedule of goods with the rates of import duties adopted by Congress is called a **tariff.** Under our tariff laws some imported articles are admitted without the payment of duties. These articles are said to be on the **free list.** Articles not on the free list are subject to an *ad valorem* duty, a *specific* duty, or to both.

2. An **ad valorem duty** is a duty reckoned according to the *value* or cost of the goods in the country from which they are imported. Thus, the duty on jewelry is 60 % of the value.

3. A **specific duty** is a tax of a specified amount on each pound, yard, gallon, bushel, etc., regardless of the cost of the goods. Thus, the duty on onions is 40¢ a bushel.

4. Certain ports are designated as *ports of entry*, where duties on cargoes are payable. A **customhouse** has been established at each port of entry for the collection of customs. The collection of customs at each port is under the direction of a government officer called the **collector of the port.**

5. The following rates of custom are from the schedule adopted by Congress in 1897, and commonly known as the Dingley Tariff.

Cheese, 6¢ per lb.
Hay, $4 per T.
Coffee, free.
Apples, Dried, 2¢ per lb.
Tea, free.
Bacon and Hams, 5¢ per lb.
Honey, 20¢ per gal.
Soap (Castile), $1\frac{1}{4}$¢ per lb.
Musical instr'ts, 45 % ad val.
Paintings, 20 % ad val.
Penholders, 25 % ad val.
Carpets (Axminster), 60¢ per sq. yd. and 40 % ad val.
Carpets (Brussels), 44¢ per sq. yd. and 40% ad val.
Oats, 15¢ per bu.
Wheat, 25¢ per bu.
Hops, 12¢ per lb.

341. Written Exercises.

1. How much is the duty on a violin worth $80?

2. A painting costing $2500 was purchased in Italy and brought into the United States. Find the amount of the duty charged for its admission.

3. What is the duty on 200 sq. yd. of Brussels carpet valued at $1.20 a square yard?

4. What per cent of the expenditures of the national government were defrayed from customs receipts in 1906? from internal revenue receipts?

BANKING

342. Give the names of some banks that you know of. Of what use to a community are banks? Why is a bank a safer place in which to keep money than a house or an office? If a person wishes to make a payment to another, he may do so by giving him the money. Do you know of any other method commonly used in making payments?

343. Savings Banks. Savings banks are banks organized under the laws of the different states for the purpose of receiving and investing the savings of people. Their capital consists of the money put into the bank by the depositors, and their profits are divided among the depositors in proportion to the amount that each has on deposit. The profits are paid to the depositors in the form of interest, usually ranging from 3% to 4% annually, which is paid monthly, quarterly, or semi-annually. If the interest is not collected by the depositor when it becomes due, it is entered to his credit on the books of the bank and it thereafter draws interest in the same manner as the ordinary deposits. Do savings banks pay compound interest?

Deposit Slip.

SAVINGS DEPOSIT

No. *1488* Bal. *$315.45*

DEPOSITED WITH

UNION SAVINGS BANK

FOR ACCOUNT OF

Charles W. Smith

Los Angeles, Cal., March 22, 1907.

	DOLLARS	
Gold		
Silver . . .	*10*	
Currency . .		
Checks . . .		
" . . .		
" . . .		

344. Savings Bank Accounts. Any reliable person who wishes to deposit money for safekeeping and investment may open an account

with a savings bank. On opening an account, the depositor is usually required to answer certain questions and to leave his signature with the bank, to protect the bank against fraudulent demands upon his accounts. The depositor then hands the "receiving teller" or the "cashier" the money which he wishes to deposit, together with a *deposit slip*, showing the amount of his deposit. He is then given a *savings bank book*, usually bearing a number, with the amount of his deposit credited to his account. This bank book must be presented whenever the depositor wishes to make a deposit or to draw against his credit in the bank. The smallest amount received for deposit is usually one dollar.

When a depositor withdraws money from a savings bank, he is required to give the bank a receipt for the amount he has withdrawn.

RECEIPT

SAVINGS ACCOUNT

LOS ANGELES, CAL., *April 16, 1907* NO. *1488*

RECEIVED FROM THE UNION SAVINGS BANK

Fifteen and $\frac{00}{100}$ ~~~~~~~~ DOLLARS $15.00

Charles W. Smith

BALANCE, *$285.15.*

As the usage of banks differs considerably in the various sections of the country, no attempt is made here to give details. Pupils should familiarize themselves with local customs. Deposit slips, receipt blanks, or check blanks, sample bank books, note forms, etc., should be examined by the pupils and their use explained to them. Where possible, a visit to a neighboring bank should be made during banking hours.

345. Illustrative page from a savings bank book, showing entries of deposits, withdrawals, and interest:

STATEMENT

DATE	WITHDRAWN		DEPOSITED		BALANCE	
Dec. 6, 1907			45	50	128	75
Dec. 21, 1907 . . .			15		143	75
Dec. 24, 1907 . . .	35				108	75
Int. to Jan. 1, 1908 .			4	74	113	49
Jan. 3, 1908			50		163	49
Feb. 2, 1908			74	50	237	99
March 7, 1908 . . .	100				137	99
March 25, 1908 . . .			80		217	99

346. Banks of Deposit. Banks other than savings banks are sometimes called *banks of deposit.* They are variously known as **national banks, state banks, commercial banks, private banks,** etc. These banks are organized for the purpose of receiving deposits, making loans, etc. National banks are organized under the national law and are under the direct supervision of the Comptroller of Currency, who is appointed by the President of the United States. In addition to carrying on a general banking business, national banks have authority to issue paper money, called **bank notes.** The payment of these notes is secured by government bonds deposited with the Secretary of the Treasury. All other banks are organized under the laws of the states in which they are located.

347. Deposits. Accounts are opened with banks of deposit in much the same manner as with savings banks. Business men and others who wish to keep money on hand for the payment of bills, etc., usually have an account with a bank against which they may draw *checks* whenever they wish. Such accounts usually do not draw interest. Each depositor receives a *bank book* in which all deposits must be entered.

348. 1. Checks. When the account is opened, the depositor receives a *check book* which he uses in making demands against his accounts.

STUB

No. 68
DATE, May 4, 1908
$36.45
PAY TO James E. Thomas
BALANCE, $314.83

CHECK

SAN FRANCISCO, CAL., May 4, 1908   No. 68

UNION NATIONAL BANK

PAY TO THE ORDER OF

James E. Thomas

Thirty-six and $\frac{45}{100}$ DOLLARS

$\$36\frac{45}{100}$ R. E. Davies.

2. Checks are indorsed in the same manner as promissory notes.

3. To cash this check, James E. Thomas will indorse it and present it at the bank, or will deposit it to his credit at the bank where his accounts are kept. The check will finally be returned to R. E. Davies after it has been paid. It will then serve as a receipt from James E. Thomas, since it bears his indorsement.

4. A depositor may draw money for himself from his bank by making his check payable to "Cash," in which case no indorsement is necessary.

349. Certificate of Deposit. When a person who does not intend to become a regular customer of a bank makes a deposit, he is given a *certificate of deposit*, showing the amount of his deposit. Such a deposit is not subject to check. The amount may be withdrawn upon the return of the certificate with the proper indorsement. Certificates of deposit are usually issued to persons who deposit money with banks when interest is paid upon the deposit.

350. Drafts. Banks usually keep money on deposit in some bank, called a **correspondence bank,** in Boston, New York, Chicago, San Francisco, or other financial center, against which to draw checks. When a person wishes to make a payment in a distant place, he may purchase the check of his local bank on a bank of correspondence, which will honor this check when presented for payment. Such a check is called a **bank draft.** Thus, Mr. A, in Seattle, Wash., may wish to pay Mr. B, in Peoria, Ill., $45.60. Mr. A may purchase a draft of a bank in Seattle on a bank in Chicago, which will honor the draft when presented either by Mr. B, or by some bank which has purchased it from Mr. B. The bank in Seattle may charge Mr. A a small amount for making this exchange of money.

351. Clearing House. Daily settlements of accounts between banks are made through an association called **a clearing house.** At a fixed hour each day representatives from each bank that is a member of the clearing house visit the clearing house and settle the accounts of their bank with other banks. All large cities have clearing houses and nearly all banks in these cities are members.

By means of the clearing house the American Exchange National Bank one day transacted a business of $18,000,000 in checks, with a balance of only 12 cents to pay. The clearing house sheets showed that $9,049,255.40 in checks, drawn by the depositors of the bank, had been turned in by other institutions. Against these the bank had $9,049,255.28 in checks of other banks belonging to the clearing house, which had been deposited with the American Exchange National Bank. The clearance was made by a payment by the bank of 12 cents to the clearing house.

LIFE INSURANCE

352. Personal Insurance. There are various kinds of personal insurance. Of these the most common are: *Accident insurance,* which is an indemnity for injuries sustained by accident; *health insurance,* which is an indemnity for loss of time caused by illness; *life insurance,* the principal forms of which are discussed on p. 279.

353. 1. Persons who insure their lives usually do so to provide for those who are dependent upon them. The cost of life insurance depends (*a*) upon the age of the person insured; and (*b*) upon the kind of policy taken out. Life insurance premiums are always stated at so much on each $1000 of insurance. The most common kinds of policies are:

2. **Ordinary Life Policies**, called also *straight life policies* and *life policies*. The insured pays a premium, usually annually, at the beginning of each year from the time he insures his life until his death. At his death, the company pays the face of the policy to the person (or persons) named in the policy as his beneficiary.

3. **Limited Payment Life Policies.** The insured pays a premium for a limited number of years, as 20 years, at the expiration of which the policy is said to be *paid up*. The face of the policy is paid to the beneficiary at the death of the insured.

4. **Endowment Policies.** Premiums are paid for a period of years, as 10, 15, or 20 years, and the face of the policy is paid at the end of the period specified, or at death if the insured should die before the expiration of the period.

5. **Term Policies.** The insurance extends for a specified period, as for 10, 15, 20 years, etc., at the expiration of which the insurance ceases. The face of the policy is paid if the insured dies within the period specified.

The amount of the annual premium on $1000 of insurance for a life policy, a limited life policy, and for an endowment policy, ages 20 years to 40 years, is given in the table on p. 280.

6. Which is the more likely to live twenty years longer, a person twenty years of age or a person forty years of age? Statistics have been carefully compiled showing the ages at which persons die. From these statistics insurance companies are able to determine the average number of years a healthy person of a given age may be expected to live. The rates of annual premiums are based upon these statistics.

Table of Annual Premiums for $ 1000 (Ages 20 years to 40 years)

Policies Non-forfeitable and Participating

Premiums may also be paid half-yearly or quarterly; and if desired, may be paid in 10, 15, or 20 years instead of during the whole term.

Ordinary Life		20 Payment Life	Endowments		
Age	Yearly		10 Year	15 Year	20 Year
20	$17.30	$24.16	$99.27	$62.34	$44.10
21	17.80	24.60	99.40	62.40	44.25
22	18.30	25.10	99.50	62.45	44.40
23	18.70	25.70	99.60	62.50	44.55
24	19.30	26.20	99.75	62.60	44.70
25	19.80	26.75	99.90	62.70	44.82
26	20.30	27.30	100.00	62.80	44.95
27	20.90	27.90	100.05	62.90	45.10
28	21.50	28.50	100.10	63.05	45.25
29	22.10	29.10	100.20	63.20	45.45
30	22.70	29.70	100.30	63.34	45.63
31	23.40	30.35	100.40	63.50	45.85
32	24.10	31.00	100.50	63.70	46.05
33	24.80	31.72	100.60	63.90	46.25
34	25.60	32.50	100.75	64.05	46.45
35	26.50	33.28	100.90	64.20	46.70
36	27.40	34.10	101.15	64.40	46.85
37	28.30	34.96	101.45	64.65	47.05
38	29.30	35.88	101.75	64.95	47.25
39	30.40	36.84	101.95	65.30	47.45
40	31.50	37.84	102.14	65.67	48.64

354. Dividends. The premium charged represents the *estimated* cost of insurance and is based upon conservative assumptions as to future death rate, the rate of interest which the company may expect to receive for loans, etc. The *actual* cost of insurance is determined by experience from year to year. The difference between the estimated cost and the actual cost is called the *profit*. Policy holders are usually allowed to

participate in the profits, either by having them applied to reduce the yearly premiums or by having them *accumulate* in the possession of the companies until the expiration of the term of insurance. An insurance policy in which it is stipulated that no dividend shall be paid until the close of the term of insurance is called a **tontine policy**.

Examine a life insurance policy. Read all its provisions.

355. Use the table in answering the following:

1. How much will it cost annually to carry an ordinary life policy for $1000, if it is taken out at the age of 20? at the age of 25? at the age of 35?

2. How much will it cost annually to carry a 20-payment life policy for $2000, if it is taken out at the age of 20? at the age of 27? at the age of 40?

3. How much will it cost annually to carry a 10-year endowment policy for $5000, if it is taken out at the age of 20? at the age of 30? at the age of 40?

4. Suppose that a young man 20 years old takes out a 20-payment life policy for $1000 and dies after paying 8 annual premiums. Find the net cost of the insurance, if dividends amounting to $40 were applied to reduce the premiums. How much would the beneficiary named in the policy receive at his death?

5. How much will it cost to carry a 20-year endowment policy for $1000 for the term of the policy, if it is taken out at the age of 30? How much would the insured receive from the insurance company at the end of the term, not including the dividends?

6. If the insured (Prob. 5) died after paying 15 premiums, how much more than the amount paid as premiums would the beneficiary receive?

7. What is meant by a non-forfeitable and participating policy? by a tontine policy?

Table of Loan and Surrender Values

356. The following table shows the loan and surrender values on a 20-payment life policy for $ 1000 taken out when the insured was 25 years of age, the annual premium being $26.95:

At End of	Loan	Cash Value	Paid-up Insurance	Extended Insurance	
				Years	Days
3d			$150	4	342
4th			200	6	291
5th	$54	$60	250	8	232
6th	68	76	300	10	317
10th	130	145	500	19	17
15th	224	249	750	26	134
19th	315	351	950	31	111
20th	342	380	Policy full-paid		

Answer the following from the above table:

1. If the insured wished to borrow money, how much would the company loan him at the end of the 10th year, if he assigned to the company his policy as security? how much at the end of the 15th year?

2. If the insured surrendered his policy at the end of the 5th year, how much would the insurance company pay him for his policy? How long would they continue his insurance without the payment of premiums?

3. Find the amount of the annual premiums for 20 years. What is the cash value of the policy at the end of 20 years? If the dividends average $6.50 a year, how much will they amount to in 20 years? What is the sum of the cash value and dividends at the end of the insurance term? How does this sum compare with the total cost of the insurance for the term?

4. Using the compound interest table on p. 320, find the amount of $26.95 (the premium) for 20 years. If money is worth 6%, find the total amount of the premiums paid at the end of 20 years, the premium being paid at the beginning of each year.

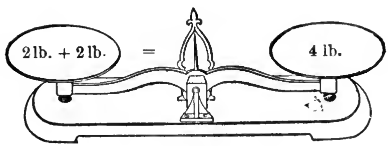
5. If the insured should die at the age of 40, how much would the beneficiary receive?

THE EQUATION

357. 1. The relation of the quantities involved in some problems can be stated in a simpler and clearer way by the use of the *equation*. In an equation, the value of the unknown quantity is usually represented by the letter x. Thus, in $6+4=x$, x is called the unknown quantity, and the expression $6+4=x$ is called an **equation**.

2. An equation may be compared to a balance scale. In an equation the quantities on the two sides are equivalent — they balance one another.

3. If a package weighing 4 lb. is placed in one pan of a balance scale, what weight must be placed in the other pan to make the scale balance?



4. A package weighing 5 lb. was placed in the pan on the right of a balance scale and a 2-lb. weight was placed in the pan on the left. What additional weight must be placed in the pan on the left to make the scales balance?

5. Would the scales as represented in the figure still balance if a 5-lb. weight were added to the weights in each pan? Would they balance if a 5-lb. weight were added to the weight in one pan?

6. Would the scales as represented in the figure still balance if 2 lb. were removed from both pans? Would they balance if 2 lb. were removed from only one pan?

7. Would the scales as represented in the figure balance if the weight on both sides were doubled? Would they balance if the weight on only one side were doubled?

8. Would the scales as represented in the figure balance if one half of the present weight were removed from each pan? Would they balance if one half of the weight were taken out of only one pan?

9. What is the value of the unknown quantity in the equation 8 lb. $= 5$ lb. $+ x$? in 6 lb. $+ x = 10$ lb.? in 7 lb. $- x =$ 2 lb.?

10. What is the value of x in $9 + x = 12$? Finding the value of the unknown quantity in an equation is called **solving the equation.**

11. If 3 is added to both sides of the equation $x + 4 = 9$, the result is $x + 7 = 12$. How does the value of x in $x + 4 = 9$ compare with the value of x in $x + 7 = 12$?

12. Write 5 equations. Add some number to both sides of each of the equations. Compare the value of x in the resulting equation with the value of x in the original equation.

13. State what effect adding the same number to both sides of the equation has upon the value of x in the equation. Prove the truth of your statement.

14. If 2 is subtracted from both sides of the equation $x + 4 = 9$, the result is $x + 2 = 7$. How does the value of x in $x + 2 = 7$ compare with the value of x in $x + 4 = 9$?

15. Write 5 equations. Subtract some number from both sides of each equation. Compare the value of x in each of the resulting equations with the value of x in the original equation.

16. State what effect upon the value of x in any equation subtracting the same number from both sides of the equation has upon the equation. Prove the truth of your statement.

17. If 4 is subtracted from both sides of the equation $x + 4 = 9$, what is the result?

18. If 3 is subtracted from both sides of the equation $x + 3 = 8$, the result is $x = 5$. If 2 is subtracted from both sides of the equation $9 = 2 + x$, the result is $7 = x$. What number must be subtracted from both sides of each of the following equations to leave x by itself on one side: $x + 5 = 8$? $x + 7 = 15$? $x + 6 = 9$? $8 = 6 + x$? $12 = x + 5$? $9 + x = 15$? $7 + x = 10$?

19. If 4 is added to both sides of the equation $x - 4 = 5$, the result is $x = 9$.

20. What number must be added to both sides of each of the following equations to leave x by itself on one side:

$x - 5 = 8$? $x - 7 = 10$? $x - 4 = 11$? $x - 9 = 2$?
$12 = x - 15$? $10 = x - 5$? $4 = x - 6$? $2 = x - 3$?

21. The value of x in $x + 4 = 9$ may be found by subtracting 4 from the left side of the equation and *indicating* the subtraction of 4 from the other side, thus: $x = 9 - 4$. Find the value of x in each of the following equations: $x + 6 = 15$; $x + 8 = 15$; $x + 9 = 16$; $x + 20 = 45$; $x + 345 = 670$.

22. The value of x in $x - 4 = 10$ may be found by adding 4 to the left side of the equation and *indicating* the addition of 4 to the other side, thus: $x = 10 + 4$. Find the value of x in each of the following equations: $x - 7 = 13$; $x - 5 = 18$; $x - 12 = 20$; $x - 14 = 17$; $x - 46 = 35$; $x - 80 = 120$.

23. Write each of the following equations with x by itself on the left side of the equation: x + 3 lb. = 10 lb.; 5 ft. + x = 13 ft.; 24 yd. + x = 45 yd.; \$7.50 + x = \$12.75; x + \$15 = \$80; x − 12 ft. = 20 ft.; x − \$3.45 = \$1.20.

24. Compare $2 + 3 = 5$ with $5 = 2 + 3$. Compare $x + 4 = 9$ with $9 = x + 4$. State what effect, if any, writing the equation with the sides changed has upon the equation.

25. Write each of the following equations so the side containing x is on the left: 45 ft. + 33 ft. = x; \$2.45 = x − \$1.20; 14 yr. = 9 yr. + x; 10 yr. = x − 7 yr.

358. Solve each of the following without using x. Then write the equation for each, using x, and find the value of x:

1. If 45 is added to a certain number, the sum is 73. What is the number?

MODEL: Let $x =$ the unknown number

$$x + 45 = 73$$
$$x = 73 - 45$$
$$x = 28$$

2. If 27 is subtracted from a certain number, the remainder is 56. What is the number?

3. If a certain number is increased by 347, the result is 591. What is the number?

4. If a certain number is diminished by 274, the result is 483. What is the number?

5. A boy deposited \$17 in a savings bank. He then had \$61 in the bank. How much money had he in the bank before depositing the \$17?

6. After drawing out \$35 from a savings bank a boy had left \$7.45 in the bank. How much money had he in the bank before drawing out the \$35?

7. After gaining 7 lb. a girl weighed 103 lb. How much did she weigh before gaining the 7 lb.?

8. George and Frank together have as much money as Walter. George has \$2.15 and Walter has \$4.10. How much money has Frank?

9. A man owns three farms amounting together to 240 acres. Two of the farms contain 80 acres and 120 acres respectively. How many acres are there in the third farm?

10. A house and lot together cost \$4500. The lot cost \$1500. Find the cost of the house.

11. The sum of two numbers is 238. One of the numbers is 79. What is the other number?

12. The sum of the three sides of a triangle is 24 in. One of the sides is 8 in. and another is 9 in. What is the length of the third side?

13. After selling 40 sheep a farmer had 236 sheep. How many sheep had he before selling the 40 sheep?

14. Write 3 problems similar to each of Probs. 1–13 and write the equation for each.

359. **1.** If x is added to both sides of the equation $7 - x = 2$, the result will be $7 = 2 + x$. What will be the result if x is added to both sides of the equation $10 - x = 7$?

2. If x is added to both sides of the equation $14 = 25 - x$, the result will be $14 + x = 25$. The value of x is found by adding to both sides of the equation some number that will leave x by itself on the left side.

3. Write each of the following equations so that x will be by itself on the left side of the equation. First, add x to both sides of the equation, then write the equation so that the side containing x will be on the left. $18 = 43 - x$; $21 = 72 - x$; $60 - x = 37$; $33 - x = 19$; $54 = 62 - x$; $68 - x = 28$.

Write each of the following statements so that x will be by itself on the left side of the equation, and solve:

4. $167 - x = 100$.

5. $x - \$36 = \$75$.

6. 15 lb. $=$ 25 lb. $- x$.

7. 78 ft. $=$ 135 ft. $- x$.

8. $74 - x = 18$.

9. $\$45.75 - x = \$30.50$.

10. $65.4 - x = 18.45$.

11. 125 da. $- x =$ 65 da.

12. Write 10 equations and find the value of x in each.

360. **1.** The sum of x and x and x, or 3 times x, is written $3x$. Write the sum of x and x. Write the product of 4 times x; of 5 times x.

2. If x is 4, what is the value of $2x$? Compare $x = 4$ and $2x = 8$. What must both terms of $x = 4$ be multiplied by to give $2x = 8$? Multiply both terms of $x = 3$ by 5. Has this changed the value of x in the equation?

3. State what effect multiplying both sides of the equation by the same number has upon the value of x in an equation. Prove the truth of your statement.

4. Divide both sides of the equation $6x = 18$ by 2; by 3; by 6. Has this changed the value of x in the several equations?

5. State what effect dividing both sides of the equation by the same number has upon the value of x in an equation. Prove the truth of your statement.

6. If $2x + 4 = 21$, what is the value of $2x$? of x? of $3x$?

7. If $x = 6$, what is the value of $7x$? of $3x$? of $5x$?

Find the value of x in each of the following equations. Where the equation shows an unknown quantity to be subtracted from one side of the equation, add this unknown quantity to both sides of the equation; then write the equation with the unknown quantity on the left side and solve:

8. $2x - 45 = 69$.
9. $24 = 78 - 2x$.
10. 45 ft. $- 4x = 13$ ft.
11. $345 - 4x = 135$.
12. 240 A. $= 880$ A. $- 4x$.
13. $146 - 3x = 83$.
14. $\$35.40 - 3x = \$10.95$.
15. $\$.85 = \$.40 + 3x$.
16. $\$90 - 6x = \$48$.
17. 24 yr. $- 3x = 6$ yr.

361. Solve each of the following without using x. Then solve each, using x.

1. If 3 times a certain number, plus 25, is 55, what is the number?

2. If 4 times a certain number, less 20, is 40, what is the number?

3. Mary is 20 years old. This is 2 years more than twice Edna's age. What is Edna's age?

4. Walter has \$45. This is \$13 more than 4 times the amount of money James has. How much money has James?

5. If 4 times a certain number, plus 3 times that number, is 28, what is the number? ($4x + 3x = 7x$.)

6. If 6 times a certain number, plus 4 times that number, is 160, what is the number?

7. If 4 times a certain number is the same as 6 times 18, what is the number?

8. A man bought three railroad tickets, each costing the same amount, and paid $1.50 for bus rides. He paid out $6.90 in all. Find the price paid for each ticket.

9. The sum of two numbers is 48, and one number is 5 times the other. What are the numbers? (Let x and $5x$ represent the numbers.)

10. A man bought two carriages. For one he paid twice what he paid for the other. Both carriages cost him $210. Find the cost of each.

11. Write problems similar to each of the above, and state the equation for each.

12. Draw an oblong whose length is twice its width. Let x represent its width. What will represent its length? its perimeter? If the perimeter of the oblong is 30 in., how wide is it? How long is it?

13. Draw two lines, one of which is 3 times the length of the other. If the sum of their lengths is 24 ft., how long is each line?

14. Two men together own 540 acres of land. One owns twice as much as the other. How many does each own?

15. A man offered to divide $10 between two boys in proportion to their ages, provided the boys could tell how much each should receive. The boys were 12 years and 13 years respectively. After solving the problem the boys stated that the younger should receive $4.50 and the older $5.50. Did they solve it correctly? If not, what is the correct answer?

16. A man offered some boys \$1.50 for weeding his garden. The boys found that they could not all work at the same time, so the man agreed to pay each boy the same wages per hour for the work done. One boy worked 7 hours, another worked 5 hours, and the third worked 3 hours. How much of the money should each boy receive?

17. A man wished to leave \$3500 to his three sons so that the second son would receive twice what the youngest received and the eldest would receive 4 times what the youngest received. How much should each son receive?

362. 1. The expression $\frac{x}{3}$ is used to denote $\frac{1}{3}$ of x. Write the expression that denotes $\frac{1}{4}$ of x; $\frac{1}{5}$ of x; $\frac{2}{3}$ of x; $\frac{4}{5}$ of x; $\frac{1}{7}$ of x.

2. By what number must $\frac{x}{4}$ be multiplied to make x? By multiplying both sides of the equation $\frac{x}{4} = 3$ by 4, the equation is changed to $x = 12$.

3. Multiply $\frac{x}{5}$ by the number that will give x as the result. Multiply $\frac{3x}{4}$ by the smallest whole number that will give a whole number of x's as the result.

4. What is the smallest number that both sides of the equation $\frac{x}{3} = 12$ can be multiplied by to leave only whole numbers in the equation? Multiplying both sides of an equation by some number that will leave the equation without fractional quantities is called **clearing the equation of fractions.**

5. Clear the following equations of fractions: $\frac{x}{6} = 8$; $\frac{x}{7} = 14$; $\frac{3x}{5} = 12$; $\frac{9x}{10} = 36$; $45 = \frac{3x}{5}$; $60 = \frac{12x}{11}$; $\frac{3x}{4} - 8 = 1$; $72 - \frac{4x}{5} = 56$.

6. Clear the following of fractions and find the value of x: $\frac{x}{2} - \frac{2x}{8} = 7$; $\frac{3x}{5} - \frac{x}{4} = 7$; $\frac{4x}{5} - \frac{2x}{15} = 20$; $\frac{x}{4} - 5 = 10$.

Solve each of the following without using x. Then solve each, using x:

7. Divide 60 into two numbers such that the first is $\frac{7}{8}$ of the second.

8. Separate 36 into two parts whose ratio is $\frac{4}{5}$.

9. Divide \$2.10 into two amounts whose ratio is the same as the ratio of 15¢ to 20¢.

10. If $\frac{1}{3}$ of a certain number, plus $\frac{3}{4}$ of it, is 39, what is the number?

11. Solve: 6 times $8 = 12$ times x; 4 times $9 = 6$ times x.

12. Solve: $\frac{6}{9} = \frac{x}{3}$; $\frac{x}{8} = \frac{12}{16}$; $\frac{x}{7} = \frac{42}{49}$.

13. The equation $\frac{6}{x} = \frac{1}{2}$ may be cleared of fractions by multiplying both sides of the equation by the least common multiple of the denominators. This is $2x$. The equation is thus changed to the form $12 = x$.

14. Solve: $\frac{8}{x} = \frac{15}{20}$; $\frac{8}{24} = \frac{7}{x}$; $\frac{10}{x} = \frac{20}{60}$; $\frac{9}{x} = \frac{24}{32}$.

15. Solve: $\frac{6}{10} = \frac{18}{x}$; $\frac{15}{20} = \frac{51}{x}$; $\frac{8}{10} = \frac{20}{x}$; $\frac{24}{4} = \frac{30}{x}$.

16. Write ten exercises similar to exercises 8–12 and find the value of x in each.

363. Proportion.

Solve each without using x. Solve each, using x:

1. The shadow of a post 5 ft. high is 3 ft. 6 in. long. How high is a telephone pole whose shadow is 28 ft. long?

a. The height of the post is $\frac{5}{3.5}$ times the length of its shadow. The height of the telephone post is $\frac{5}{3.5}$ times 28 ft. Explain.

b. The length of the shadow of the post is in the same ratio to the height of the post as the length of the shadow of the telephone pole is to the height of the pole. The equality of these ratios may be expressed thus:

$$\frac{3.5}{5} = \frac{28}{x}.$$

Solve to find the value of x, the number of feet in the height of the telephone pole.

c. The equality of the two ratios may be expressed thus: 3.5 ft. : 5 ft. : : 28 ft. : x, which is read, 3.5 ft. is to 5 ft. as 28 ft. is to x. The first and last terms (as 3.5 ft. and x) of a

proportion are called the **extremes**, and the two middle terms the **means**. *The product of the extremes in a proportion is always equal to the product of the means.* Hence, 3.5 times x $= 5$ times 28, or $3.5\,x = 140$. Solve to find the value of x, the number of feet in the height of the telephone pole. This method of solving a proportion differs only in *form* of expression from the method (*b*) given on p. 291.

2. How high is a tree whose shadow is 34 ft. 6 in., if the shadow of a boy whose height is 4 ft. 9 in. is 3 ft. 3 in.?

3. If the distance traveled by a train in 1 hr. 45 min. is 80 mi., how long, at the same rate of speed, will it take the train to travel 475 mi.?

4. Find by the method used in solving Probs. 1 and 2 the height of objects near the schoolhouse.

MEASUREMENT OF SURFACES AND SOLIDS

364. Areas of Surfaces.

1. Draw a vertical line; a horizontal line; an oblique line.

2. Draw a line perpendicular to another line; parallel to another line.

3. Draw a right angle; an acute angle; an obtuse angle.

4. Draw a rectangle. Is a rectangle a parallelogram? Draw a parallelogram that is not a rectangle.

5. How many dimensions has a rectangle? Is a rectangle a quadrilateral? Draw a quadrilateral that is not a parallelogram.

6. State how the area of a parallelogram is found. Find the area of a parallelogram whose base is 20 ft. and whose altitude is 18 ft.

7. A quadrilateral that has only two parallel sides is called a **trapezoid**.

8. State how the area of a trapezoid is found. Draw a trapezoid. Assign its dimensions and find its area.

9. What is a triangle? Draw a right triangle; an acute-angled triangle; an obtuse-angled triangle.

10. State how the area of a triangle is found. Draw a triangle. Assign its dimensions and find its area.

11. Make a drawing to show the relation of the area of a triangle to the area of a parallelogram having the same base and altitude.

12. Draw a parallelogram. Draw its diagonals. Do they cross at the middle of the parallelogram?

13. What is meant by the perimeter of a figure? Find the perimeter of your schoolroom.

14. Draw a circle. Draw its radius; its diameter. Point to its circumference.

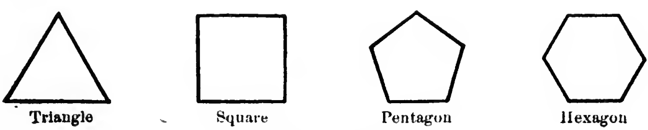
15. State how the circumference of a circle is found when the length of its radius is known. State how the diameter of a circle is found when the length of its circumference is known.

16. State how the area of a circle is found. Assign the necessary dimensions and find the area of a circle.

17. State how the area of the convex surface of a cylinder is found. Find the area (including the ends) of a cylinder whose diameter is 6 ft. and whose length is 8 ft.

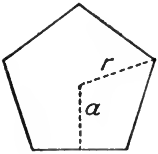
365. Regular Polygons.

1. Mention a surface that is a *plane* surface. A plane figure bounded by straight sides is called a **polygon.** A polygon whose sides are all equal and whose angles are all equal is called a **regular polygon.**



REGULAR POLYGONS

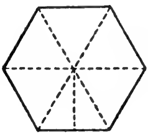
2. A regular polygon of three sides is called an **equilateral triangle**; of four sides, a **square**; of five sides, a **pentagon**; of six sides, a **hexagon**; of seven sides, a **heptagon**; of eight sides, an **octagon**. Draw an octagon.



3. A straight line from the center of a regular polygon to any vertex is called its **radius** (r).

4. The perpendicular from the center of a regular polygon to any side is called its **apothem** (a).

5. The area of a regular polygon is the sum of the areas of the triangles formed by its radii and sides. The apothem is the altitude of each of the triangles, and the perimeter is the sum of the bases of the triangles. Hence,



The area of a regular polygon is equal to one half the product of its perimeter and apothem.

6. Draw a pentagon. Assign its dimensions and find its area.

7. Draw a hexagon. Assign its dimensions and find its area.

8. Draw an octagon. Assign its dimensions and find its area.

9. The area of a circle is one half the product of its radius and circumference. Compare the method of finding the area of a regular polygon with this method of finding the area of a circle.

366. Solids.

1. How many dimensions has a plane surface? Name them.

2. How many dimensions has a solid? Name them.

3. What name is given to a solid whose faces are all rectangles? to a solid whose faces are equal squares?

4. What name is given to a solid whose ends are triangles and whose sides are rectangles?

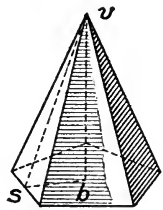
5. State how the volume of a prism is found. Draw a prism. Assign its dimensions and find its volume.

6. Name solids that are rectangular prisms.

7. State how the volume of a cylinder is found. Draw a cylinder. Assign its dimensions and find its volume. Find its area, including the ends.

367. Pyramids and Cones.

1. A solid whose base is a polygon and whose faces are triangles meeting at a point (vertex) is called a **pyramid.**



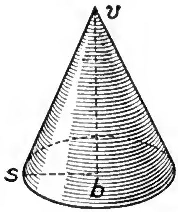
2. The area of the surface of a pyramid is the sum of the areas of the triangular faces.

3. The perpendicular distance from the base to the vertex of a pyramid is called its **altitude** (*vb*).

4. The altitude of one of the triangular faces of a pyramid is called its **slant height** (*vs*).

5. Construct a pyramid of cardboard. Which is the greater, the altitude of a pyramid or its slant height? The apothem of a polygon forming the base of a pyramid may be regarded as the base of a right triangle (*bs*), the altitude as the other leg (*vb*), and the slant height as the hypotenuse (*vs*). How may the altitude be found when the apothem of the base and the slant height are given?

6. Draw a regular polygon. Draw its radius and the apothem of an adjacent side. The figure formed by the radius, apothem, and one half of the adjacent side is what kind of a triangle? If the radius and side of a regular polygon are given, how may the apothem be found?

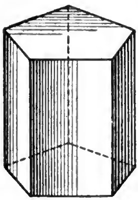


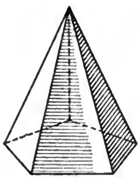
7. If the altitude of a pyramid, the radius of its base, and the adjacent side are given, how may the slant height be found?

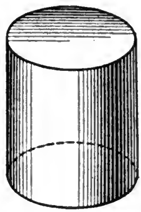
8. A solid whose base is a circle and which tapers to a point called the vertex or apex, is called a **cone**.

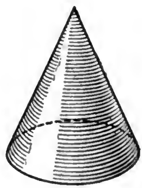
9. A cone may be regarded as a pyramid whose surface is an infinite number of narrow triangles. Its altitude and slant height correspond to the altitude and slant height of a pyramid.

The area of the surface of a pyramid or a cone is equal to one half the product of its slant height and the perimeter of its base.


PRISM


PYRAMID


CYLINDER


CONE

10. The volume of a pyramid is equal to one third the volume of a prism of the same base and altitude, and the volume of a cone is equal to one third the volume of a cylinder of the same base and altitude. Hence,

The volume of a pyramid or a cone is equal to one third the product of its altitude and the area of its base.

11. A cylindrical granite stone 3 ft. in diameter and 4 ft. in height was cut down into a cone of the same base and altitude. What part of the stone was cut away?

368. Spheres.

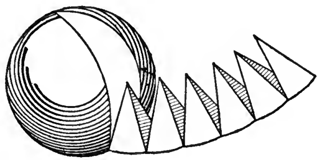
The area of the surface of a sphere is four times the area of a great circle (πr^2) *of the sphere.*

1. As $(2r)^2$, or $4r^2$ is equal to d^2, $4\pi r^2$ is equal to πd^2.

The area of the surface of a sphere is equal to the square of the diameter $\times$ π, *or* πd^2.

2. Which is the greater and how much, the area of a cube whose side is 1 ft. or the area of a sphere whose diameter is 1 ft.?

3. A sphere may be divided into an infinite number of figures that are essentially pyramids. The combined volume of these pyramids is the volume of the sphere. The convex surface of the sphere may be regarded as the sum of the bases of the pyramids and the radius of the sphere as the altitude of the pyramids. Hence,



The volume of a sphere is equal to one third the product of its radius and its convex surface, or $\frac{4}{3}\pi r^3$ ($\frac{1}{3}$ of $r \times 4\pi r^2$).

4. As d^3, or $(2r)^3$ is equal to $8r^3$, $\frac{4}{3}\pi r^3$ is equal to $\frac{1}{6}\pi d^3$. Hence,

To find the volume of a sphere, multiply the cube of its diameter by .5236 ($\frac{1}{6}$ *of* π).

5. The earth is how many times the size of the moon, if the diameter of the earth is 8000 mi. and the diameter of the moon is 2000 mi.?

Volumes of spheres are to each other as the cubes of their like dimensions. The ratio of the earth and moon is 8^3 (8000^3) to 2^3 (2000^3), or 4^3 to 1^3.

MEASUREMENT OF PUBLIC LANDS

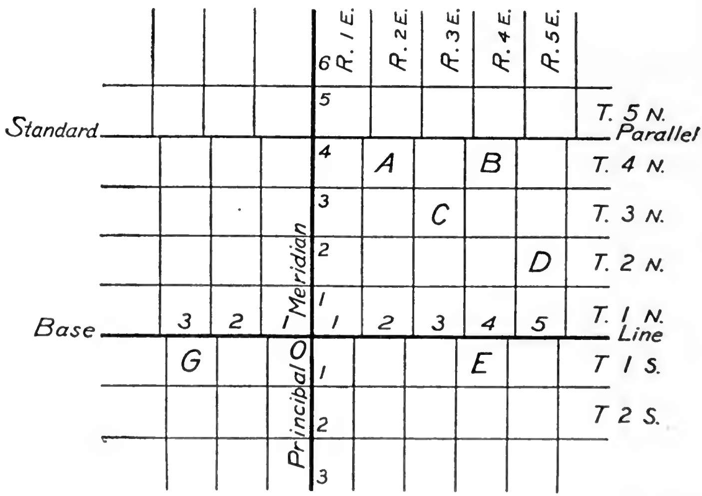
369. 1. At the time the colonial settlements were made, no uniform system of measuring lands was used. Generally, each settler was permitted to occupy whatever lands he wished, and the boundary lines were often designated by such convenient natural objects as rocks, streams, trees, hilltops, etc. Later these boundaries were recorded as the legal "metes and bounds" of their several possessions. These tracts of land

were often so irregular in shape as to make it difficult to fix their exact boundaries and to determine their exact areas.

2. Shortly after the close of the Revolutionary War, the Continental Congress appointed a committee, of which Thomas Jefferson was chairman, to draw up some plan for the survey of public lands. This committee reported a plan which, after being slightly amended, was adopted by Congress in 1785, and thus became the government system of measuring public lands.

3. In accordance with this system, all public lands, except "waste and useless lands," have been laid out in tracts 6 miles square called **townships.** The exact location of each township is determined by north and south lines called **principal meridians,** and by east and west lines called **base lines.**

Study the following diagram:



4. In surveying a tract of land, a prominent point that is easily identified and is visible for some distance is established astronomically, and is known as the **initial** (beginning) **point.** In the figure, the initial point is at *O*.

5. A line extending north or south, or both north and south, from the initial point is taken as a **principal meridian.** The principal meridian is the true meridian at the initial point. Locate the principal meridian in the figure.

6. A line extending either east or west, or both east and west, through the initial point, or a line perpendicular to the principal meridian, is taken as a **base line.** The base line is always a true parallel of latitude. Locate the base line in the figure.

7. East and west lines 6 miles apart, called **town lines,** are run parallel to the base line, and north and south meridian lines 6 miles apart, called **range lines.** These lines divide the tract into townships 6 miles square. Point to the township lines in the figure. How far apart are these lines? Point to the range lines. How far apart are these lines?

8. Point to a township in the first tier of townships north of the base line. Point to a township in the second tier of townships north of the base line. Point to a township in the first tier of townships south of the base line.

9. A township in the third tier of townships north of a base line is said to be in township 3, north (T. 3 N.). A township in the second tier of townships south of a base line is said to be in township 2, south (T. 2 S.).

10. Point to the first north and south row of townships, east of the principal meridian. These townships are said to be in **range** 1, east (R. 1 E.). Point to a township in range 3, east; in range 2, west.

11. The township marked *A* is numbered township 4 north, range 2 east (T. 4 N., R. 2 E.). Describe the location of townships *B, C, D, E,* and *G.* Write the description of each, using abbreviations.

12. Locate in the figure each of the following described townships: T. 2 N., R. 3 E.; T. 4 N., R. 5 E.; T. 1 N., R. 1 W.; T. 2 S., R. 4 E.; T. 1 S., R. 3 W.; T. 4 N., R. 2 W.; T. 2 S., R. 2 E.

13. Draw a diagram showing a principal meridian, a base line, and townships and ranges as in the figure on p. 298. In your diagram, locate the following: **T. 4 S., R. 1 E.; T. 6 N.,** R. 5 W.; T. 6 N., R. 6 E.

14. Locate on a map a principal meridian and a base line from which ranges and townships in your state are numbered, if the land has been measured by this system.* Give the number of the township in which you live. Can you tell the width of the state in which you live from the number of townships along the base line? Is there any similarity between the method of locating townships by means of principal meridians and base lines and the method of locating places on the earth's surface by means of degrees of longitude and latitude?

15. The lands of Florida, Alabama, Mississippi, of the states west of Pennsylvania and north of the Ohio River, and of all states west of the Mississippi River, except Texas, have been surveyed in the manner described. Can you tell from your study of United States History why the lands of the other states were not surveyed in this manner?

16. The initial points are located somewhat arbitrarily. Sometimes they are located on the east or west boundaries of states, at other times they are located at the junction of rivers, or on the summits of elevations. They are at irregular intervals apart. Consequently, the land in a single state may be measured from more than one principal meridian, or a single meridian may be used for measuring the land in several states. Much care is taken to preserve the exact location of all initial points.

"An initial point should have a conspicuous location, visible from distant points on lines; it should be perpetuated by an indestructible monument, preferably a copper bolt firmly set in a rock ledge; and it should be witnessed by rock bearings, without relying on anything perishable like wood." Manual of Surveying Instructions, 1902.

* Unmounted land maps of the various states may be purchased from the Department of Interior, Washington, for a few cents.

17. As the lines that bound the ranges on the east and west are true meridians, they converge as they extend north from a base line. As a result, townships are not true squares. To correct the effect of the convergency of the meridians, *standard parallels* (formerly called *correction lines*) are established at regular intervals (now 24 miles apart) from the base line, and new meridians are established 6 miles apart on the standard parallels. Guide meridians are also established at intervals (now 24 miles apart), east and west of the principal meridians, to correct inaccuracies in measurement.

370. Townships.

1. A **township** is a tract of land 6 miles square. It contains 36 square miles of land. A square mile of land is called a **section.** The sections of a township are numbered as shown in the diagram. The sections of a township are numbered, respectively, beginning with number 1 in the northeast section and numbering west and east alternately. Draw a township and number the sections.

6	5	4	3	2	1
7	8	9	10	11	12
18	17		15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	

A TOWNSHIP DIVIDED INTO SECTIONS.

2. Section 16 of each township in the state was granted by Congress to the states for educational purposes. This section is therefore commonly known as the *school section,* and all moneys derived from the rent or sale of these sections is placed in the public school fund of the state. States that have been organized since 1852 have been granted two sections in each township for the support of public schools, sections 16 and 36.

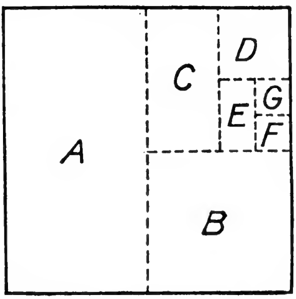
Owing to the convergency of the meridians that bound the townships on the east and west, a township is never exactly 6 miles from east to west, and does not therefore contain 36 full

sections of 640 acres each. The survey of the sections in each township is begun in the southeast corner of the township, and all sections except those along the western and northern boundaries of the township are 1 mile square, and contain 640 acres each. All excess or deficiency is added to or deducted from the sections along the western and northern boundaries of the township. These sections generally contain less than 640 acres. The sections along the western boundary of a township often contain less than 630 acres. Section 6 is frequently reduced to about 620 acres.

371. Sections.

1. A section is subdivided into quarter sections, and these are again subdivided into quarters, etc., as shown in the diagram.

2. The part of the section marked *A* is described as the west one half (W. $\frac{1}{2}$) of the section, and contains 320 acres. The part marked *B* is described as the southeast quarter of the section, and contains 160 acres. *C* is the west one half of the northeast one fourth of the section (W. $\frac{1}{2}$ of N.E. $\frac{1}{4}$). How many acres does it contain?



A SECTION SUBDIVIDED.

The part marked *F* is described as the S.E. $\frac{1}{4}$ of the S.E. $\frac{1}{4}$ of the N.E. $\frac{1}{4}$ of the section. How many acres does it contain?

3. Describe the part marked *G* and tell how many acres it contains.

4. Describe the part marked *E* and tell how many acres it contains.

5. Describe the part marked *D* and tell how many acres it contains.

6. Draw a section and subdivide it to show the following and give the number of acres in each:

7. N.W. $\frac{1}{4}$ of the N.E. $\frac{1}{4}$. **8.** S.W. $\frac{1}{4}$ of the N.W. $\frac{1}{4}$.

9. E. $\frac{1}{2}$ of the S.W. $\frac{1}{4}$. 10. W. $\frac{1}{2}$ of the N.W. $\frac{1}{4}$.

11. S.E. $\frac{1}{4}$ of the S.W. $\frac{1}{4}$ of the N.E. $\frac{1}{4}$.

12. S. $\frac{1}{2}$ of the S.E. $\frac{1}{4}$ of the N.E. $\frac{1}{4}$.

372. Using the scale 1 in. = 1 mi., draw a plot to represent a township, say T. 6 N., R. 4 E.; locate and find the area of each of the following:

1. E. $\frac{1}{2}$ of the S.E. $\frac{1}{4}$ of Sec. 9, T. 6 N., R. 4 E.
2. N.W. $\frac{1}{4}$ of the S.E. $\frac{1}{4}$ of Sec. 22, T. 6 N., R. 4 E.
3. S.E. $\frac{1}{4}$ of the S.W. $\frac{1}{4}$ of the S.E. $\frac{1}{4}$ of Sec. 32, T. 6 N., R. 4 E.
4. E. $\frac{1}{2}$ of the S.W. $\frac{1}{4}$ of the N.E. $\frac{1}{4}$ of Sec. 24, T. 6 N., R. 4 E., which is a farm owned by Mr. Thomas.
5. S.E. $\frac{1}{4}$ of the N.W. $\frac{1}{4}$ of Sec. 18, T. 6 N., R. 4 E., which is the description of a piece of property on which Mr. White pays taxes.

373. Review.

1. The unit of land measure is the township, which is theoretically 6 miles square. The word *town* is commonly used for township.
2. What are initial points? principal meridians? base lines?
3. What is a range? How many sections are there in a township? How are they numbered?
4. How many acres are there in a section? in a quarter section?
5. Public lands are generally sold in sections, half sections, quarter sections, and in half quarter sections. What part of a section is 80 acres? 40 acres? 20 acres?
6. How many acres are there in a full township? in a full section?
7. What are standard parallels, or correction lines?
8. Which are the school sections? Why are they so called?

9. Can you tell from your study of United States history why some uniform system of surveying public lands was necessary soon after the close of the Revolutionary War?

10. What sections generally contain less than 640 acres? Why?

11. Locate the principal meridian and the base line used in measuring the land in which your schoolhouse is located.

12. A new standard parallel is located at intervals of 24 miles north or south of the base line, and a new guide meridian is located at intervals of 24 miles east and west of a principal meridian. Make a diagram showing these lines.

THE METRIC SYSTEM OF WEIGHTS AND MEASURES

374. 1. The system of denominate units of measure in common use in the United States is practically the same as that in use in Great Britain, with the exception of the units used in measuring value. Nearly all the other civilized nations use a decimal system of denominate numbers, called the **metric system**. The metric system has been legalized by the United States and Great Britain, and has been adopted as the system for use in the Philippines and Porto Rico. It is extensively used in scientific work.

2. A little more than a century ago the French government invited the nations of the world to a conference to consider an international system of weights and measures. Later, the French government appointed a committee to devise a convenient system of denominate units. The committee originated what is known as the Metric System of Weights and Measures. The metric system includes measures of length, surface, capacity, volume, and weight. The primary unit of linear measure is the **meter** The primary unit of each of the other measures is based upon the meter.

3. One ten-millionth part of the distance from the equator to the North Pole, measured on the meridian of Paris, was

selected as the primary unit of linear measure. This unit is called the **meter.** *Meter* is the French word for measure. The meter is a little longer than the yard. As it is based upon a measurement of the earth's polar circumference, the meter is a fixed natural unit.*

4. An International Bureau of Weights and Measures has been established in Paris, and is now supported by the contributions of more than twenty nations. A standard meter, made from an alloy of platinum and iridium, is carefully preserved by this bureau. All the nations of the world have been furnished with duplicates of this standard meter. These duplicates are made of the most durable and least expansible metals known. The United States Bureau of Standards has fixed the legal equivalent of the meter as 39.37 inches.

5. The metric system is a decimal system. Units larger than the primary units are 10 times the primary units, 100 times the primary units, and 1000 times the primary units; units smaller than the primary units are $\frac{1}{10}$ the primary units, $\frac{1}{100}$ the primary units, and $\frac{1}{1000}$ the primary units. Units of any given denomination are therefore reduced to units of a larger denomination by dividing by 10, by 100, and by 1000; and units are reduced to units of a smaller denomination by multiplying by 10, by 100, and by 1000. Quantities are not generally expressed in terms of two or more units, but in some single unit, parts of the unit being expressed as a decimal of the unit, as 6.35 meters.

6. Names of units larger than the primary units are formed by prefixing to the names of the primary units prefixes derived from the Greek words meaning ten, one hundred, and one thousand, etc.; and names of units smaller than the primary units are formed by prefixing to the names of the primary units prefixes derived from the Latin words meaning ten, one hundred, and one thousand, as follows:

* Subsequent calculations have shown that the meter is not exactly a ten-millionth part of the distance from the equator to the North Pole.

Greek Prefixes

deka, meaning 10; dekameter, meaning 10 meters.
hekto, meaning 100; hektometer, meaning 100 meters.
kilo, meaning 1000; kilometer, meaning 1000 meters.
myria, meaning 10,000; myriameter, meaning 10,000 meters.

The prefixes *deka* and *hekto* are sometimes written *deca* and *hecto*.

Latin Prefixes

deci, meaning 10; decimeter, meaning .1 meter.
centi, meaning 100; centimeter, meaning .01 meter.
milli, meaning 1000; millimeter, meaning .001 meter.

Very small linear measurements are expressed in *mikrons*. *Mikron* is a Greek word meaning *small*.

375. Measures of Length.

In the following exercises use a meter stick on which the centimeters and millimeters are marked off. Practice drawing these units until you can estimate their lengths quite accurately. Test all estimates by actual measurements.

1. Draw on the blackboard a line 1 meter long; 2 meters long; 3 meters long.

2. Fix two points on the floor 1 meter apart; 2 meters apart; 3 meters apart; 4 meters apart.

3. Estimate the length, width, and height of your schoolroom in meters.

4. Measure the length of a blackboard in meters. Express fractional parts as a decimal of a meter, thus: if the blackboard is 4 meters and 12 centimeters long, its length may be stated as 4.12 meters.

5. Estimate the length and width of the school yard in meters.

6. Draw a line 1 decimeter in length. Name some object in the schoolroom that is one decimeter in length, width, or thickness.

7. Draw a line 1 centimeter in length, 2 centimeters in length, 3 centimeters in length.

8. Measure the length and width of this book in centimeters. Express fractional parts as a decimal of a centimeter.

9. Measure the thickness of this book in millimeters. How many millimeters make a centimeter? a decimeter? a meter?

10. A kilometer is 1000 meters. It is equivalent to about $\frac{5}{8}$ of a mile. Select some place that is about 1 kilometer from the schoolhouse.

11. Using rulers on which the units are marked off, compare the millimeter with $\frac{1}{16}$ of an inch.

12. Which is the longer, a centimeter or an inch?

376. Reduction of Linear Units.

1. A meter is how many decimeters? how many centimeters? how many millimeters?

2. 67 centimeters may be expressed as a decimal of a meter, thus: .67 meter. Express as meters: 34 centimeters, 15 centimeters, 76 centimeters.

3. A decimeter is what part of a meter? 4 decimeters may be expressed as a decimal of a meter, thus: .4 meter. Express as meters: 3 meters and 4 decimeters; 7 meters and 32 centimeters; 9 meters, 2 decimeters, and 4 centimeters.

4. Write a millimeter as a decimal of a meter. Write 8 millimeters as a decimal of a meter. Write 3 centimeters and 8 millimeters as a decimal of a meter.

5. Write 2 kilometers as meters. Write 2 kilometers and 430 meters as meters. Write as meters: 24.5 kilometers; 4.25 kilometers.

6. Reduce to meters: 304 centimeters; 2.467 kilometers; 245.376 kilometers; 30 centimeters.

377. Table of Measures of Length.

The following is the complete table of linear measure. The units most commonly used are the millimeter, centimeter, meter, and kilometer.

1000 mikrons (μ)	= 1 **millimeter** (mm.)
10 mm.	= 1 **centimeter** (cm.)
10 cm.	= 1 decimeter (dm.)
10 dm.	= 1 **meter** (m.)
10 m.	= 1 dekameter (Dm.)
10 Dm.	= 1 hektometer (Hm.)
10 Hm.	= 1 **kilometer** (Km.)
10 Km.	= 1 myriameter

Abbreviations of the names of the units that are multiples of the primary unit are written with capital letters to distinguish them from the abbreviations of the names of the units that are parts of the primary unit.

378. Measures of Surface.

1. Draw on the blackboard a square whose side is 1 meter in length. This is called a square meter.

2. Divide a square meter into square decimeters. How many square decimeters are there in a square meter?

3. Divide a square decimeter into square centimeters. How many square centimeters are there in a square decimeter?

4. How many square millimeters are there in a square centimeter?

5. How many square centimeters are there in a square meter?

6. In what square unit should you express the area of the surface of the cover of this book? of the floor of your schoolroom?

7. Draw on the school grounds a square whose side is 10 meters. This is called an **are**. It is the primary unit of land measure.

The **are** is equivalent to 119.6 square yards.

8. A square whose side is 100 meters is called a **hektare.**

9. A square whose side is 1 kilometer is called a **square kilometer.**

The area of gardens, etc. is usually given in ares; of fields, etc. in hektares; and of countries, etc. in square kilometers.

10. Estimate the number of square meters in the surface of the floor of your schoolroom. Test your estimate.

11. Estimate the number of ares in the school yard. Test your estimate.

12. How long is the side of a hektare? of a square kilometer?

The hektare is nearly $2\frac{1}{2}$ acres.

379. Table of Measures of Surface.

100 square millimeters (qmm.)	= 1 square centimeter (qcm.)
100 qcm.	= 1 square decimeter (qdm.)
100 qdm.	= 1 **square meter** (qm.)
100 qm.	= 1 square dekameter (qDm.)
100 qDm.	= 1 square hektometer (qHm.)
100 qHm.	= 1 **square kilometer** (qKm.)

380. Table of Land Measure.

100 centares (ca.)	= 1 are (a.)
100 a.	= 1 **hektare** (Ha.)

381. Measures of Volume.

1. From a piece of cardboard construct a cube whose edges are each 1 decimeter. This is called a cubic decimeter.

2. How many cubic decimeters are there in 1 cubic meter?

3. From a piece of cardboard construct a cubic centimeter. Estimate the capacity of a crayon box in cubic centimeters.

4. Estimate the number of cubic meters of air in your schoolroom. Using a meter stick, make an approximate test of your estimate.

5. The primary unit of volume is the **cubic meter.**

The cubic meter is equivalent to 1.308 cubic yards.

6. The primary unit of **wood measure** is the **stere,** which is a cubic meter.

382. Table of Measures of Volume.

1000 cubic millimeters (cu. mm.)	= 1 **cubic centimeter** (cu. cm.)
1000 cu. cm.	= 1 **cubic decimeter** (cu. dm.)
1000 cu. dm.	= 1 **cubic meter** (cu. m.)

Units higher than the cubic meter are seldom used.

383. Measures of Capacity.

1. The primary unit of capacity for both liquid and dry measure is the **liter,** which contains 1 cubic decimeter. Using the measures, compare the capacity of a liter and a quart.

The **liter** is equivalent to 1.0567 liquid quarts or .908 dry quart.

2. How many cubic centimeters are equivalent to 1 liter?

3. 100 liters are 1 **hektoliter.** The liter is used to measure comparatively small quantities; the hektoliter is used to measure grain, produce, etc., in large quantities.

The **hektoliter** is equivalent to 2.8377 bushels.

4. Mention some things that are bought or sold by the quart, dry measure; by the quart or gallon, liquid measure. Where the metric system is used, these are bought and sold by the liter, or by the hektoliter if the quantities are large.

5. How many liters of water will a tank hold whose inside dimensions are 3.45 m. by 80 cm. by 60 cm.?

$\frac{345 \times 80 \times 60}{1000}$, number of liters in the tank. Explain.

6. Find the capacity in liters of a cylindrical tank whose diameter is 2.85 m. and whose altitude is 3.68 m.

384. Table cf Measures of Capacity.

10 milliliters	= 1 centiliter (cl.)
10 cl.	= 1 deciliter (dl.)
10 dl.	= 1 **liter** (l.)
10 l.	= 1 dekaliter (Dl.)
10 Dl.	= 1 **hektoliter** (Hl.)

385. Measures of Weight.

1. The primary unit of weight is the **gram**, which is the weight of 1 cu. cm. of pure water at its greatest density.

2. Heft a gram weight. How many grams does a liter of pure water at its greatest density weigh?

3. The weight of 1000 cubic centimeters of water (a liter) is called a **kilogram**, or a **kilo**. Heft a kilogram weight.

A **kilogram** is equivalent to 2.2046 pounds avoirdupois. How many grams are equivalent to an ounce avoirdupois?

4. The gram is used in weighing precious metals, medicines, etc.; the kilogram in weighing meat, groceries, etc. Express your weight in kilograms, calling 2.2 pounds 1 kilogram.

5. 100 kilograms are 1 **metric quintal**, and 1000 kilograms 1 **metric ton**.

A **metric ton** is equivalent to 2205 pounds or 1.1023 tons.

6. Express as grams: 2.125 Kg.; 3.4 Kg. Express as kilograms: 245 g.; 28 g.; 362 M. T.; 4.25 M. T.; 4 Kg. 72 g.

386. Table of Measures of Weight.

10 milligrams (mg.)	= 1 centigram (cg.)
10 cg.	= 1 decigram (dg.)
10 dg.	= 1 **gram** (g.)
10 g.	= 1 dekagram (Dg.)
10 Dg.	= 1 hektogram (Hg.)
10 Hg.	= 1 **kilogram** (Kg.)
10 Kg.	= 1 myriagram (Mg.)
100 Kg.	= 1 **metric quintal** (Q.)
1000 Kg.	= 1 **metric ton** (M. T.)

387. Equivalents of Metric Units.

The following equivalents are given for comparison and for reference:

Metric to Common		Common to Metric	
1 m.	= 39.37 in., or 1.0936 yd.	1 yd.	= .9144 m.
1 Km.	= .62137 mi.	1 mi.	= 1.60935 Km.
1 sq. m.	= 1.196 sq. yd.	1 sq. yd.	= .836 sq. m.
1 Ha.	= 2.471 A.	1 A.	= .4047 Ha.
1 cu. m.	= 1.308 cu. yd.	1 cu. yd.	= .765 cu. m.
1 l.	= .908 qt. (dry)	1 qt. (dry)	= 1.1012 l.
1 l.	= 1.0567 qt. (liquid)	1 qt. (liquid)	= .94636 l.
1 Hl.	= 2.8377 bu.	1 bu.	= .35239 Hl.
1 g.	= 15.43 gr. (troy)	1 oz. (troy)	= 31.10348 g.
1 Kg.	= 32.1507 oz. (troy)	1 lb. (av.)	= .45359 Kg.
1 Kg.	= 2.2046 lb. (av.)		
1 M. T.	= 1.1023 T.	1 T.	= .90718 M. T.

TABLES OF DENOMINATE MEASURES

(For Reference)

388. Measures of Time.

60 seconds	= 1 minute	365 days	= 1 year
60 minutes	= 1 hour	366 days	= 1 leap year
24 hours	= 1 day	10 years	= 1 decade
7 days	= 1 week	100 years	= 1 century

1. The day is the primary unit of time measure. It is the time taken by the earth to make one rotation on its axis. Is it a natural or an artificial unit? The earth revolves around the sun in 365 days 5 hours 48 minutes 46 seconds (nearly $365\frac{1}{4}$ days). This period is the **solar** (sun) **year**.

2. As the exact period taken for the earth to make a revolution around the sun is a little less than $365\frac{1}{4}$ days, an extra day (Feb. 29) is added to the common year once in four years (leap year), except in centennial years not exactly divisible by 400.

3. *Centennial years divisible by 400 and other years divisible by 4 are* **leap years.**

Was 1700 a leap year? Will 2000 be a leap year?

4. More than four thousand years ago the Chaldeans, a people living in the valley of the Euphrates, calculated the length of the year to be 360 days. They believed that the sun traveled around the earth in a circle in this period. They therefore divided the circular path of the sun into 360 equal parts, called degrees — one for the part traversed each day. Hence there are 360 degrees in a circle. They observed twelve clusters of stars (constellations) in the zone in the heavens (zodiac) in which the paths of the sun and planets lie, and the occurrence of twelve full moons in successive parts of the zodiac each year. They therefore divided the course of the sun into twelve equal parts, one for each constellation. Hence there are twelve months in a year. The exact length of the lunar month is 29.53059 days. The Chaldeans divided the day into twelve "double hours." The number 60 was used by them as a unit, and they therefore divided the hour and the degree into 60 minutes; and the minute into 60 seconds.

5. Seven days were made to constitute a unit of time measure (week), either in accordance with the Mosaic law or from the fact that seven planets were known to the ancients. The days of the week were originally named after seven heavenly bodies. The English names of the days of the week are derived from the Saxons, a Germanic people who invaded and conquered England in the fifth and sixth centuries. The Saxons borrowed the week from some eastern nation and substituted the names of their own divinities for those of the Grecian deities.

Names of the Days of the Week

Latin	Saxon	English
Dies Solis (Sun)	Sun's day	Sunday
Dies Lunae (Moon)	Moon's day	Monday
Dies Martis (Mars)	Tiw's day	Tuesday
Dies Mercurii (Mercury)	Woden's day	Wednesday
Dies Jovis (Jupiter)	Thor's day	Thursday
Dies Veneris (Venus)	Friga's day	Friday
Dies Saturni (Saturn)	Seterne's day	Saturday

6. Until the time of Julius Cæsar (46 B.C.) the calendar was in almost constant state of confusion, owing to the fact that the number of days allowed for a year was more or less than the actual number of days taken for one revolution of the earth in its orbit. As a result of this error, in the time of Julius Cæsar the winter months had been carried back into autumn, and the autumn months into summer. To correct the error, Cæsar decreed that 90 days should be added to the year to restore the time of the vernal equinox, and that the year should consist of $365\frac{1}{4}$ days. He ordered that the common year should thereafter consist of 365 days and that every fourth year should consist of 366 days. The extra day was added to February, which at that time had 29 days. This arrangement is known as the **Julian Calendar,** or **Old Style.** The month of July was named after Julius Cæsar.

7. Augustus Cæsar ordered that the month following that which bore the name of Julius (July) should be named after himself; and in order that the month bearing his name should have as many days as the month bearing the name of Julius, he ordered that one day be taken from February and added to the month which should bear his name. Hence the eighth month is named August and consists of as many days as July.

8. The year established by the Julian Calendar ($365\frac{1}{4}$ days) was .00778 of a day longer than the actual time taken for one revolution of the earth in its orbit. This error had amounted to 10 days by 1582, when Pope Gregory XIII undertook the correction of the calendar. To adjust the time of the vernal equinox, Pope Gregory ordered that ten days be skipped, from October 5th to the 15th, and that only centennial years that are exactly divisible by 400 and other years that are exactly divisible by 4 be made leap years. This arrangement is known as the **Gregorian Calendar,** or **New Style,** and is the one in common use. Russia still follows the Julian or Old Style. The error in the Gregorian Calendar will amount to one day in about 5000 years.

389. Measures of Length.

12 inches	= 1 foot
3 feet	= 1 yard
$16\frac{1}{2}$ feet ($5\frac{1}{2}$ yd.)	= 1 rod
320 rods	= 1 mile
1 mile = 1760 yards	= 5280 feet

1. The yard is the primary unit of length. All the other units of length are derived from it.

2. A **furlong** is $\frac{1}{8}$ mile. It is little used at the present time.

3. A **hand**, used in measuring the height of horses at the shoulder, is 4 inches.

4. A **fathom**, used in measuring the depth of the sea, is 6 feet.

5. A **knot, or nautical mile**, used in measuring distances at sea, is 6080.27 feet, or approximately 1.15 (about $1\frac{1}{6}$) miles. The speed of vessels is expressed in knots. A vessel that travels 18 knots an hour travels about 21 miles an hour (18 mi. plus $\frac{1}{6}$ of 18 mi.).

6. For the supposed origin of the inch, foot, fathom, etc., consult a dictionary or an encyclopedia.

390. Measures of Surface.

144 square inches	= 1 square foot
9 square feet	= 1 square yard
$30\frac{1}{4}$ square yards	= 1 square rod
160 square rods	= 1 acre
640 acres	= 1 square mile

1. A square **acre** is 208.71+ feet on a side.

2. A tract of land 1 mile square is called a **section**. A **township** is a tract of land 6 miles square and consists of 36 sections.

3. 100 square feet of flooring, roofing, or slating is called a **square**.

391. Measures of Volume.

1728 cubic inches	= 1 cubic foot
27 cubic feet	= 1 cubic yard

1. A pile of wood 8 feet long, 4 feet wide, and 4 feet high, or 128 cubic feet of wood, is called a **cord**. For the origin of the name, consult the dictionary. Stonework is sometimes measured by the cord.

2. In measuring stonework, a pile of stone $16\frac{1}{2}$ feet long, $1\frac{1}{2}$ feet wide, and 1 foot high, or $24\frac{3}{4}$ cubic feet of stone, is called a **perch**.

392. Surveyors' Measures of Length.

100 links (l.)	= 1 chain (ch.)
80 chains	= 1 mile

The chain in common use is called **Gunter's chain**. It is 4 rods, or 66 feet long. A link is .66 foot. Links are written as hundredths of a chain, thus: 30 chains 45 links is written 30.45 chains.

393. Surveyors' Measures of Surface.

10 square chains	= 1 acre
640 acres	= 1 square mile

Square chains are reduced to acres by moving the decimal point one place toward the left. Explain.

394. Avoirdupois Weight.

16 ounces	= 1 pound
100 pounds	= 1 hundredweight
2000 pounds	= 1 ton

1. The English ton, known in the United States as the **long ton**, is 2240 pounds. It is used in United States custom-houses and in weighing coal and mineral products at the mines and sometimes in retailing coal.

2. The smallest unit of weight is the **grain**. A pound avoirdupois is 7000 grains. Consult a dictionary for an explanation of the origin of the name.

3. Grains, vegetables, etc., are commonly sold by weight or measure. The weight of 1 bushel of the most common of these articles is as follows:

wheat = 60 lb.	oats = 32 lb.
beans = 60 lb.	barley = 48 lb.
peas = 60 lb.	sweet potatoes = 55 lb.
clover seed = 60 lb.	rye = 56 lb.
Irish potatoes = 60 lb.	shelled corn = 56 lb.

395. Troy Weight.

Troy weight is used in weighing precious metals.

24 grains	= 1 pennyweight
20 pennyweights	= 1 ounce
12 ounces	= 1 pound

A pound troy is 5760 grains. It is $\frac{5760}{7000}$ pound avoirdupois.

Precious stones and pearls are weighed by the **carat.** A carat equals $3\frac{1}{5}$ grains troy. The term *carat* is used also to express the proportion of gold in an alloy. It then signifies a twenty-fourth part. Thus, gold that is 18 carats fine is $\frac{18}{24}$, or $\frac{3}{4}$ pure gold.

396. Apothecaries' Weight.

Consult a dictionary for the meaning of the word apothecary. This system of weights is used to some extent in filling prescriptions. The pound, ounce, and grain are the same as in troy weight, but the ounce is subdivided differently.

20 grains (gr.)	= 1 scruple . . .	sc. or ℈
3 scruples	= 1 dram . . .	dr. or ʒ
8 drams	= 1 ounce . . .	oz. or ℥
12 ounces	= 1 pound . . .	lb. or ℔

397. Apothecaries' Liquid Measures.

60 drops (gtt.) or minims (♏)	= 1 fluid dram . . .	fʒ
8 fluid drams	= 1 fluid ounce . . .	f℥
16 fluid ounces	= 1 pint	O.
8 pints	= 1 gallon	Cong.

398. Liquid Measures.

4 gills = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon

Quart means one fourth. A quart is one fourth of a gallon. A gallon is 231 cubic inches. A gallon of water weighs about $8\frac{1}{3}$ pounds. A cubic foot of water (about $7\frac{1}{2}$ gal.) weighs about $62\frac{1}{2}$ pounds. In measuring the capacity of cisterns, etc., $31\frac{1}{2}$ gallons are called a **barrel**.

399. Dry Measures.

This system is but little used in some parts of the United States. Where it is not used, articles are usually sold by weight.

2 pints = 1 quart
8 quarts = 1 peck
4 pecks = 1 bushel

The dry quart contains 67.20 cubic inches, the fluid quart 57.75 cubic inches. A bushel contains 2150.42 cubic inches. The standard bushel in the United States is the **Winchester bushel.** It is the volume of a cylinder $18\frac{1}{2}$ inches in internal diameter and 8 inches in depth.

400. Measures of Angles and Arcs.

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
360 degrees = 4 right angles, or 1 circumference
90° of angle = 1 right angle ; 90° of arc = 1 quadrant

For an explanation of the origin of 360 degrees in a circumference, etc., see Measures of Time, p. 313.

401. Counting Table.

2 units = 1 pair
12 units = 1 dozen
20 units = 1 score
12 dozen = 1 gross
12 gross = 1 great gross

402. Measures of Value — United States Money.

10 mills = 1 cent	10 dimes = 1 dollar
10 cents = 1 dime	10 dollars = 1 eagle

The standard unit of value is the gold dollar. A gold dollar (no longer coined) contains 23.22 grains of pure gold and 2.58 grains of alloy. A silver dollar contains 371.25 grains of pure silver and 41.25 grains of alloy. The symbol for dollar is \$, which is taken from U.S.

The coins of the United States are bronze, 1¢; nickel, 5¢; silver, 10¢, 25¢, 50¢, \$1; and gold \$2½, \$5, \$10, and \$20. The *mill* is not coined. These are coined at mints located in Philadelphia, New Orleans, Denver, and San Francisco.

The paper currency is issued in the denominations of \$1, \$2, \$5, \$10, \$20, \$50, \$100, \$500, and \$1000. Paper currency consists of bank notes, silver certificates, and gold certificates. Examine some paper currency. The provision made for the redemption of each piece of paper currency is printed on each bill.

Paper currency issued by national banks is commonly called **bank notes.** Their payment is guaranteed by deposits of government bonds with the national government.

403. Values of Common Coins.

Country	Monetary Unit	Value in Terms of U.S. Gold Dollar	Rough Equivalent
Austria-Hungary	Crown	\$ .203	\$.20
British Possessions, N. A. (except Newfoundland)	Dollar (\$)	\$1.000	\$1.00
France	Franc (F.)	\$ .193	\$ .20
German Empire	Mark (M.)	\$ .238	\$ .25
Great Britain	Pound Sterling (£)	\$4.866½	\$5.00
Italy	Lira (L.)	\$ .193	\$ .20
Japan	Yen (Y.)	\$ .498	\$ .50
Mexico	Peso	\$ .498	\$ .50
Philippine Islands	Peso	\$ .500	\$ .50
Russia	Ruble	\$ .515	\$ .50

404. Table of Compound Interest.

Amount of $ 1, at various rates, interest compounded annually.

Years	1%	1½%	2%	2½%	3%	3½%
1	1.010000	1.015000	1.020000	1.025000	1.030000	1.035000
2	1.020100	1.030225	1.040400	1.050625	1.060900	1.071225
3	1.030301	1.045678	1.061208	1.076891	1.092727	1.108718
4	1.040604	1.061364	1.082432	1.103813	1.125509	1.147523
5	1.051010	1.077284	1.104081	1.131408	1.159274	1.187686
6	1.061520	1.093443	1.126162	1.159693	1.194052	1.229255
7	1.072135	1.109845	1.148686	1.188686	1.229874	1.272279
8	1.082857	1.126493	1.171659	1.218403	1.266770	1.316809
9	1.093685	1.143390	1.195093	1.248863	1.304773	1.362897
10	1.104622	1.160541	1.218994	1.280085	1.343916	1.410599
11	1.115668	1.177949	1.243374	1.312087	1.384234	1.459970
12	1.126825	1.195618	1.268242	1.344889	1.425761	1.511069
13	1.138093	1.213552	1.293607	1.378511	1.468534	1.563956
14	1.149474	1.231756	1.319479	1.412974	1.512590	1.618695
15	1.160969	1.250232	1.345868	1.448298	1.557967	1.675349
16	1.172579	1.268986	1.372786	1.484506	1.604706	1.733986
17	1.184304	1.288020	1.400241	1.521618	1.652848	1.794676
18	1.196148	1.307341	1.428246	1.559659	1.702433	1.857489
19	1.208109	1.326951	1.456811	1.598650	1.753506	1.922501
20	1.220190	1.346855	1.485947	1.638616	1.806111	1.989789

Years	4%	4½%	5%	6%	7%	8%
1	1.040000	1.045000	1.050000	1.060000	1.070000	1.080000
2	1.081600	1.092025	1.102500	1.123600	1.144900	1.166400
3	1.124864	1.141166	1.157625	1.191016	1.225043	1.259712
4	1.169859	1.192519	1.215506	1.262477	1.310796	1.360489
5	1.216653	1.246182	1.276282	1.338226	1.402552	1.469328
6	1.265319	1.302260	1.340096	1.418519	1.500730	1.586874
7	1.315932	1.360862	1.407100	1.503630	1.605782	1.713824
8	1.368569	1.422101	1.477455	1.593848	1.718186	1.850930
9	1.423312	1.486095	1.551328	1.689479	1.838459	1.999005
10	1.480244	1.552969	1.628895	1.790848	1.967151	2.158925
11	1.539454	1.622853	1.710339	1.898299	2.104852	2.331639
12	1.601032	1.695881	1.795856	2.012197	2.252192	2.518170
13	1.665074	1.772196	1.885649	2.132928	2.409845	2.719624
14	1.731676	1.851945	1.979932	2.260904	2.578534	2.937194
15	1.800944	1.935282	2.078928	2.396558	2.759032	3.172169
16	1.872981	2.022370	2.182875	2.540352	2.952164	3.425943
17	1.947901	2.113377	2.292018	2.692773	3.158815	3.700018
18	2.025817	2.208479	2.406619	2.854339	3.379932	3.996020
19	2.106849	2.307860	2.526950	3.025600	3.616528	4.315701
20	2.191123	2.411714	2.653298	3.207136	3.869684	4.660957

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